Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.7-Miscellaneous/135-4.7.1-c-trig-^m-d-trig-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [254]. This is test number [135].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (254)	0.00 (0)
Mathematica	99.21 (252)	0.79 (2)
Maple	88.98 (226)	11.02 (28)
Fricas	87.01 (221)	12.99 (33)
Mupad	66.54 (169)	33.46 (85)
Giac	63.39 (161)	36.61 (93)
Maxima	62.60 (159)	37.40 (95)
Sympy	26.77 (68)	73.23 (186)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

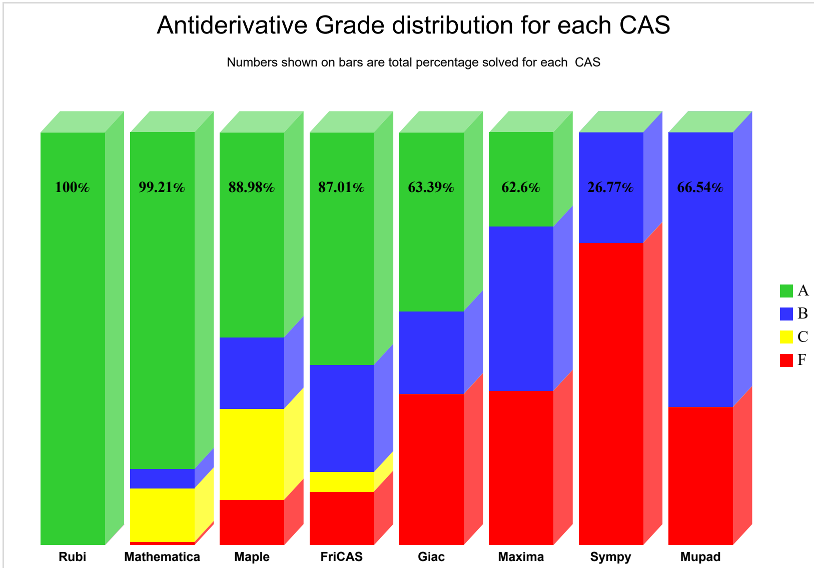
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

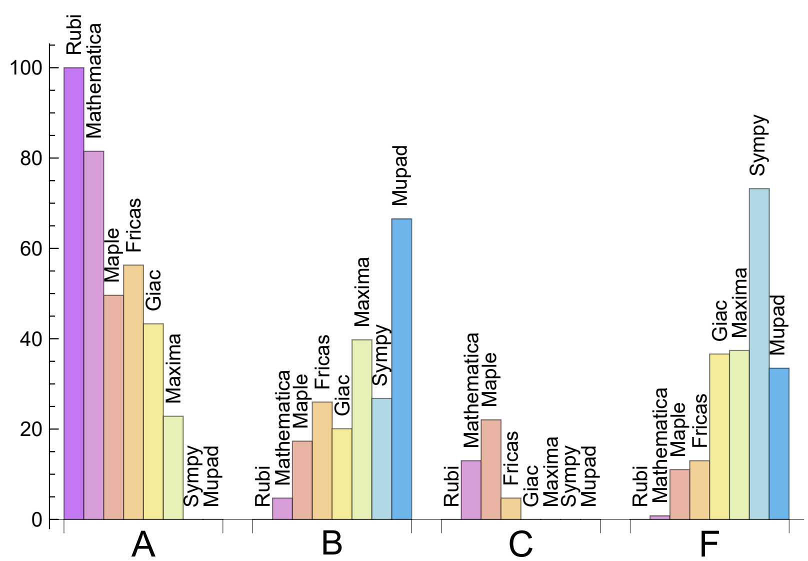
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	81.496	4.724	12.992	0.787
Fricas	56.299	25.984	4.724	12.992
Maple	49.606	17.323	22.047	11.024
Giac	43.307	20.079	0.000	36.614
Maxima	22.835	39.764	0.000	37.402
Mupad	0.000	66.535	0.000	33.465
Sympy	0.000	26.772	0.000	73.228

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Maple	28	67.86	32.14	0.00
Fricas	33	100.00	0.00	0.00
Mupad	85	0.00	100.00	0.00
Giac	93	87.10	10.75	2.15
Maxima	95	100.00	0.00	0.00
Sympy	186	17.74	80.11	2.15

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.25
Maxima	0.27
Rubi	0.34
Mathematica	0.66
Giac	4.67
Sympy	10.31
Mupad	16.21
Maple	19.78

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	73.96	1.01	54.00	1.00
Mathematica	92.55	1.40	61.00	0.95
Fricas	113.73	1.85	74.00	1.35
Mupad	123.10	1.96	49.00	1.00
Maxima	702.83	10.38	124.00	6.38
Giac	722.25	15.96	68.00	1.09
Sympy	2569.85	140.74	385.50	8.80
Maple	18208773.98	235613.24	87.00	1.54

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

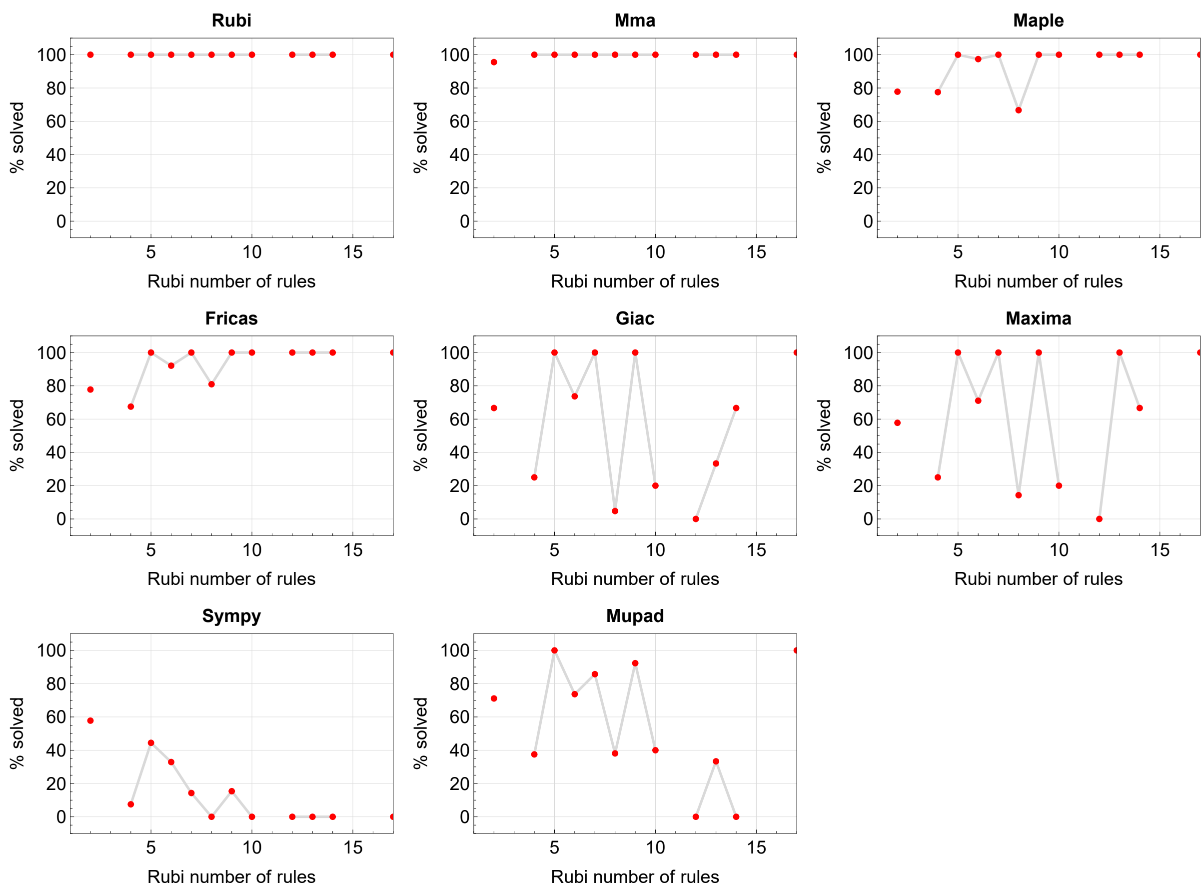


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

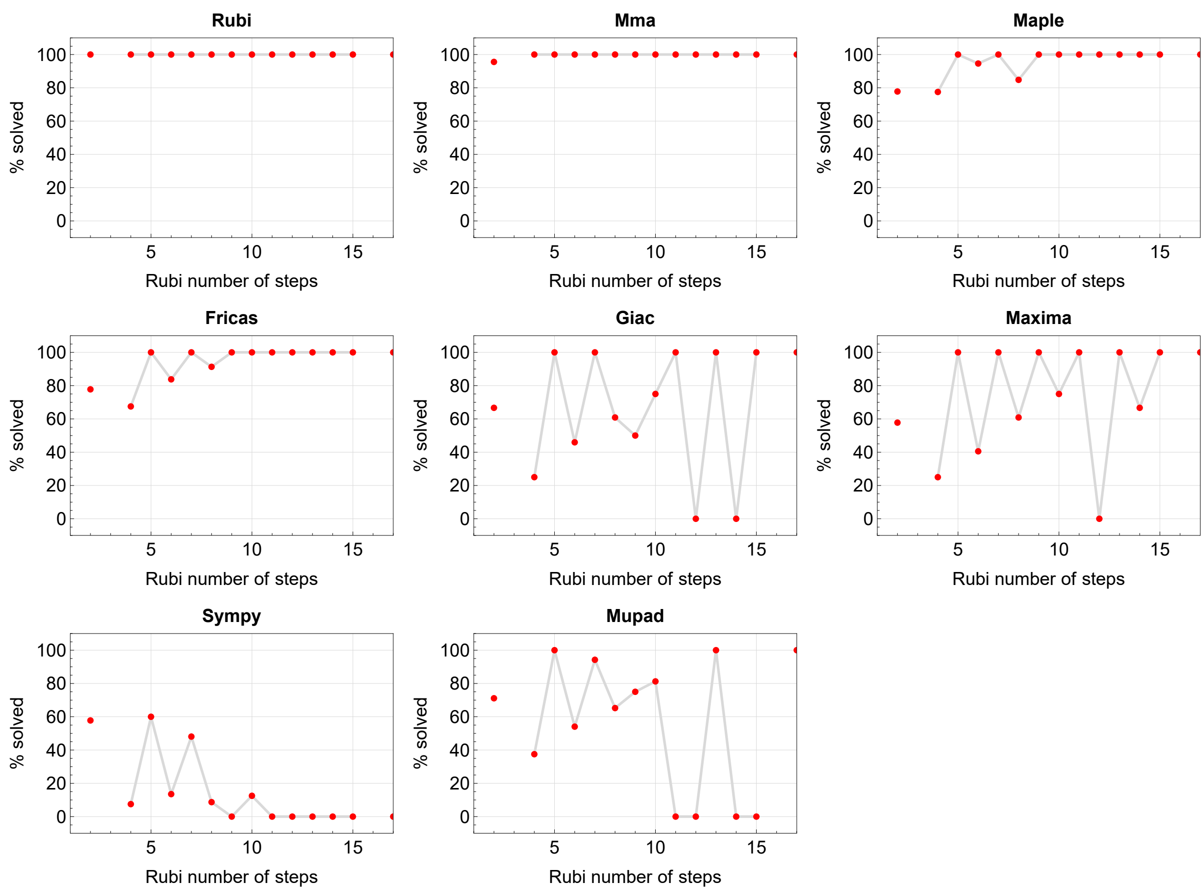


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

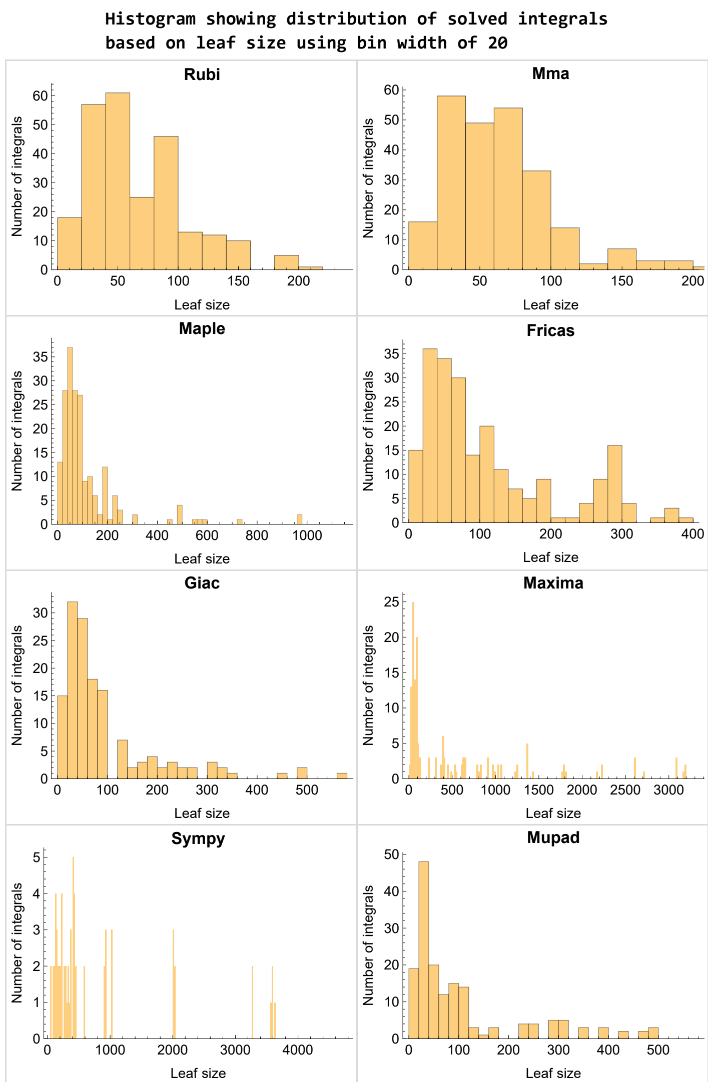


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

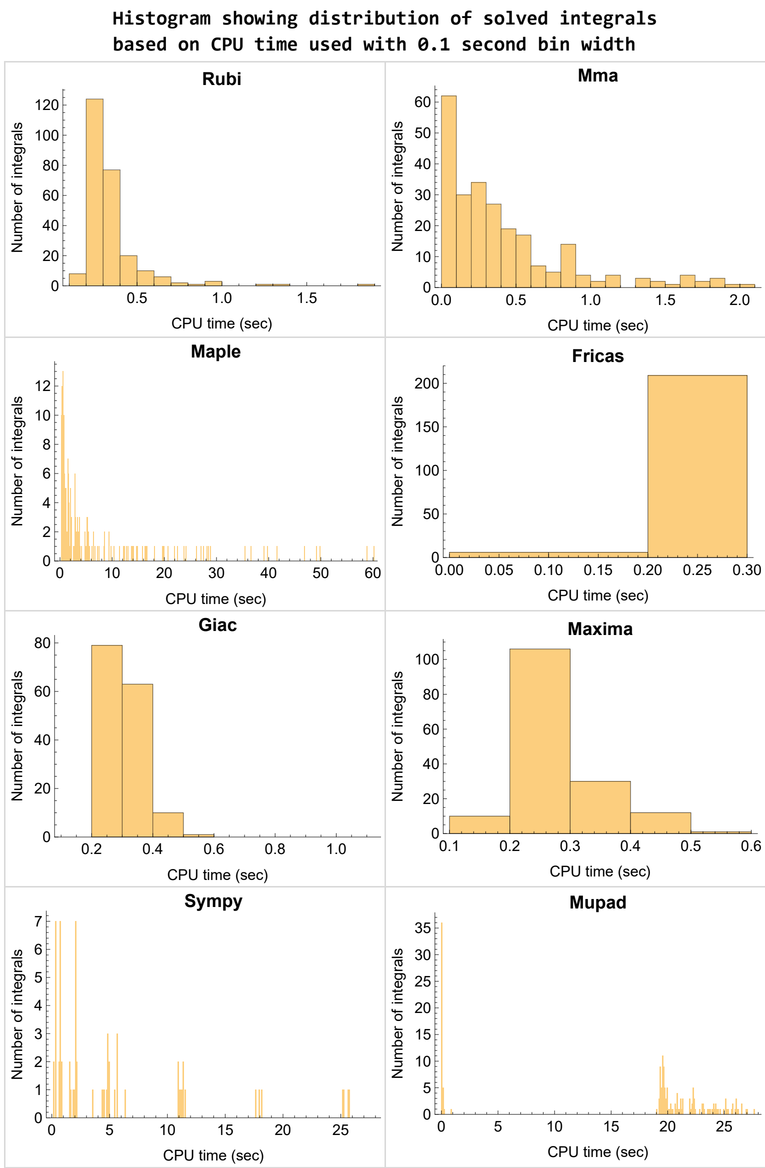


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

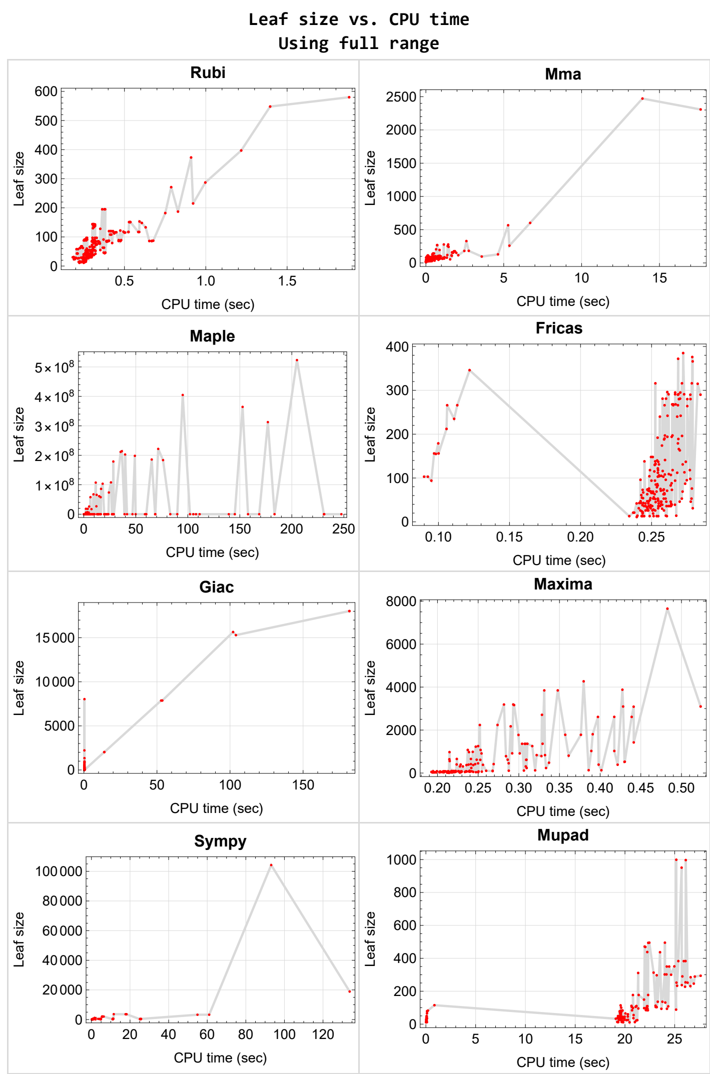


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{22, 55, 57, 150\}$

Mathematica $\{123, 125, 126, 128, 187, 189, 191, 209, 219, 223\}$

Maple $\{73, 74, 75, 76, 77, 82, 83, 84, 85, 86, 89, 90, 91, 92, 98, 99, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 177, 178, 179, 180\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

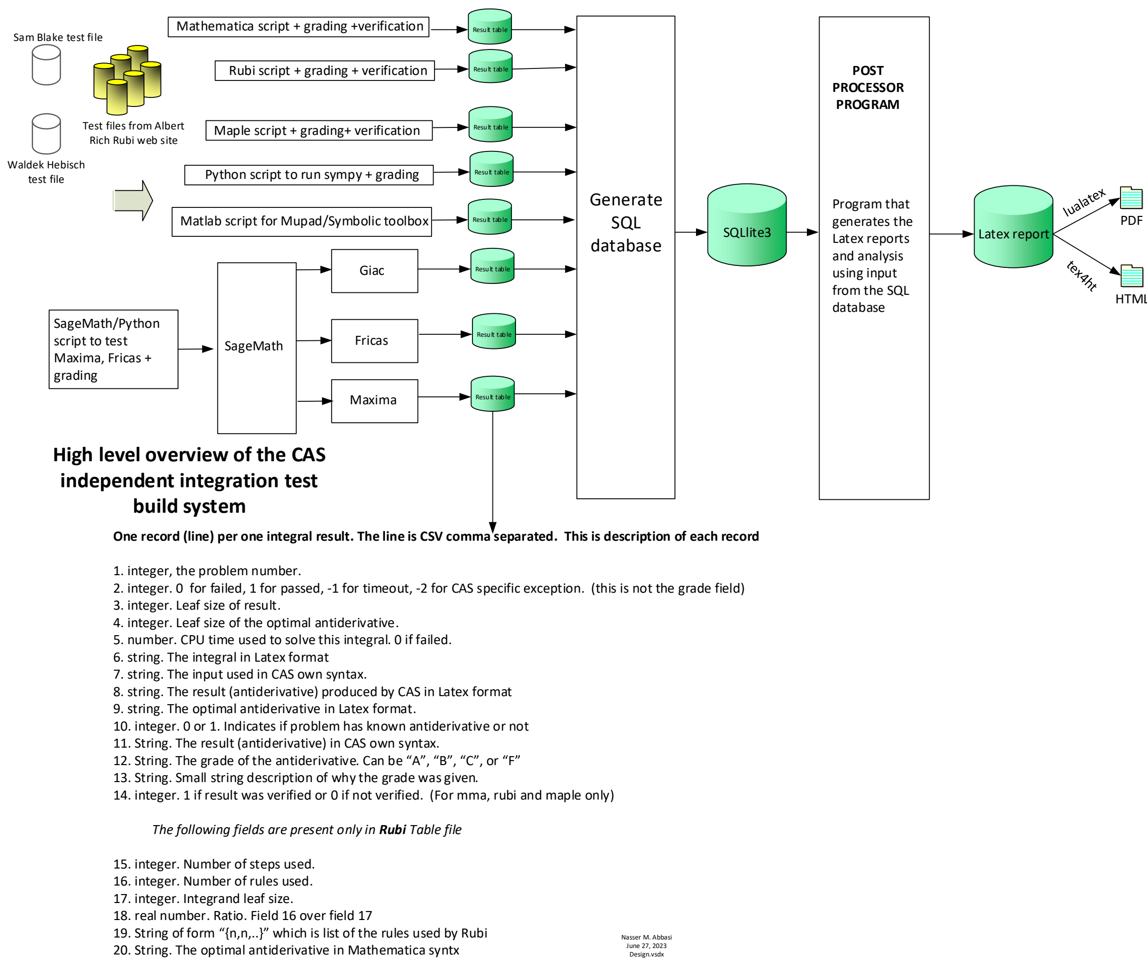
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 127, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 229, 230, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 252, 253, 254 }

B grade { 11, 40, 42, 44, 52, 61, 72, 136, 138, 140, 158, 160 }

C grade { 10, 12, 32, 41, 43, 69, 71, 108, 110, 123, 125, 126, 128, 137, 139, 159, 173, 175, 187, 189, 196, 214, 227, 228, 231, 232, 233, 234, 242, 248, 249, 250, 251 }

F normal fail { 201, 205 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 87, 105, 106, 107, 108, 110, 112, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 152, 153, 154, 156, 157, 158, 159, 160, 175, 190, 192, 193, 194, 202, 203, 204, 206, 207, 208, 210, 211, 212, 215, 217, 220, 221, 222, 224, 225, 226, 229, 238, 239, 240, 243, 244, 245, 246 }

B grade { 1, 3, 17, 23, 27, 73, 74, 75, 76, 77, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 98, 99, 109, 111, 129, 131, 145, 151, 155, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180 }

C grade { 21, 54, 56, 58, 78, 79, 93, 94, 97, 100, 101, 102, 103, 104, 115, 116, 117, 118, 119, 120, 121, 122, 149, 166, 167, 168, 181, 182, 185, 186, 195, 196, 197, 198, 199, 200, 213, 214, 216, 218, 227, 228, 230, 231, 232, 233, 234, 235, 241, 242, 247, 248, 249, 250, 251, 252 }

F normal fail { 123, 124, 125, 126, 127, 128, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

F(-1) timedout fail { 80, 81, 95, 96, 113, 114, 169, 183, 184 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 72, 77, 78, 79, 80, 93, 94, 95, 96, 101, 102, 103, 104, 115, 116, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 149, 151, 152, 153, 154, 155, 156, 157, 165, 166, 167, 168, 182, 183, 184, 190, 192, 193, 194, 195, 196, 197, 199, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 227, 229, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

B grade { 8, 9, 20, 22, 27, 30, 41, 42, 43, 53, 55, 57, 69, 70, 71, 73, 74, 75, 76, 89, 90, 91, 92, 97, 98, 99, 100, 117, 118, 119, 136, 137, 138, 139, 148, 150, 158, 159, 160, 161, 162, 163, 164, 177, 178, 179, 180, 181, 185, 186, 198, 200, 214, 228, 230, 231, 232, 233, 234, 235, 247, 248, 249, 250, 251, 252 }

C grade { 86, 87, 88, 109, 110, 111, 112, 113, 114, 174, 175, 176 }

F normal fail { 81, 82, 83, 84, 85, 105, 106, 107, 108, 123, 124, 125, 126, 127, 128, 169, 170, 171, 172, 173, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 66, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 151, 152, 153, 154, 190, 194, 212, 240 }

B grade { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 52, 53, 54, 55, 56, 57, 58, 65, 67, 68, 69, 70, 71, 72, 136, 137, 138, 139, 140, 146, 147, 148, 149, 150, 155, 156, 157, 158, 159, 160, 192, 193, 195, 196, 197, 198, 199, 200, 202, 203, 204, 206, 207, 208, 210, 211, 213, 214, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252 }

C grade { }

F normal fail { 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 34, 36, 38, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 62, 64, 66, 68, 69, 71, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 190, 192, 193, 194, 202, 203, 204, 206, 207, 208, 210, 211, 212, 220, 221, 222, 224, 225, 226, 238, 239, 240, 244, 245, 246 }

B grade { 8, 9, 11, 29, 31, 33, 35, 37, 39, 42, 44, 59, 61, 63, 65, 67, 70, 72, 77, 78, 79, 93, 136, 165, 166, 167, 181, 195, 196, 197, 198, 199, 200, 213, 214, 215, 216, 217, 218, 227, 228, 229, 233, 234, 235, 241, 242, 243, 250, 251, 252 }

C grade { }

F normal fail { 73, 74, 75, 76, 81, 82, 83, 84, 86, 87, 88, 89, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 185, 186, 187, 188, 189, 191, 201, 205, 209, 219, 223, 230, 231, 232, 236, 237, 247, 248, 249, 253, 254 }

F(-1) timedout fail { 80, 85, 91, 94, 95, 96, 168, 182, 183, 184 }

F(-2) exception fail { 90, 178 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 93, 94, 95, 96, 101, 102, 103, 104, 119, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 165, 166, 167, 168, 181, 182, 183, 184, 190, 192, 193, 194, 195, 196, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 214, 220, 221, 222, 224, 225, 226, 227, 228, 231, 232, 233, 234, 238, 239, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251 }

C grade { }

F normal fail { }

F(-1) timedout fail { 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 98, 99, 100, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 209, 215, 216, 217, 218, 219, 223, 229, 230, 235, 236, 237, 243, 247, 252, 253, 254 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, 24, 25, 26, 27, 39, 40, 52, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 151, 152, 153, 154, 155, 190, 192, 193, 194, 195, 196, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 220, 221, 222, 224, 225, 226, 227, 228, 238, 239, 240, 241, 244, 245, 246 }

C grade { }

F normal fail { 41, 42, 43, 44, 53, 54, 55, 56, 57, 58, 69, 70, 71, 72, 126, 127, 128, 230, 231, 232, 233, 234, 235, 236, 237, 247, 248, 249, 250, 251, 252, 253, 254 }

F(-1) timedout fail { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 136, 137, 138, 139, 140, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 209, 216, 217, 218, 219, 223, 229 }

F(-2) exception fail { 214, 215, 242, 243 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	47	111	91	83	269	46	45
N.S.	1	0.89	0.77	1.82	1.49	1.36	4.41	0.75	0.74
time (sec)	N/A	0.265	0.526	9.371	0.220	0.254	25.134	0.277	0.082

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	47	97	80	46	235	80	46
N.S.	1	0.89	0.77	1.59	1.31	0.75	3.85	1.31	0.75
time (sec)	N/A	0.279	0.455	5.260	0.229	0.257	10.963	0.279	0.074

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	83	69	63	197	36	36
N.S.	1	0.91	0.80	1.80	1.50	1.37	4.28	0.78	0.78
time (sec)	N/A	0.272	0.244	2.862	0.203	0.258	4.740	0.272	0.082

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	69	58	36	163	58	36
N.S.	1	0.91	0.80	1.50	1.26	0.78	3.54	1.26	0.78
time (sec)	N/A	0.274	0.141	1.549	0.223	0.255	1.993	0.266	19.386

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	27	55	47	41	126	26	26
N.S.	1	0.97	0.87	1.77	1.52	1.32	4.06	0.84	0.84
time (sec)	N/A	0.265	0.095	0.904	0.204	0.264	0.777	0.376	0.053

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	27	41	36	26	92	36	26
N.S.	1	0.97	0.87	1.32	1.16	0.84	2.97	1.16	0.84
time (sec)	N/A	0.265	0.062	0.542	0.197	0.248	0.362	0.302	0.058

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	15	26	26	21	51	13	44
N.S.	1	1.00	0.50	0.87	0.87	0.70	1.70	0.43	1.47
time (sec)	N/A	0.187	0.031	0.377	0.196	0.237	0.168	0.259	19.636

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	20	115	28	0	28	12
N.S.	1	1.00	1.00	1.43	8.21	2.00	0.00	2.00	0.86
time (sec)	N/A	0.219	0.005	0.385	0.306	0.251	0.000	0.262	19.326

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	23	50	31	236	52	0	52	26
N.S.	1	0.82	1.79	1.11	8.43	1.86	0.00	1.86	0.93
time (sec)	N/A	0.252	0.229	0.773	0.242	0.253	0.000	0.300	0.069

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	29	51	808	85	0	63	48
N.S.	1	1.10	0.59	1.04	16.49	1.73	0.00	1.29	0.98
time (sec)	N/A	0.271	0.021	1.453	0.361	0.253	0.000	0.330	19.308

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	205	71	2174	112	0	160	60
N.S.	1	1.00	3.11	1.08	32.94	1.70	0.00	2.42	0.91
time (sec)	N/A	0.294	0.656	2.234	0.290	0.254	0.000	0.276	20.293

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	97	31	87	3088	140	0	85	79
N.S.	1	1.09	0.35	0.98	34.70	1.57	0.00	0.96	0.89
time (sec)	N/A	0.307	0.035	3.763	0.441	0.254	0.000	0.273	0.130

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	42	68	74	72	46	593	36	46
N.S.	1	0.95	1.55	1.68	1.64	1.05	13.48	0.82	1.05
time (sec)	N/A	0.288	0.376	3.017	0.235	0.257	11.201	0.266	19.376

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	86	62	66	65	67	434	68	110
N.S.	1	1.13	0.82	0.87	0.86	0.88	5.71	0.89	1.45
time (sec)	N/A	0.393	0.240	1.702	0.211	0.251	4.806	0.291	21.153

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	30	48	52	50	36	359	26	33
N.S.	1	1.03	1.66	1.79	1.72	1.24	12.38	0.90	1.14
time (sec)	N/A	0.281	0.126	0.934	0.198	0.256	2.009	0.377	19.078

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	40	42	41	46	231	44	43
N.S.	1	1.10	0.82	0.86	0.84	0.94	4.71	0.90	0.88
time (sec)	N/A	0.307	0.066	0.525	0.219	0.256	0.828	0.292	19.373

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	26	24	131	13	13
N.S.	1	1.00	1.00	2.00	1.73	1.60	8.73	0.87	0.87
time (sec)	N/A	0.240	0.006	0.327	0.198	0.243	0.366	0.259	19.260

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	55	14	0	18	12
N.S.	1	1.00	1.00	0.93	3.93	1.00	0.00	1.29	0.86
time (sec)	N/A	0.224	0.016	0.309	0.222	0.267	0.000	0.259	0.108

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	53	19	0	11	11
N.S.	1	1.00	1.00	0.92	4.08	1.46	0.00	0.85	0.85
time (sec)	N/A	0.239	0.007	1.115	0.207	0.244	0.000	0.304	20.462

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	27	43	24	641	56	0	41	35
N.S.	1	0.90	1.43	0.80	21.37	1.87	0.00	1.37	1.17
time (sec)	N/A	0.264	0.026	0.990	0.215	0.252	0.000	0.337	0.108

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	36	48	46	308	43	0	32	33
N.S.	1	0.86	1.14	1.10	7.33	1.02	0.00	0.76	0.79
time (sec)	N/A	0.281	0.336	4.834	0.228	0.244	0.000	0.285	19.515

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	49	56	62	3164	112	0	74	74
N.S.	1	0.82	0.93	1.03	52.73	1.87	0.00	1.23	1.23
time (sec)	N/A	0.290	0.254	2.913	0.294	0.268	0.000	0.286	19.636

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	97	80	73	447	36	36
N.S.	1	0.91	0.80	2.11	1.74	1.59	9.72	0.78	0.78
time (sec)	N/A	0.282	0.360	9.740	0.242	0.244	25.241	0.283	0.088

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	47	71	69	46	366	69	46
N.S.	1	0.89	0.77	1.16	1.13	0.75	6.00	1.13	0.75
time (sec)	N/A	0.282	0.207	5.657	0.230	0.260	11.110	0.299	19.421

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	27	55	47	53	284	26	26
N.S.	1	0.97	0.87	1.77	1.52	1.71	9.16	0.84	0.84
time (sec)	N/A	0.271	0.156	2.893	0.227	0.259	4.827	0.402	0.063

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	55	47	36	202	47	36
N.S.	1	0.91	0.80	1.20	1.02	0.78	4.39	1.02	0.78
time (sec)	N/A	0.275	0.109	1.507	0.216	0.245	2.066	0.316	19.234

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	34	31	117	13	13
N.S.	1	1.00	1.00	2.73	2.27	2.07	7.80	0.87	0.87
time (sec)	N/A	0.234	0.009	0.762	0.217	0.241	0.788	0.262	19.228

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	23	27	29	124	36	0	36	23
N.S.	1	0.82	0.96	1.04	4.43	1.29	0.00	1.29	0.82
time (sec)	N/A	0.240	0.019	0.694	0.320	0.246	0.000	0.264	0.070

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	83	13	0	28	13
N.S.	1	1.00	1.00	1.08	6.38	1.00	0.00	2.15	1.00
time (sec)	N/A	0.237	0.014	1.718	0.208	0.245	0.000	0.306	0.032

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	38	37	480	61	0	48	36
N.S.	1	1.12	1.12	1.09	14.12	1.79	0.00	1.41	1.06
time (sec)	N/A	0.298	0.008	2.891	0.337	0.243	0.000	0.340	0.064

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	35	61	41	987	67	0	98	37
N.S.	1	0.81	1.42	0.95	22.95	1.56	0.00	2.28	0.86
time (sec)	N/A	0.267	0.140	6.727	0.243	0.244	0.000	0.295	0.070

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	85	29	69	1805	95	0	73	67
N.S.	1	1.21	0.41	0.99	25.79	1.36	0.00	1.04	0.96
time (sec)	N/A	0.290	0.042	13.984	0.391	0.275	0.000	0.278	19.512

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	119	47	91	46	0	270	46
N.S.	1	0.89	1.95	0.77	1.49	0.75	0.00	4.43	0.75
time (sec)	N/A	0.284	0.419	19.920	0.268	0.278	0.000	0.301	20.285

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	61	47	80	73	0	46	45
N.S.	1	0.89	1.00	0.77	1.31	1.20	0.00	0.75	0.74
time (sec)	N/A	0.288	0.044	8.563	0.235	0.247	0.000	0.275	19.971

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	89	37	69	36	0	204	36
N.S.	1	0.91	1.93	0.80	1.50	0.78	0.00	4.43	0.78
time (sec)	N/A	0.277	0.260	6.194	0.203	0.264	0.000	0.286	0.086

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	46	37	58	53	0	36	36
N.S.	1	0.91	1.00	0.80	1.26	1.15	0.00	0.78	0.78
time (sec)	N/A	0.275	0.037	2.293	0.218	0.251	0.000	0.270	0.071

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	59	27	47	26	0	138	26
N.S.	1	0.97	1.90	0.87	1.52	0.84	0.00	4.45	0.84
time (sec)	N/A	0.272	0.158	1.728	0.208	0.253	0.000	0.275	0.060

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	31	27	36	33	0	26	26
N.S.	1	0.97	1.00	0.87	1.16	1.06	0.00	0.84	0.84
time (sec)	N/A	0.269	0.024	0.626	0.217	0.247	0.000	0.430	0.061

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	13	104225	52	13
N.S.	1	1.00	1.00	0.93	1.53	0.87	6948.33	3.47	0.87
time (sec)	N/A	0.244	0.009	0.481	0.227	0.261	93.149	0.299	0.045

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	11	11	3636	11	11
N.S.	1	1.00	2.09	1.09	1.00	1.00	330.55	1.00	1.00
time (sec)	N/A	0.212	0.007	0.271	0.214	0.240	11.569	0.263	0.025

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	23	29	31	233	50	0	38	26
N.S.	1	0.82	1.04	1.11	8.32	1.79	0.00	1.36	0.93
time (sec)	N/A	0.248	0.021	0.402	0.333	0.249	0.000	0.256	19.501

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	143	53	974	96	0	137	49
N.S.	1	1.10	2.92	1.08	19.88	1.96	0.00	2.80	1.00
time (sec)	N/A	0.269	0.459	0.513	0.239	0.242	0.000	0.296	19.307

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	31	69	1780	130	0	72	61
N.S.	1	1.00	0.47	1.05	26.97	1.97	0.00	1.09	0.92
time (sec)	N/A	0.291	0.022	0.547	0.357	0.252	0.000	0.387	19.537

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	97	268	89	3846	148	0	206	78
N.S.	1	1.09	3.01	1.00	43.21	1.66	0.00	2.31	0.88
time (sec)	N/A	0.314	0.719	0.705	0.348	0.250	0.000	0.285	19.466

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	187	85	103	87	87	0	95	149
N.S.	1	1.21	0.55	0.66	0.56	0.56	0.00	0.61	0.96
time (sec)	N/A	0.827	0.933	46.860	0.226	0.274	0.000	0.311	21.914

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	42	59	47	72	36	0	46	35
N.S.	1	0.95	1.34	1.07	1.64	0.82	0.00	1.05	0.80
time (sec)	N/A	0.286	0.050	28.895	0.219	0.248	0.000	0.279	19.910

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	133	62	75	65	66	0	75	109
N.S.	1	1.20	0.56	0.68	0.59	0.59	0.00	0.68	0.98
time (sec)	N/A	0.603	0.194	15.880	0.254	0.251	0.000	0.290	22.221

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	30	48	27	50	26	0	36	25
N.S.	1	1.03	1.66	0.93	1.72	0.90	0.00	1.24	0.86
time (sec)	N/A	0.274	0.123	8.577	0.234	0.246	0.000	0.273	0.062

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	79	40	45	43	47	0	55	65
N.S.	1	1.32	0.67	0.75	0.72	0.78	0.00	0.92	1.08
time (sec)	N/A	0.398	0.094	3.555	0.208	0.248	0.000	0.278	20.042

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	26	13	0	13	13
N.S.	1	1.00	1.00	1.08	2.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.246	0.009	1.533	0.213	0.249	0.000	0.438	0.051

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	20	18	18	22	0	29	17
N.S.	1	1.29	0.95	0.86	0.86	1.05	0.00	1.38	0.81
time (sec)	N/A	0.241	0.024	0.772	0.212	0.244	0.000	0.306	19.762

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	14	25	13	81	14	18894	13	13
N.S.	1	1.17	2.08	1.08	6.75	1.17	1574.50	1.08	1.08
time (sec)	N/A	0.225	0.009	0.588	0.208	0.240	133.805	0.266	19.667

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	27	44	24	656	65	0	37	36
N.S.	1	0.90	1.47	0.80	21.87	2.17	0.00	1.23	1.20
time (sec)	N/A	0.268	0.041	0.457	0.241	0.259	0.000	0.255	19.508

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	34	48	46	308	54	0	35	37
N.S.	1	0.81	1.14	1.10	7.33	1.29	0.00	0.83	0.88
time (sec)	N/A	0.278	0.373	0.790	0.309	0.244	0.000	0.304	19.412

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	47	54	62	3188	138	0	74	82
N.S.	1	0.78	0.90	1.03	53.13	2.30	0.00	1.23	1.37
time (sec)	N/A	0.291	0.372	0.689	0.293	0.253	0.000	0.328	0.208

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	60	90	68	1227	86	0	56	55
N.S.	1	0.83	1.25	0.94	17.04	1.19	0.00	0.78	0.76
time (sec)	N/A	0.297	0.236	1.094	0.247	0.254	0.000	0.294	20.868

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	73	76	98	7650	194	0	94	114
N.S.	1	0.81	0.84	1.09	85.00	2.16	0.00	1.04	1.27
time (sec)	N/A	0.312	0.434	1.165	0.483	0.261	0.000	0.274	19.563

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	80	132	90	2710	118	0	76	83
N.S.	1	0.78	1.29	0.88	26.57	1.16	0.00	0.75	0.81
time (sec)	N/A	0.305	0.418	1.797	0.328	0.249	0.000	0.299	19.760

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	119	47	91	46	0	314	46
N.S.	1	0.89	1.95	0.77	1.49	0.75	0.00	5.15	0.75
time (sec)	N/A	0.296	1.378	168.671	0.222	0.270	0.000	0.374	19.629

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	66	76	57	91	83	0	56	55
N.S.	1	0.87	1.00	0.75	1.20	1.09	0.00	0.74	0.72
time (sec)	N/A	0.299	0.108	111.372	0.217	0.263	0.000	0.338	19.547

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	104	37	80	36	0	248	36
N.S.	1	0.91	2.26	0.80	1.74	0.78	0.00	5.39	0.78
time (sec)	N/A	0.278	0.785	68.809	0.214	0.263	0.000	0.337	19.809

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	61	47	69	63	0	46	45
N.S.	1	0.89	1.00	0.77	1.13	1.03	0.00	0.75	0.74
time (sec)	N/A	0.290	0.045	41.565	0.236	0.252	0.000	0.318	0.068

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	27	27	47	26	0	182	26
N.S.	1	0.97	0.87	0.87	1.52	0.84	0.00	5.87	0.84
time (sec)	N/A	0.281	0.143	23.729	0.229	0.242	0.000	0.309	20.307

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	37	47	43	0	36	36
N.S.	1	0.91	0.80	0.80	1.02	0.93	0.00	0.78	0.78
time (sec)	N/A	0.278	0.102	12.738	0.228	0.241	0.000	0.293	0.065

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	34	13	0	74	13
N.S.	1	1.00	1.00	0.93	2.27	0.87	0.00	4.93	0.87
time (sec)	N/A	0.253	0.008	5.188	0.214	0.251	0.000	0.297	0.047

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	26	28	22	23	21	0	22	24
N.S.	1	0.96	1.04	0.81	0.85	0.78	0.00	0.81	0.89
time (sec)	N/A	0.252	0.011	2.803	0.215	0.238	0.000	0.455	19.668

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	21	44	29	92	38	0	54	22
N.S.	1	0.88	1.83	1.21	3.83	1.58	0.00	2.25	0.92
time (sec)	N/A	0.253	0.049	2.026	0.220	0.244	0.000	0.316	21.141

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	84	13	0	13	13
N.S.	1	1.00	1.00	1.27	7.64	1.18	0.00	1.18	1.18
time (sec)	N/A	0.236	0.014	1.230	0.199	0.234	0.000	0.267	0.042

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	35	31	41	834	94	0	52	38
N.S.	1	0.81	0.72	0.95	19.40	2.19	0.00	1.21	0.88
time (sec)	N/A	0.271	0.025	0.560	0.332	0.252	0.000	0.264	19.907

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	85	129	71	2237	132	0	160	66
N.S.	1	1.21	1.84	1.01	31.96	1.89	0.00	2.29	0.94
time (sec)	N/A	0.296	4.630	0.684	0.252	0.253	0.000	0.319	0.114

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	78	31	87	3095	166	0	82	71
N.S.	1	0.96	0.38	1.07	38.21	2.05	0.00	1.01	0.88
time (sec)	N/A	0.298	0.043	0.866	0.428	0.269	0.000	0.350	19.498

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	128	278	107	4268	194	0	268	100
N.S.	1	1.14	2.48	0.96	38.11	1.73	0.00	2.39	0.89
time (sec)	N/A	0.336	1.154	1.192	0.380	0.264	0.000	0.307	19.624

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	136	151	98	183661410	0	291	0	0	0
N.S.	1	1.11	0.72	1350451.54	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.530	0.462	76.221	0.000	0.262	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	120	86	73677072	0	280	0	0	0
N.S.	1	1.09	0.78	669791.56	0.00	2.55	0.00	0.00	0.00
time (sec)	N/A	0.403	0.317	24.149	0.000	0.277	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	89	72	6800166	0	266	0	0	0
N.S.	1	1.06	0.86	80954.36	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.299	0.178	3.391	0.000	0.269	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	58	58	50	18281230	0	240	0	0	0
N.S.	1	1.00	0.86	315193.62	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.211	0.150	3.743	0.000	0.256	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	23	23	22	65687946	0	39	0	2029	34
N.S.	1	1.00	0.96	2855997.65	0.00	1.70	0.00	88.22	1.48
time (sec)	N/A	0.187	0.019	12.355	0.000	0.266	0.000	14.018	19.947

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	597	0	69	0	7875	108
N.S.	1	1.00	0.81	11.26	0.00	1.30	0.00	148.58	2.04
time (sec)	N/A	0.276	0.305	27.099	0.000	0.272	0.000	52.847	24.348

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	84	52	308	0	88	0	18022	131
N.S.	1	1.06	0.66	3.90	0.00	1.11	0.00	228.13	1.66
time (sec)	N/A	0.384	0.444	247.658	0.000	0.264	0.000	181.626	24.051

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	115	67	0	0	113	0	0	351
N.S.	1	1.10	0.64	0.00	0.00	1.08	0.00	0.00	3.34
time (sec)	N/A	0.507	0.500	0.000	0.000	0.273	0.000	0.000	24.963

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	108	96	0	0	0	0	0	0
N.S.	1	1.10	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	3.589	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	69	74	66	312731247	0	0	0	0	0
N.S.	1	1.07	0.96	4532336.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.421	176.980	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	69	74	76	185748620	0	0	0	0	0
N.S.	1	1.07	1.10	2692008.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.464	65.320	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	40	40	34	17183759	0	0	0	0	0
N.S.	1	1.00	0.85	429593.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	0.236	7.405	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	40	40	75	61245868	0	0	0	0	0
N.S.	1	1.00	1.88	1531146.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.383	14.818	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	45	45	41	108031867	0	156	0	0	0
N.S.	1	1.00	0.91	2400708.16	0.00	3.47	0.00	0.00	0.00
time (sec)	N/A	0.258	0.267	26.139	0.000	0.100	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	83	123	0	94	0	0	0
N.S.	1	1.00	1.73	2.56	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.266	0.412	28.151	0.000	0.095	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	78	66	227	0	212	0	0	0
N.S.	1	1.01	0.86	2.95	0.00	2.75	0.00	0.00	0.00
time (sec)	N/A	0.334	1.177	58.819	0.000	0.106	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	136	151	98	404845695	0	290	0	0	0
N.S.	1	1.11	0.72	2976806.58	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.528	0.536	95.121	0.000	0.269	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	120	86	57690707	0	281	0	0	0
N.S.	1	1.09	0.78	524460.97	0.00	2.55	0.00	0.00	0.00
time (sec)	N/A	0.413	0.365	16.218	0.000	0.260	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	89	74	202764172	0	268	0	0	0
N.S.	1	1.06	0.88	2413859.19	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.307	0.290	39.736	0.000	0.267	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	81	86	72	178923370	0	296	0	0	0
N.S.	1	1.06	0.89	2208930.49	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	0.393	0.265	28.412	0.000	0.261	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	727	0	48	0	15648	85
N.S.	1	1.00	0.96	25.96	0.00	1.71	0.00	558.86	3.04
time (sec)	N/A	0.195	0.262	83.161	0.000	0.255	0.000	102.081	22.201

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	35	4684	0	55	0	0	88
N.S.	1	1.00	0.64	85.16	0.00	1.00	0.00	0.00	1.60
time (sec)	N/A	0.286	0.422	183.395	0.000	0.243	0.000	0.000	25.137

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	86	55	0	0	79	0	0	300
N.S.	1	1.06	0.68	0.00	0.00	0.98	0.00	0.00	3.70
time (sec)	N/A	0.383	0.509	0.000	0.000	0.254	0.000	0.000	24.635

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	117	62	0	0	98	0	0	383
N.S.	1	1.09	0.58	0.00	0.00	0.92	0.00	0.00	3.58
time (sec)	N/A	0.513	0.878	0.000	0.000	0.277	0.000	0.000	25.910

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	153	98	973	0	290	0	0	0
N.S.	1	1.12	0.72	7.15	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.586	0.567	7.124	0.000	0.284	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	122	86	26835015	0	281	0	0	0
N.S.	1	1.11	0.78	243954.68	0.00	2.55	0.00	0.00	0.00
time (sec)	N/A	0.463	0.330	12.119	0.000	0.270	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	81	91	70	5540414	0	266	0	0	0
N.S.	1	1.12	0.86	68400.17	0.00	3.28	0.00	0.00	0.00
time (sec)	N/A	0.354	0.201	4.736	0.000	0.265	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	60	52	157	0	242	0	0	0
N.S.	1	1.13	0.98	2.96	0.00	4.57	0.00	0.00	0.00
time (sec)	N/A	0.269	0.130	2.643	0.000	0.270	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	308	0	39	0	0	24
N.S.	1	1.00	0.96	12.83	0.00	1.62	0.00	0.00	1.00
time (sec)	N/A	0.189	0.138	3.383	0.000	0.264	0.000	0.000	21.243

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	55	43	194	0	74	0	0	103
N.S.	1	1.04	0.81	3.66	0.00	1.40	0.00	0.00	1.94
time (sec)	N/A	0.338	0.223	5.353	0.000	0.259	0.000	0.000	23.192

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	86	52	481	0	103	0	0	136
N.S.	1	1.09	0.66	6.09	0.00	1.30	0.00	0.00	1.72
time (sec)	N/A	0.446	0.259	16.418	0.000	0.258	0.000	0.000	23.761

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	117	67	222	0	118	0	0	350
N.S.	1	1.11	0.64	2.11	0.00	1.12	0.00	0.00	3.33
time (sec)	N/A	0.575	0.328	89.889	0.000	0.255	0.000	0.000	24.442

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	116	66	204	0	0	0	0	0
N.S.	1	1.09	0.62	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	0.875	22.559	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	116	76	139	0	0	0	0	0
N.S.	1	1.09	0.72	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.507	20.796	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	82	34	137	0	0	0	0	0
N.S.	1	1.09	0.45	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.291	19.631	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	77	52	111	0	0	0	0	0
N.S.	1	1.10	0.74	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.424	4.717	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	37	176	0	155	0	0	0
N.S.	1	1.00	0.84	4.00	0.00	3.52	0.00	0.00	0.00
time (sec)	N/A	0.263	0.278	4.158	0.000	0.098	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	62	123	0	103	0	0	0
N.S.	1	1.00	1.29	2.56	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.259	0.361	6.490	0.000	0.093	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	78	64	227	0	266	0	0	0
N.S.	1	1.01	0.83	2.95	0.00	3.45	0.00	0.00	0.00
time (sec)	N/A	0.343	0.865	21.935	0.000	0.113	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	82	66	154	0	179	0	0	0
N.S.	1	1.06	0.86	2.00	0.00	2.32	0.00	0.00	0.00
time (sec)	N/A	0.339	0.912	39.194	0.000	0.100	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	112	85	0	0	346	0	0	0
N.S.	1	1.06	0.80	0.00	0.00	3.26	0.00	0.00	0.00
time (sec)	N/A	0.430	1.198	0.000	0.000	0.122	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	116	86	0	0	235	0	0	0
N.S.	1	1.09	0.81	0.00	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.439	0.839	0.000	0.000	0.111	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	215	100	441	0	291	0	0	0
N.S.	1	1.13	0.53	2.32	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.932	0.956	105.301	0.000	0.273	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	182	84	973	0	280	0	0	0
N.S.	1	1.11	0.51	5.93	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.741	0.550	102.092	0.000	0.269	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	148	70	243	0	268	0	0	0
N.S.	1	1.17	0.55	1.91	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.613	0.328	107.882	0.000	0.265	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	118	68	542	0	295	0	0	0
N.S.	1	1.13	0.65	5.21	0.00	2.84	0.00	0.00	0.00
time (sec)	N/A	0.486	0.273	14.649	0.000	0.267	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	53	0	0	95
N.S.	1	1.00	0.96	6.86	0.00	1.89	0.00	0.00	3.39
time (sec)	N/A	0.191	0.198	13.021	0.000	0.271	0.000	0.000	21.343

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	35	482	0	76	0	0	93
N.S.	1	1.00	0.64	8.76	0.00	1.38	0.00	0.00	1.69
time (sec)	N/A	0.280	0.242	13.756	0.000	0.248	0.000	0.000	24.217

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	86	55	222	0	104	0	0	302
N.S.	1	1.06	0.68	2.74	0.00	1.28	0.00	0.00	3.73
time (sec)	N/A	0.475	0.292	60.106	0.000	0.248	0.000	0.000	24.172

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	117	62	560	0	131	0	0	383
N.S.	1	1.09	0.58	5.23	0.00	1.22	0.00	0.00	3.58
time (sec)	N/A	0.588	0.360	139.424	0.000	0.272	0.000	0.000	26.084

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	602	0	0	0	0	0	0
N.S.	1	1.00	7.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	6.697	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	65	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.587	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	278	0	0	0	0	0	0
N.S.	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	1.433	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	254	0	0	0	0	0	0
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	1.398	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	61	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	2308	0	0	0	0	0	0
N.S.	1	1.00	27.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	17.643	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	47	111	91	46	270	46	46
N.S.	1	0.89	0.77	1.82	1.49	0.75	4.43	0.75	0.75
time (sec)	N/A	0.273	0.807	4.973	0.218	0.263	25.606	0.315	19.646

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	47	97	80	73	233	80	45
N.S.	1	0.89	0.77	1.59	1.31	1.20	3.82	1.31	0.74
time (sec)	N/A	0.283	0.380	2.831	0.207	0.264	11.059	0.300	19.708

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	83	69	36	199	36	36
N.S.	1	0.91	0.80	1.80	1.50	0.78	4.33	0.78	0.78
time (sec)	N/A	0.268	0.300	1.531	0.207	0.255	4.835	0.283	19.471

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	69	58	53	162	58	36
N.S.	1	0.91	0.80	1.50	1.26	1.15	3.52	1.26	0.78
time (sec)	N/A	0.274	0.151	0.891	0.195	0.254	2.042	0.287	0.052

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	27	55	47	26	128	26	26
N.S.	1	0.97	0.87	1.77	1.52	0.84	4.13	0.84	0.84
time (sec)	N/A	0.266	0.091	0.500	0.193	0.259	0.769	0.459	19.392

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	27	41	36	33	90	36	26
N.S.	1	0.97	0.87	1.32	1.16	1.06	2.90	1.16	0.84
time (sec)	N/A	0.265	0.071	0.345	0.207	0.249	0.359	0.316	0.035

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	15	27	26	13	53	13	43
N.S.	1	1.00	0.50	0.90	0.87	0.43	1.77	0.43	1.43
time (sec)	N/A	0.178	0.009	0.224	0.194	0.244	0.173	0.293	19.789

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	42	22	84	30	0	28	12
N.S.	1	1.00	3.00	1.57	6.00	2.14	0.00	2.00	0.86
time (sec)	N/A	0.215	0.019	0.345	0.195	0.247	0.000	0.300	20.996

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	23	29	31	233	50	0	38	26
N.S.	1	0.82	1.04	1.11	8.32	1.79	0.00	1.36	0.93
time (sec)	N/A	0.244	0.061	0.569	0.310	0.255	0.000	0.328	0.061

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	143	53	974	96	0	63	49
N.S.	1	1.10	2.92	1.08	19.88	1.96	0.00	1.29	1.00
time (sec)	N/A	0.268	0.522	1.181	0.215	0.260	0.000	0.412	0.077

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	31	69	1780	130	0	72	61
N.S.	1	1.00	0.47	1.05	26.97	1.97	0.00	1.09	0.92
time (sec)	N/A	0.279	0.031	2.510	0.376	0.252	0.000	0.297	19.601

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	97	268	89	3846	148	0	85	78
N.S.	1	1.09	3.01	1.00	43.21	1.66	0.00	0.96	0.88
time (sec)	N/A	0.300	0.733	5.290	0.331	0.251	0.000	0.286	0.128

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	42	68	74	72	36	597	36	35
N.S.	1	0.95	1.55	1.68	1.64	0.82	13.57	0.82	0.80
time (sec)	N/A	0.289	0.413	2.984	0.220	0.262	11.304	0.304	19.369

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	86	62	66	65	66	434	68	109
N.S.	1	1.13	0.82	0.87	0.86	0.87	5.71	0.89	1.43
time (sec)	N/A	0.382	0.143	1.654	0.209	0.251	4.913	0.296	21.142

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	30	48	52	50	26	362	26	26
N.S.	1	1.07	1.71	1.86	1.79	0.93	12.93	0.93	0.93
time (sec)	N/A	0.274	0.091	0.941	0.193	0.251	2.053	0.463	0.053

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	40	44	43	47	231	46	43
N.S.	1	1.10	0.82	0.90	0.88	0.96	4.71	0.94	0.88
time (sec)	N/A	0.303	0.077	0.536	0.221	0.247	0.822	0.313	19.742

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	26	13	133	13	13
N.S.	1	1.00	1.00	2.00	1.73	0.87	8.87	0.87	0.87
time (sec)	N/A	0.232	0.007	0.350	0.218	0.243	0.367	0.277	0.051

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	16	27	13	82	14	0	13	14
N.S.	1	1.14	1.93	0.93	5.86	1.00	0.00	0.93	1.00
time (sec)	N/A	0.222	0.008	0.475	0.226	0.260	0.000	0.278	0.061

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	53	19	0	13	11
N.S.	1	1.00	1.00	0.92	4.08	1.46	0.00	1.00	0.85
time (sec)	N/A	0.231	0.011	1.240	0.244	0.251	0.000	0.346	19.813

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	27	43	24	656	65	0	41	36
N.S.	1	0.90	1.43	0.80	21.87	2.17	0.00	1.37	1.20
time (sec)	N/A	0.258	0.038	0.919	0.224	0.259	0.000	0.432	19.641

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	34	48	46	308	54	0	35	37
N.S.	1	0.81	1.14	1.10	7.33	1.29	0.00	0.83	0.88
time (sec)	N/A	0.278	0.353	5.418	0.231	0.249	0.000	0.322	19.599

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	47	54	62	3188	138	0	74	82
N.S.	1	0.78	0.90	1.03	53.13	2.30	0.00	1.23	1.37
time (sec)	N/A	0.285	0.388	2.806	0.282	0.245	0.000	0.297	19.599

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	82	80	36	447	36	36
N.S.	1	0.91	0.80	1.78	1.74	0.78	9.72	0.78	0.78
time (sec)	N/A	0.291	0.396	5.408	0.210	0.261	25.731	0.341	20.849

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	54	47	70	69	63	366	69	45
N.S.	1	0.89	0.77	1.15	1.13	1.03	6.00	1.13	0.74
time (sec)	N/A	0.290	0.221	3.102	0.228	0.259	11.326	0.324	0.068

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	27	49	47	26	284	26	26
N.S.	1	0.97	0.87	1.58	1.52	0.84	9.16	0.84	0.84
time (sec)	N/A	0.283	0.103	1.610	0.204	0.247	4.921	0.527	19.941

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	48	47	43	202	47	36
N.S.	1	0.91	0.80	1.04	1.02	0.93	4.39	1.02	0.78
time (sec)	N/A	0.282	0.066	0.849	0.206	0.243	2.102	0.358	0.069

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	34	13	117	13	13
N.S.	1	1.00	1.00	2.73	2.27	0.87	7.80	0.87	0.87
time (sec)	N/A	0.238	0.009	0.454	0.202	0.254	0.794	0.310	0.053

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	23	46	29	92	38	0	36	22
N.S.	1	0.82	1.64	1.04	3.29	1.36	0.00	1.29	0.79
time (sec)	N/A	0.250	0.017	1.060	0.222	0.249	0.000	0.283	0.063

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	84	13	0	13	13
N.S.	1	1.00	1.00	1.08	6.46	1.00	0.00	1.00	1.00
time (sec)	N/A	0.250	0.011	2.234	0.213	0.255	0.000	0.348	0.026

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	79	39	558	72	0	48	36
N.S.	1	1.12	2.32	1.15	16.41	2.12	0.00	1.41	1.06
time (sec)	N/A	0.303	0.013	5.292	0.215	0.258	0.000	0.437	19.815

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	35	31	41	834	94	0	52	38
N.S.	1	0.81	0.72	0.95	19.40	2.19	0.00	1.21	0.88
time (sec)	N/A	0.275	0.031	10.375	0.322	0.267	0.000	0.328	19.306

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	85	195	71	2237	132	0	73	66
N.S.	1	1.21	2.79	1.01	31.96	1.89	0.00	1.04	0.94
time (sec)	N/A	0.297	0.879	19.832	0.274	0.253	0.000	0.291	19.572

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	136	151	98	221760772	0	290	0	0	0
N.S.	1	1.11	0.72	1630593.91	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.525	0.605	71.553	0.000	0.277	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	120	86	85939054	0	281	0	0	0
N.S.	1	1.09	0.78	781264.13	0.00	2.55	0.00	0.00	0.00
time (sec)	N/A	0.402	0.321	16.618	0.000	0.258	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	89	70	5537888	0	266	0	0	0
N.S.	1	1.06	0.83	65927.24	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.304	0.202	2.019	0.000	0.259	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	58	58	52	18441891	0	242	0	0	0
N.S.	1	1.00	0.90	317963.64	0.00	4.17	0.00	0.00	0.00
time (sec)	N/A	0.208	0.156	2.068	0.000	0.261	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	24	24	23	57905011	0	39	0	2029	24
N.S.	1	1.00	0.96	2412708.79	0.00	1.62	0.00	84.54	1.00
time (sec)	N/A	0.193	0.023	6.415	0.000	0.258	0.000	13.871	20.158

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	194	0	74	0	7875	104
N.S.	1	1.00	0.81	3.66	0.00	1.40	0.00	148.58	1.96
time (sec)	N/A	0.279	0.298	14.079	0.000	0.247	0.000	53.653	23.087

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	84	52	481	0	103	0	18022	136
N.S.	1	1.06	0.66	6.09	0.00	1.30	0.00	228.13	1.72
time (sec)	N/A	0.382	0.385	231.201	0.000	0.254	0.000	181.760	23.298

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	115	67	222	0	118	0	0	350
N.S.	1	1.10	0.64	2.11	0.00	1.12	0.00	0.00	3.33
time (sec)	N/A	0.499	0.509	157.895	0.000	0.277	0.000	0.000	24.201

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	108	96	0	0	0	0	0	0
N.S.	1	1.10	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.722	0.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	69	74	66	364390662	0	0	0	0	0
N.S.	1	1.07	0.96	5281024.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	0.568	152.693	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	69	74	76	198204147	0	0	0	0	0
N.S.	1	1.07	1.10	2872523.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.776	49.126	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	40	40	34	26159851	0	0	0	0	0
N.S.	1	1.00	0.85	653996.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.352	5.164	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	40	40	54	67488705	0	0	0	0	0
N.S.	1	1.00	1.35	1687217.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.259	1.559	9.333	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	39	107396897	0	156	0	0	0
N.S.	1	1.00	0.85	2334715.15	0.00	3.39	0.00	0.00	0.00
time (sec)	N/A	0.264	0.348	11.489	0.000	0.097	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	62	123	0	103	0	0	0
N.S.	1	1.00	1.29	2.56	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.265	0.888	27.506	0.000	0.090	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	78	64	227	0	266	0	0	0
N.S.	1	1.01	0.83	2.95	0.00	3.45	0.00	0.00	0.00
time (sec)	N/A	0.343	1.088	145.204	0.000	0.106	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	136	151	99	523275790	0	291	0	0	0
N.S.	1	1.11	0.73	3847616.10	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.528	0.628	204.947	0.000	0.267	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	120	84	103475646	0	280	0	0	0
N.S.	1	1.09	0.76	940687.69	0.00	2.55	0.00	0.00	0.00
time (sec)	N/A	0.411	0.355	18.148	0.000	0.275	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	89	73	213968312	0	268	0	0	0
N.S.	1	1.06	0.87	2547241.81	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.304	0.345	36.608	0.000	0.265	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	82	87	70	211562444	0	295	0	0	0
N.S.	1	1.06	0.85	2580029.80	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.400	0.347	35.435	0.000	0.270	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	53	0	15292	94
N.S.	1	1.00	0.96	6.86	0.00	1.89	0.00	546.14	3.36
time (sec)	N/A	0.198	0.378	68.588	0.000	0.260	0.000	104.028	22.263

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	35	482	0	76	0	0	93
N.S.	1	1.00	0.64	8.76	0.00	1.38	0.00	0.00	1.69
time (sec)	N/A	0.285	0.562	230.854	0.000	0.278	0.000	0.000	23.660

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	86	55	0	0	104	0	0	302
N.S.	1	1.06	0.68	0.00	0.00	1.28	0.00	0.00	3.73
time (sec)	N/A	0.394	0.803	0.000	0.000	0.255	0.000	0.000	23.989

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	117	62	0	0	131	0	0	383
N.S.	1	1.09	0.58	0.00	0.00	1.22	0.00	0.00	3.58
time (sec)	N/A	0.515	1.054	0.000	0.000	0.255	0.000	0.000	25.365

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	137	0	0	0
N.S.	1	1.00	0.94	3.16	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.181	0.054	0.629	0.000	0.255	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	33	25	99	0	137	0	0	0
N.S.	1	1.32	1.00	3.96	0.00	5.48	0.00	0.00	0.00
time (sec)	N/A	0.241	0.043	0.497	0.000	0.274	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	2472	0	0	0	0	0	0
N.S.	1	1.00	29.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	13.908	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	63	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.693	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	567	0	0	0	0	0	0
N.S.	1	1.00	6.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	5.282	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	37	60	58	36	318	36	36
N.S.	1	0.91	0.80	1.30	1.26	0.78	6.91	0.78	0.78
time (sec)	N/A	0.313	0.274	3.158	0.240	0.250	10.970	0.334	19.541

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	293	287	182	0	0	0	0	0	0
N.S.	1	0.98	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.983	2.756	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	916	122	921	84	311
N.S.	1	1.00	0.95	0.92	10.07	1.34	10.12	0.92	3.42
time (sec)	N/A	0.266	0.631	0.799	0.292	0.254	1.853	0.279	21.328

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	414	71	408	56	98
N.S.	1	1.00	1.11	0.92	6.68	1.15	6.58	0.90	1.58
time (sec)	N/A	0.234	0.810	0.532	0.246	0.257	0.667	0.296	20.801

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	153	40	84
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.56	0.93	1.95
time (sec)	N/A	0.203	0.231	0.343	0.247	0.248	0.308	0.287	20.713

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	28	26	68	108	31	333	236	111
N.S.	1	1.08	1.00	2.62	4.15	1.19	12.81	9.08	4.27
time (sec)	N/A	0.232	0.184	1.061	0.288	0.279	4.450	0.303	20.651

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	115	454	71	3264	349	252
N.S.	1	1.00	2.50	3.19	12.61	1.97	90.67	9.69	7.00
time (sec)	N/A	0.280	0.125	1.375	0.288	0.261	54.943	0.316	25.174

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	63	399	47	0	145	0
N.S.	1	1.00	0.90	1.62	10.23	1.21	0.00	3.72	0.00
time (sec)	N/A	0.291	0.226	1.697	0.224	0.242	0.000	0.310	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	63	67	189	1773	141	0	2221	0
N.S.	1	0.94	1.00	2.82	26.46	2.10	0.00	33.15	0.00
time (sec)	N/A	0.366	0.632	3.848	0.300	0.259	0.000	0.324	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	52	58	97	1076	75	0	301	0
N.S.	1	0.87	0.97	1.62	17.93	1.25	0.00	5.02	0.00
time (sec)	N/A	0.314	0.430	3.747	0.237	0.256	0.000	0.320	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	89	79	257	3879	197	0	8035	0
N.S.	1	0.95	0.84	2.73	41.27	2.10	0.00	85.48	0.00
time (sec)	N/A	0.468	1.347	13.880	0.427	0.259	0.000	0.364	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	410	397	0	0	0	0	0	0	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.162	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	80	63	371	69	410	61	105
N.S.	1	1.00	1.18	0.93	5.46	1.01	6.03	0.90	1.54
time (sec)	N/A	0.237	0.410	0.559	0.236	0.251	0.695	0.285	20.682

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	89	620	118	1027	80	177
N.S.	1	1.00	1.20	1.01	7.05	1.34	11.67	0.91	2.01
time (sec)	N/A	0.260	0.855	0.820	0.248	0.265	1.525	0.283	20.801

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	192	2003	129	469
N.S.	1	1.00	1.10	0.92	9.46	1.33	13.91	0.90	3.26
time (sec)	N/A	0.315	1.681	1.634	0.304	0.254	5.433	0.290	22.053

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	600	580	0	0	0	0	0	0	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.839	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	91	90	789	115	932	89	494
N.S.	1	1.00	0.94	0.93	8.13	1.19	9.61	0.92	5.09
time (sec)	N/A	0.266	0.596	0.767	0.328	0.252	2.085	0.277	22.340

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	189	2030	124	437
N.S.	1	1.00	1.11	0.92	9.86	1.37	14.71	0.90	3.17
time (sec)	N/A	0.307	1.740	1.632	0.307	0.269	6.366	0.293	23.522

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	177	190	2612	291	3584	181	997
N.S.	1	1.00	0.91	0.97	13.39	1.49	18.38	0.93	5.11
time (sec)	N/A	0.368	1.863	3.316	0.439	0.269	18.196	0.286	26.109

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	277	271	181	0	0	0	0	0	0
N.S.	1	0.98	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	2.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	87	84	912	106	918	84	297
N.S.	1	1.00	0.96	0.92	10.02	1.16	10.09	0.92	3.26
time (sec)	N/A	0.255	0.678	0.759	0.254	0.255	2.118	0.302	23.191

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	71	57	414	66	405	56	97
N.S.	1	1.00	1.15	0.92	6.68	1.06	6.53	0.90	1.56
time (sec)	N/A	0.228	0.884	0.488	0.241	0.250	0.766	0.315	21.926

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	155	40	85
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.60	0.93	1.98
time (sec)	N/A	0.206	0.258	0.341	0.228	0.255	0.348	0.299	22.102

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	69	73	31	435	158	112
N.S.	1	1.00	1.00	2.56	2.70	1.15	16.11	5.85	4.15
time (sec)	N/A	0.213	0.295	1.101	0.239	0.255	4.323	0.299	23.072

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	88	115	387	69	0	248	254
N.S.	1	1.00	2.59	3.38	11.38	2.03	0.00	7.29	7.47
time (sec)	N/A	0.270	0.144	1.950	0.422	0.250	0.000	0.306	26.084

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	34	40	391	42	0	174	0
N.S.	1	1.00	0.89	1.05	10.29	1.11	0.00	4.58	0.00
time (sec)	N/A	0.291	0.204	2.115	0.238	0.246	0.000	0.308	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	63	64	191	1424	94	0	495	0
N.S.	1	0.94	0.96	2.85	21.25	1.40	0.00	7.39	0.00
time (sec)	N/A	0.366	0.540	8.446	0.441	0.250	0.000	0.312	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	48	76	1074	53	0	327	0
N.S.	1	0.92	0.81	1.29	18.20	0.90	0.00	5.54	0.00
time (sec)	N/A	0.307	0.401	5.163	0.253	0.278	0.000	0.318	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	89	78	259	3096	107	0	756	0
N.S.	1	0.95	0.83	2.76	32.94	1.14	0.00	8.04	0.00
time (sec)	N/A	0.475	1.114	49.844	0.524	0.264	0.000	0.322	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	386	373	242	0	0	0	0	0	0
N.S.	1	0.97	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.906	1.459	0.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	76	63	371	70	410	61	105
N.S.	1	1.00	1.12	0.93	5.46	1.03	6.03	0.90	1.54
time (sec)	N/A	0.234	0.915	0.540	0.244	0.249	0.720	0.277	21.078

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	108	89	620	108	1027	80	177
N.S.	1	1.00	1.23	1.01	7.05	1.23	11.67	0.91	2.01
time (sec)	N/A	0.254	0.894	0.841	0.309	0.259	1.630	0.285	22.329

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	174	2003	129	495
N.S.	1	1.00	1.10	0.92	9.46	1.21	13.91	0.90	3.44
time (sec)	N/A	0.305	1.767	1.556	0.329	0.257	5.685	0.300	22.447

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	568	548	329	0	0	0	0	0	0
N.S.	1	0.96	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.369	2.602	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	785	116	932	89	471
N.S.	1	1.00	0.93	0.93	8.09	1.20	9.61	0.92	4.86
time (sec)	N/A	0.264	0.564	0.723	0.284	0.258	2.038	0.289	21.981

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	179	2020	124	438
N.S.	1	1.00	1.11	0.92	9.86	1.30	14.64	0.90	3.17
time (sec)	N/A	0.301	1.952	1.635	0.308	0.269	5.640	0.288	22.242

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	190	2612	264	3577	181	951
N.S.	1	1.00	0.90	0.97	13.39	1.35	18.34	0.93	4.88
time (sec)	N/A	0.367	1.820	2.969	0.397	0.275	17.956	0.290	25.692

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	29	58	70	106	30	333	482	115
N.S.	1	1.07	2.15	2.59	3.93	1.11	12.33	17.85	4.26
time (sec)	N/A	0.225	0.278	1.086	0.239	0.257	4.536	0.332	0.861

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	90	115	450	71	3264	893	252
N.S.	1	1.00	2.57	3.29	12.86	2.03	93.26	25.51	7.20
time (sec)	N/A	0.271	0.200	1.484	0.250	0.248	61.014	0.415	26.291

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	36	395	45	0	327	0
N.S.	1	1.00	0.92	0.95	10.39	1.18	0.00	8.61	0.00
time (sec)	N/A	0.293	0.246	1.580	0.228	0.253	0.000	0.328	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	86	70	186	1027	376	0	0	0
N.S.	1	1.19	0.97	2.58	14.26	5.22	0.00	0.00	0.00
time (sec)	N/A	0.661	0.421	0.273	0.417	0.279	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	109	143	520	315	0	0	294
N.S.	1	1.00	2.48	3.25	11.82	7.16	0.00	0.00	6.68
time (sec)	N/A	0.369	0.115	0.218	0.430	0.282	0.000	0.000	27.602

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	94	99	131	188	0	0	227
N.S.	1	1.00	3.24	3.41	4.52	6.48	0.00	0.00	7.83
time (sec)	N/A	0.239	0.058	0.210	0.386	0.259	0.000	0.000	26.036

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	95	105	197	0	226	233
N.S.	1	1.00	3.21	3.28	3.62	6.79	0.00	7.79	8.03
time (sec)	N/A	0.237	0.092	0.224	0.260	0.265	0.000	0.298	25.237

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	111	143	612	316	0	577	290
N.S.	1	1.00	2.41	3.11	13.30	6.87	0.00	12.54	6.30
time (sec)	N/A	0.375	0.149	0.303	0.309	0.253	0.000	0.312	25.766

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	87	71	184	1254	372	0	870	0
N.S.	1	1.18	0.96	2.49	16.95	5.03	0.00	11.76	0.00
time (sec)	N/A	0.672	0.426	0.404	0.317	0.269	0.000	0.359	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	116	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	2.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	260	0	0	0	0	0	0
N.S.	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	5.365	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	914	109	918	84	313
N.S.	1	1.00	0.93	0.92	10.04	1.20	10.09	0.92	3.44
time (sec)	N/A	0.260	0.532	0.743	0.301	0.262	2.017	0.292	22.875

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	416	63	405	56	98
N.S.	1	1.00	1.11	0.92	6.71	1.02	6.53	0.90	1.58
time (sec)	N/A	0.227	0.860	0.518	0.269	0.247	0.761	0.292	22.135

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	153	40	84
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.56	0.93	1.95
time (sec)	N/A	0.204	0.196	0.301	0.250	0.239	0.318	0.291	22.303

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	66	74	31	435	440	109
N.S.	1	1.00	1.00	2.54	2.85	1.19	16.73	16.92	4.19
time (sec)	N/A	0.211	0.145	1.076	0.250	0.249	3.516	0.337	22.258

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	89	119	391	69	0	1341	246
N.S.	1	1.00	2.54	3.40	11.17	1.97	0.00	38.31	7.03
time (sec)	N/A	0.260	0.120	2.015	0.398	0.253	0.000	0.381	26.915

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	26	382	40	0	315	0
N.S.	1	1.00	0.92	0.68	10.05	1.05	0.00	8.29	0.00
time (sec)	N/A	0.287	0.233	2.056	0.256	0.257	0.000	0.339	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	163	2006	129	495
N.S.	1	1.00	1.10	0.92	9.46	1.13	13.93	0.90	3.44
time (sec)	N/A	0.298	1.683	1.516	0.311	0.270	5.658	0.308	24.014

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	105	89	620	105	1027	80	177
N.S.	1	1.00	1.19	1.01	7.05	1.19	11.67	0.91	2.01
time (sec)	N/A	0.248	0.861	0.806	0.286	0.257	1.598	0.284	21.378

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	190	2614	240	3584	181	999
N.S.	1	1.00	0.90	0.97	13.41	1.23	18.38	0.93	5.12
time (sec)	N/A	0.360	1.801	3.503	0.417	0.268	17.687	0.285	25.158

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	86	70	181	1027	366	0	0	0
N.S.	1	1.19	0.97	2.51	14.26	5.08	0.00	0.00	0.00
time (sec)	N/A	0.657	0.449	0.345	0.389	0.279	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	111	149	526	316	0	0	285
N.S.	1	1.00	2.41	3.24	11.43	6.87	0.00	0.00	6.20
time (sec)	N/A	0.374	0.122	0.260	0.430	0.271	0.000	0.000	26.595

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	93	97	131	196	0	0	237
N.S.	1	1.00	3.10	3.23	4.37	6.53	0.00	0.00	7.90
time (sec)	N/A	0.241	0.060	0.207	0.402	0.271	0.000	0.000	25.712

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	94	93	105	190	0	234	231
N.S.	1	1.00	3.24	3.21	3.62	6.55	0.00	8.07	7.97
time (sec)	N/A	0.237	0.069	0.241	0.268	0.259	0.000	0.324	26.573

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	112	145	613	316	0	627	289
N.S.	1	1.00	2.43	3.15	13.33	6.87	0.00	13.63	6.28
time (sec)	N/A	0.382	0.123	0.292	0.253	0.278	0.000	0.326	27.018

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	86	71	179	1254	385	0	963	0
N.S.	1	1.18	0.97	2.45	17.18	5.27	0.00	13.19	0.00
time (sec)	N/A	0.649	0.390	0.405	0.250	0.272	0.000	0.375	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	1.645	0.000	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	108	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	1.667	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [235] had the largest ratio of [.9333329999999999968]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	0.89	18	0.333
2	A	7	6	0.89	18	0.333
3	A	7	6	0.91	18	0.333
4	A	7	6	0.91	18	0.333
5	A	7	6	0.97	18	0.333
6	A	7	6	0.97	18	0.333
7	A	2	2	1.00	16	0.125
8	A	4	4	1.00	16	0.250
9	A	8	7	0.82	18	0.389
10	A	8	7	1.10	18	0.389
11	A	8	7	1.00	18	0.389
12	A	10	9	1.09	18	0.500
13	A	8	7	0.95	20	0.350
14	A	10	9	1.13	20	0.450
15	A	7	6	1.03	20	0.300
16	A	8	7	1.10	20	0.350
17	A	6	5	1.00	18	0.278
18	A	4	4	1.00	18	0.222
19	A	6	5	1.00	20	0.250
20	A	7	6	0.90	20	0.300
21	A	7	6	0.86	20	0.300
22	A	8	7	0.82	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	0.91	20	0.300
24	A	7	6	0.89	20	0.300
25	A	7	6	0.97	20	0.300
26	A	7	6	0.91	20	0.300
27	A	6	5	1.00	18	0.278
28	A	7	6	0.82	18	0.333
29	A	6	5	1.00	20	0.250
30	A	6	6	1.12	20	0.300
31	A	8	7	0.81	20	0.350
32	A	10	9	1.21	20	0.450
33	A	7	6	0.89	18	0.333
34	A	7	6	0.89	18	0.333
35	A	7	6	0.91	18	0.333
36	A	7	6	0.91	18	0.333
37	A	7	6	0.97	18	0.333
38	A	7	6	0.97	18	0.333
39	A	6	5	1.00	18	0.278
40	A	4	4	1.00	16	0.250
41	A	8	7	0.82	16	0.438
42	A	8	7	1.10	18	0.389
43	A	8	7	1.00	18	0.389
44	A	10	9	1.09	18	0.500
45	A	17	17	1.21	20	0.850
46	A	8	7	0.95	20	0.350
47	A	13	13	1.20	20	0.650
48	A	7	6	1.03	20	0.300
49	A	9	9	1.32	20	0.450
50	A	6	5	1.00	20	0.250
51	A	5	5	1.29	20	0.250
52	A	5	5	1.17	18	0.278
53	A	7	6	0.90	18	0.333
54	A	7	6	0.81	20	0.300
55	A	8	7	0.78	20	0.350
56	A	7	6	0.83	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	8	7	0.81	20	0.350
58	A	7	6	0.78	20	0.300
59	A	7	6	0.89	20	0.300
60	A	7	6	0.87	20	0.300
61	A	7	6	0.91	20	0.300
62	A	7	6	0.89	20	0.300
63	A	7	6	0.97	20	0.300
64	A	7	6	0.91	20	0.300
65	A	6	5	1.00	20	0.250
66	A	6	5	0.96	20	0.250
67	A	8	7	0.88	20	0.350
68	A	7	6	1.00	18	0.333
69	A	8	7	0.81	18	0.389
70	A	10	9	1.21	20	0.450
71	A	8	7	0.96	20	0.350
72	A	10	9	1.14	20	0.450
73	A	8	8	1.11	20	0.400
74	A	6	6	1.09	20	0.300
75	A	4	4	1.06	20	0.200
76	A	2	2	1.00	20	0.100
77	A	2	2	1.00	20	0.100
78	A	4	4	1.00	20	0.200
79	A	6	6	1.06	20	0.300
80	A	8	8	1.10	20	0.400
81	A	8	8	1.10	22	0.364
82	A	6	6	1.07	22	0.273
83	A	6	6	1.07	22	0.273
84	A	4	4	1.00	22	0.182
85	A	4	4	1.00	22	0.182
86	A	4	4	1.00	22	0.182
87	A	4	4	1.00	22	0.182
88	A	6	6	1.01	22	0.273
89	A	8	8	1.11	22	0.364
90	A	6	6	1.09	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	4	4	1.06	22	0.182
92	A	6	6	1.06	22	0.273
93	A	2	2	1.00	22	0.091
94	A	4	4	1.00	22	0.182
95	A	6	6	1.06	22	0.273
96	A	8	8	1.09	22	0.364
97	A	10	10	1.12	20	0.500
98	A	8	8	1.11	20	0.400
99	A	6	6	1.12	20	0.300
100	A	4	4	1.13	20	0.200
101	A	2	2	1.00	20	0.100
102	A	6	6	1.04	20	0.300
103	A	8	8	1.09	20	0.400
104	A	10	10	1.11	20	0.500
105	A	8	8	1.09	22	0.364
106	A	8	8	1.09	22	0.364
107	A	6	6	1.09	22	0.273
108	A	6	6	1.10	22	0.273
109	A	4	4	1.00	22	0.182
110	A	4	4	1.00	22	0.182
111	A	6	6	1.01	22	0.273
112	A	6	6	1.06	22	0.273
113	A	8	8	1.06	22	0.364
114	A	8	8	1.09	22	0.364
115	A	14	14	1.13	22	0.636
116	A	12	12	1.11	22	0.545
117	A	10	10	1.17	22	0.455
118	A	8	8	1.13	22	0.364
119	A	2	2	1.00	22	0.091
120	A	4	4	1.00	22	0.182
121	A	8	8	1.06	22	0.364
122	A	10	10	1.09	22	0.455
123	A	4	4	1.00	20	0.200
124	A	4	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	4	4	1.00	18	0.222
126	A	4	4	1.00	18	0.222
127	A	4	4	1.00	20	0.200
128	A	4	4	1.00	20	0.200
129	A	7	6	0.89	18	0.333
130	A	7	6	0.89	18	0.333
131	A	7	6	0.91	18	0.333
132	A	7	6	0.91	18	0.333
133	A	7	6	0.97	18	0.333
134	A	7	6	0.97	18	0.333
135	A	2	2	1.00	16	0.125
136	A	4	4	1.00	16	0.250
137	A	8	7	0.82	18	0.389
138	A	8	7	1.10	18	0.389
139	A	8	7	1.00	18	0.389
140	A	10	9	1.09	18	0.500
141	A	8	7	0.95	20	0.350
142	A	10	9	1.13	20	0.450
143	A	7	6	1.07	20	0.300
144	A	8	7	1.10	20	0.350
145	A	6	5	1.00	18	0.278
146	A	5	5	1.14	18	0.278
147	A	6	5	1.00	20	0.250
148	A	7	6	0.90	20	0.300
149	A	7	6	0.81	20	0.300
150	A	8	7	0.78	20	0.350
151	A	7	6	0.91	20	0.300
152	A	7	6	0.89	20	0.300
153	A	7	6	0.97	20	0.300
154	A	7	6	0.91	20	0.300
155	A	6	5	1.00	18	0.278
156	A	8	7	0.82	18	0.389
157	A	7	6	1.00	20	0.300
158	A	6	6	1.12	20	0.300

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	8	7	0.81	20	0.350
160	A	10	9	1.21	20	0.450
161	A	8	8	1.11	20	0.400
162	A	6	6	1.09	20	0.300
163	A	4	4	1.06	20	0.200
164	A	2	2	1.00	20	0.100
165	A	2	2	1.00	20	0.100
166	A	4	4	1.00	20	0.200
167	A	6	6	1.06	20	0.300
168	A	8	8	1.10	20	0.400
169	A	8	8	1.10	22	0.364
170	A	6	6	1.07	22	0.273
171	A	6	6	1.07	22	0.273
172	A	4	4	1.00	22	0.182
173	A	4	4	1.00	22	0.182
174	A	4	4	1.00	22	0.182
175	A	4	4	1.00	22	0.182
176	A	6	6	1.01	22	0.273
177	A	8	8	1.11	22	0.364
178	A	6	6	1.09	22	0.273
179	A	4	4	1.06	22	0.182
180	A	6	6	1.06	22	0.273
181	A	2	2	1.00	22	0.091
182	A	4	4	1.00	22	0.182
183	A	6	6	1.06	22	0.273
184	A	8	8	1.09	22	0.364
185	A	2	2	1.00	11	0.182
186	A	4	4	1.32	11	0.364
187	A	4	4	1.00	20	0.200
188	A	4	4	1.00	20	0.200
189	A	4	4	1.00	18	0.222
190	A	7	6	0.91	28	0.214
191	A	2	2	0.98	15	0.133
192	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	2	2	1.00	15	0.133
194	A	2	2	1.00	13	0.154
195	A	5	5	1.08	13	0.385
196	A	7	6	1.00	15	0.400
197	A	8	7	1.00	15	0.467
198	A	9	8	0.94	15	0.533
199	A	8	7	0.87	15	0.467
200	A	11	10	0.95	15	0.667
201	A	2	2	0.97	17	0.118
202	A	2	2	1.00	15	0.133
203	A	2	2	1.00	17	0.118
204	A	2	2	1.00	17	0.118
205	A	2	2	0.97	17	0.118
206	A	2	2	1.00	15	0.133
207	A	2	2	1.00	17	0.118
208	A	2	2	1.00	17	0.118
209	A	2	2	0.98	15	0.133
210	A	2	2	1.00	15	0.133
211	A	2	2	1.00	15	0.133
212	A	2	2	1.00	13	0.154
213	A	4	4	1.00	13	0.308
214	A	6	5	1.00	15	0.333
215	A	7	6	1.00	15	0.400
216	A	8	7	0.94	15	0.467
217	A	7	6	0.92	15	0.400
218	A	10	9	0.95	15	0.600
219	A	2	2	0.97	17	0.118
220	A	2	2	1.00	15	0.133
221	A	2	2	1.00	17	0.118
222	A	2	2	1.00	17	0.118
223	A	2	2	0.96	17	0.118
224	A	2	2	1.00	15	0.133
225	A	2	2	1.00	17	0.118
226	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	5	5	1.07	13	0.385
228	A	7	6	1.00	15	0.400
229	A	8	7	1.00	15	0.467
230	A	14	13	1.19	15	0.867
231	A	9	8	1.00	15	0.533
232	A	4	4	1.00	13	0.308
233	A	4	4	1.00	13	0.308
234	A	10	9	1.00	15	0.600
235	A	15	14	1.18	15	0.933
236	A	2	2	1.00	13	0.154
237	A	2	2	1.00	13	0.154
238	A	2	2	1.00	15	0.133
239	A	2	2	1.00	15	0.133
240	A	2	2	1.00	13	0.154
241	A	4	4	1.00	13	0.308
242	A	6	5	1.00	15	0.333
243	A	7	6	1.00	15	0.400
244	A	2	2	1.00	17	0.118
245	A	2	2	1.00	17	0.118
246	A	2	2	1.00	17	0.118
247	A	14	13	1.19	15	0.867
248	A	9	8	1.00	15	0.533
249	A	4	4	1.00	13	0.308
250	A	4	4	1.00	13	0.308
251	A	10	9	1.00	15	0.600
252	A	15	14	1.18	15	0.933
253	A	2	2	1.00	13	0.154
254	A	2	2	1.00	13	0.154

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sin(a+bx)\sin^7(2a+2bx)\,dx$

3.1.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = \frac{128\sin^9(a+bx)}{9b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{128\sin^{15}(a+bx)}{15b}$$

output
```
128/9*sin(b*x+a)^9/b-384/11*sin(b*x+a)^11/b+384/13*sin(b*x+a)^13/b-128/15*
sin(b*x+a)^15/b
```

3.1.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = \frac{4(8330+10755\cos(2(a+bx))+3366\cos(4(a+bx))+429\cos(6(a+bx)))\sin^9(a+bx)}{6435b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(4*(8330 + 10755*Cos[2*(a + b*x)] + 3366*Cos[4*(a + b*x)] + 429*Cos[6*(a +
 b*x)])*Sin[a + b*x]^9)/(6435*b)
```

3.1.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(a+bx)\sin(2a+2bx)^7dx$$

$$\Big\downarrow \; 4776$$

$$128\int \cos^7(a+bx)\sin^8(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$128\int \cos(a+bx)^7\sin(a+bx)^8dx$$

$$\Big\downarrow \; 3044$$

$$\frac{128\int \sin^8(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$\frac{128\int \left(-\sin^{14}(a+bx)+3\sin^{12}(a+bx)-3\sin^{10}(a+bx)+\sin^8(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{128\left(-\frac{1}{15}\sin^{15}(a+bx)+\frac{3}{13}\sin^{13}(a+bx)-\frac{3}{11}\sin^{11}(a+bx)+\frac{1}{9}\sin^9(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*(Sin[a + b*x]^9/9 - (3*Sin[a + b*x]^11)/11 + (3*Sin[a + b*x]^13)/13 -
 Sin[a + b*x]^15/15))/b
```

3.1.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.1.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(53) = 106$.

Time = 9.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

method	result
default	$\frac{35\sin(xb+a)}{128b} - \frac{35\sin(3xb+3a)}{384b} - \frac{21\sin(5xb+5a)}{640b} + \frac{3\sin(7xb+7a)}{128b} + \frac{7\sin(9xb+9a)}{1152b} - \frac{7\sin(11xb+11a)}{1408b} - \frac{\sin(13xb\text{-}}{1664}$
risch	$\frac{35\sin(xb+a)}{128b} - \frac{35\sin(3xb+3a)}{384b} - \frac{21\sin(5xb+5a)}{640b} + \frac{3\sin(7xb+7a)}{128b} + \frac{7\sin(9xb+9a)}{1152b} - \frac{7\sin(11xb+11a)}{1408b} - \frac{\sin(13xb\text{-}}{1664}$
parallelrisch	$\frac{\left(-2048\tan(xb+a)^{13}-13312\tan(xb+a)^{11}-36608\tan(xb+a)^{9}-54912\tan(xb+a)^{7}-36608\tan(xb+a)^{5}-13312\tan(xb+a)^{3}-20\right.}{}$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
35/128*sin(b*x+a)/b-35/384*sin(3*b*x+3*a)/b-21/640/b*sin(5*b*x+5*a)+3/128/
b*sin(7*b*x+7*a)+7/1152/b*sin(9*b*x+9*a)-7/1408/b*sin(11*b*x+11*a)-1/1664/
b*sin(13*b*x+13*a)+1/1920/b*sin(15*b*x+15*a)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{128\left(429\cos\left(bx+a\right)^{14}-1518\cos\left(bx+a\right)^{12}+1854\cos\left(bx+a\right)^{10}-800\cos\left(bx+a\right)^{8}+5\cos\left(bx+a\right)^{6}\right.}{6435\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
128/6435*(429*cos(b*x + a)^14 - 1518*cos(b*x + a)^12 + 1854*cos(b*x + a)^1
0 - 800*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x +
 a)^2 + 16)*sin(b*x + a)/b
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(53) = 106.

Time = 25.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.41

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{3838\sin\left(a+bx\right)\sin^{6}\left(2a+2bx\right)\cos\left(2a+2bx\right)}{6435b} - \frac{1648\sin\left(a+bx\right)\sin^{4}\left(2a+2bx\right)\cos^{3}\left(2a+2bx\right)}{1287b} - \frac{768\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{5}\left(2a+2b\right.}{715b} \\ x\sin\left(a\right)\sin^{7}\left(2a\right) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**7,x)
```

output
```
Piecewise((-3838*sin(a + b*x)*sin(2*a + 2*b*x)**6*cos(2*a + 2*b*x)/(6435*b
) - 1648*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)**3/(1287*b) - 7
68*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**5/(715*b) - 2048*sin
(a + b*x)*cos(2*a + 2*b*x)**7/(6435*b) + 1241*sin(2*a + 2*b*x)**7*cos(a +
b*x)/(6435*b) + 376*sin(2*a + 2*b*x)**5*cos(a + b*x)*cos(2*a + 2*b*x)**2/(
715*b) + 640*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**4/(1287*b)
 + 1024*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**6/(6435*b), Ne(b,
0)), (x*sin(a)*sin(2*a)**7, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{429\sin(15bx+15a) - 495\sin(13bx+13a) - 4095\sin(11bx+11a) + 5005\sin(9bx+9a) + 19305\,\mathrm{s}}{823680\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
1/823680*(429*sin(15*b*x + 15*a) - 495*sin(13*b*x + 13*a) - 4095*sin(11*b*
x + 11*a) + 5005*sin(9*b*x + 9*a) + 19305*sin(7*b*x + 7*a) - 27027*sin(5*b
*x + 5*a) - 75075*sin(3*b*x + 3*a) + 225225*sin(b*x + a))/b
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx =$$
$$-\frac{128\left(429\sin(bx+a)^{15} - 1485\sin(bx+a)^{13} + 1755\sin(bx+a)^{11} - 715\sin(bx+a)^{9}\right)}{6435\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
-128/6435*(429*sin(b*x + a)^15 - 1485*sin(b*x + a)^13 + 1755*sin(b*x + a)^
11 - 715*sin(b*x + a)^9)/b
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = \frac{-\frac{128\sin(a+b\,x)^{15}}{15}+\frac{384\sin(a+b\,x)^{13}}{13}-\frac{384\sin(a+b\,x)^{11}}{11}+\frac{128\sin(a+b\,x)^9}{9}}{b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^7,x)
```

output
```
((128*sin(a + b*x)^9)/9 - (384*sin(a + b*x)^11)/11 + (384*sin(a + b*x)^13)
/13 - (128*sin(a + b*x)^15)/15)/b
```

3.2 $\int \sin(a+bx)\sin^6(2a+2bx)\,dx$

3.2.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{3b} - \frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^{13}(a+bx)}{13b}$$

output
```
-64/7*cos(b*x+a)^7/b+64/3*cos(b*x+a)^9/b-192/11*cos(b*x+a)^11/b+64/13*cos(
b*x+a)^13/b
```

3.2.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx = \frac{2\cos^7(a+bx)(-5230+6377\cos(2(a+bx))-1890\cos(4(a+bx))+231\cos(6(a+bx)))}{3003b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(2*Cos[a + b*x]^7*(-5230 + 6377*Cos[2*(a + b*x)] - 1890*Cos[4*(a + b*x)] +
 231*Cos[6*(a + b*x)]))/(3003*b)
```

3.2.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^6dx$$

$$\Big\downarrow \text{4776}$$

$$64\int \cos^6(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^6\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{64\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{64\int \left(-\cos^{12}(a+bx)+3\cos^{10}(a+bx)-3\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{64\left(-\frac{1}{13}\cos^{13}(a+bx)+\frac{3}{11}\cos^{11}(a+bx)-\frac{1}{3}\cos^9(a+bx)+\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(-64*(Cos[a + b*x]^7/7 - Cos[a + b*x]^9/3 + (3*Cos[a + b*x]^11)/11 - Cos[a
 + b*x]^13/13))/b
```

3.2.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.2.4 Maple [A] (verified)

Time = 5.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

method	result
default	$-\frac{5\cos(xb+a)}{16b}-\frac{5\cos(3xb+3a)}{64b}+\frac{3\cos(5xb+5a)}{64b}+\frac{3\cos(7xb+7a)}{224b}-\frac{\cos(9xb+9a)}{96b}-\frac{\cos(11xb+11a)}{704b}+\frac{\cos(13xb+1}{832b}$
risch	$-\frac{5\cos(xb+a)}{16b}-\frac{5\cos(3xb+3a)}{64b}+\frac{3\cos(5xb+5a)}{64b}+\frac{3\cos(7xb+7a)}{224b}-\frac{\cos(9xb+9a)}{96b}-\frac{\cos(11xb+11a)}{704b}+\frac{\cos(13xb+1}{832b}$
parallelrisch	$\frac{\left(-512\tan(xb+a)^{10}-2688\tan(xb+a)^{8}-5696\tan(xb+a)^{6}-2688\tan(xb+a)^{4}-512\tan(xb+a)^{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}+\left(2048\tan(xb+\right.}{}$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
-5/16*cos(b*x+a)/b-5/64*cos(3*b*x+3*a)/b+3/64*cos(5*b*x+5*a)/b+3/224*cos(7
*b*x+7*a)/b-1/96*cos(9*b*x+9*a)/b-1/704*cos(11*b*x+11*a)/b+1/832*cos(13*b*
x+13*a)/b
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx = \frac{64\left(231\cos(bx+a)^{13}-819\cos(bx+a)^{11}+1001\cos(bx+a)^{9}-429\cos(bx+a)^{7}\right)}{3003\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
64/3003*(231*cos(b*x + a)^13 - 819*cos(b*x + a)^11 + 1001*cos(b*x + a)^9 -
 429*cos(b*x + a)^7)/b
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(53) = 106$.

Time = 10.96 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.85

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx = \begin{cases} -\frac{1084\sin(a+bx)\sin^5(2a+2bx)\cos(2a+2bx)}{3003b} - \frac{64\sin(a+bx)\sin^3(2a+2bx)\cos^3(2a+2bx)}{143b} - \frac{512\sin(a+bx)\sin(2a+2bx)\cos^5(2a+2bx)}{3003b} \\ x\sin(a)\sin^6(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**6,x)
```

output
```
Piecewise((-1084*sin(a + b*x)*sin(2*a + 2*b*x)**5*cos(2*a + 2*b*x)/(3003*b
) - 64*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**3/(143*b) - 512*
sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**5/(3003*b) - 835*sin(2*a +
 2*b*x)**6*cos(a + b*x)/(3003*b) - 2776*sin(2*a + 2*b*x)**4*cos(a + b*x)*c
os(2*a + 2*b*x)**2/(3003*b) - 2944*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*
a + 2*b*x)**4/(3003*b) - 1024*cos(a + b*x)*cos(2*a + 2*b*x)**6/(3003*b), N
e(b, 0)), (x*sin(a)*sin(2*a)**6, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx$$
$$= \frac{231\cos(13\,bx+13\,a)-273\cos(11\,bx+11\,a)-2002\cos(9\,bx+9\,a)+2574\cos(7\,bx+7\,a)+9009\cos}{192192\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
1/192192*(231*cos(13*b*x + 13*a) - 273*cos(11*b*x + 11*a) - 2002*cos(9*b*x
 + 9*a) + 2574*cos(7*b*x + 7*a) + 9009*cos(5*b*x + 5*a) - 15015*cos(3*b*x
+ 3*a) - 60060*cos(b*x + a))/b
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx$$
$$= \frac{231\cos(13\,bx+13\,a)-273\cos(11\,bx+11\,a)-2002\cos(9\,bx+9\,a)+2574\cos(7\,bx+7\,a)+9009\cos}{192192\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
1/192192*(231*cos(13*b*x + 13*a) - 273*cos(11*b*x + 11*a) - 2002*cos(9*b*x
 + 9*a) + 2574*cos(7*b*x + 7*a) + 9009*cos(5*b*x + 5*a) - 15015*cos(3*b*x
+ 3*a) - 60060*cos(b*x + a))/b
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx = -\frac{-\frac{64\cos(a+b\,x)^{13}}{13}+\frac{192\cos(a+b\,x)^{11}}{11}-\frac{64\cos(a+b\,x)^{9}}{3}+\frac{64\cos(a+b\,x)^{7}}{7}}{b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^6,x)
```

output
```
-((64*cos(a + b*x)^7)/7 - (64*cos(a + b*x)^9)/3 + (192*cos(a + b*x)^11)/11
 - (64*cos(a + b*x)^13)/13)/b
```

3.2. $\int \sin(a+bx)\sin^6(2a+2bx)\,dx$

3.3 $\int \sin(a+bx)\sin^5(2a+2bx)\,dx$

3.3.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx=\frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^{11}(a+bx)}{11b}$$

output
```
32/7*sin(b*x+a)^7/b-64/9*sin(b*x+a)^9/b+32/11*sin(b*x+a)^11/b
```

3.3.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx =\frac{4(365+364\cos(2(a+bx))+63\cos(4(a+bx)))\sin^7(a+bx)}{693b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*(365 + 364*Cos[2*(a + b*x)] + 63*Cos[4*(a + b*x)])*Sin[a + b*x]^7)/(693
*b)
```

3.3.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^5dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^6dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{32\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{32\int \left(\sin^{10}(a+bx)-2\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{32\left(\frac{1}{11}\sin^{11}(a+bx)-\frac{2}{9}\sin^9(a+bx)+\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*(Sin[a + b*x]^7/7 - (2*Sin[a + b*x]^9)/9 + Sin[a + b*x]^11/11))/b
```

3.3.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.3.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.

Time = 2.86 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

method	result
default	$\frac{5\sin(xb+a)}{16b}-\frac{5\sin(3xb+3a)}{48b}-\frac{\sin(5xb+5a)}{32b}+\frac{5\sin(7xb+7a)}{224b}+\frac{\sin(9xb+9a)}{288b}-\frac{\sin(11xb+11a)}{352b}$
risch	$\frac{5\sin(xb+a)}{16b}-\frac{5\sin(3xb+3a)}{48b}-\frac{\sin(5xb+5a)}{32b}+\frac{5\sin(7xb+7a)}{224b}+\frac{\sin(9xb+9a)}{288b}-\frac{\sin(11xb+11a)}{352b}$
parallelrisch	$\left(-256\tan(xb+a)^9-1152\tan(xb+a)^7-2016\tan(xb+a)^5-1152\tan(xb+a)^3-256\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2+\left(512\tan(xb+a)^1\right.$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
5/16*sin(b*x+a)/b-5/48*sin(3*b*x+3*a)/b-1/32/b*sin(5*b*x+5*a)+5/224/b*sin(
7*b*x+7*a)+1/288/b*sin(9*b*x+9*a)-1/352/b*sin(11*b*x+11*a)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\,\left(63\,\cos\left(bx+a\right)^{10}-161\,\cos\left(bx+a\right)^{8}+113\,\cos\left(bx+a\right)^{6}-3\,\cos\left(bx+a\right)^{4}-4\,\cos\left(bx+a\right)^{2}-8\right)\sin\left(bx+a\right)}{693\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-32/693*(63*cos(b*x + a)^10 - 161*cos(b*x + a)^8 + 113*cos(b*x + a)^6 - 3*
cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*x + a)/b
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(39) = 78.

Time = 4.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.28

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \begin{cases} -\frac{422\sin\left(a+bx\right)\sin^{4}\left(2a+2bx\right)\cos\left(2a+2bx\right)}{693b} - \frac{608\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{3}\left(2a+2bx\right)}{693b} - \frac{256\sin\left(a+bx\right)\cos^{5}\left(2a+2bx\right)}{693b} + \frac{151\sin^{5}\left(2a+2bx\right)\cos\left(a+bx\right)}{693b} + \cdots \\ x\sin\left(a\right)\sin^{5}\left(2a\right) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-422*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)/(693*b)
- 608*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**3/(693*b) - 256*s
in(a + b*x)*cos(2*a + 2*b*x)**5/(693*b) + 151*sin(2*a + 2*b*x)**5*cos(a +
b*x)/(693*b) + 272*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(6
93*b) + 128*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(693*b), Ne(
b, 0)), (x*sin(a)*sin(2*a)**5, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = -\frac{63\sin(11bx+11a)-77\sin(9bx+9a)-495\sin(7bx+7a)+693\sin(5bx+5a)+2310\sin(3bx}{22176b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/22176*(63*sin(11*b*x + 11*a) - 77*sin(9*b*x + 9*a) - 495*sin(7*b*x + 7*
a) + 693*sin(5*b*x + 5*a) + 2310*sin(3*b*x + 3*a) - 6930*sin(b*x + a))/b
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(63\sin(bx+a)^{11}-154\sin(bx+a)^9+99\sin(bx+a)^7\right)}{693b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
32/693*(63*sin(b*x + a)^11 - 154*sin(b*x + a)^9 + 99*sin(b*x + a)^7)/b
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(63\sin(a+bx)^{11}-154\sin(a+bx)^9+99\sin(a+bx)^7\right)}{693b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^5,x)
```

output
```
(32*(99*sin(a + b*x)^7 - 154*sin(a + b*x)^9 + 63*sin(a + b*x)^11))/(693*b)
```

3.4 $\int \sin(a+bx)\sin^4(2a+2bx)\,dx$

3.4.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx=-\frac{16\cos^5(a+bx)}{5b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{9b}$$

output
```
-16/5*cos(b*x+a)^5/b+32/7*cos(b*x+a)^7/b-16/9*cos(b*x+a)^9/b
```

3.4.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = \frac{2\cos^5(a+bx)(-249+220\cos(2(a+bx))-35\cos(4(a+bx)))}{315b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(2*Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(31
5*b)
```

3.4.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{4776}$$

$$16\int \cos^4(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^4\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{16\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{16\int \left(\cos^8(a+bx)-2\cos^6(a+bx)+\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(\frac{1}{9}\cos^9(a+bx)-\frac{2}{7}\cos^7(a+bx)+\frac{1}{5}\cos^5(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*(Cos[a + b*x]^5/5 - (2*Cos[a + b*x]^7)/7 + Cos[a + b*x]^9/9))/b
```

3.4.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.4.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

method	result
default	$-\frac{3\cos(xb+a)}{8b}-\frac{\cos(3xb+3a)}{12b}+\frac{\cos(5xb+5a)}{20b}+\frac{\cos(7xb+7a)}{112b}-\frac{\cos(9xb+9a)}{144b}$
risch	$-\frac{3\cos(xb+a)}{8b}-\frac{\cos(3xb+3a)}{12b}+\frac{\cos(5xb+5a)}{20b}+\frac{\cos(7xb+7a)}{112b}-\frac{\cos(9xb+9a)}{144b}$
parallelrisch	$\frac{\left(-64\tan(xb+a)^{6}-208\tan(xb+a)^{4}-64\tan(xb+a)^{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}+\left(256\tan(xb+a)^{7}+896\tan(xb+a)^{5}-896\tan(xb+a)^{3}-25}{315b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)\left(1+\tan\right.}$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b+1/112*cos(7*
b*x+7*a)/b-1/144*cos(9*b*x+9*a)/b
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\left(35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5\right)}{315\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-16/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(39) = 78$.

Time = 1.99 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.54

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = \begin{cases} -\frac{104\sin(a+bx)\sin^3(2a+2bx)\cos(2a+2bx)}{315b} - \frac{64\sin(a+bx)\sin(2a+2bx)\cos^3(2a+2bx)}{315b} - \frac{107\sin^4(2a+2bx)\cos(a+bx)}{315b} - \frac{16\sin^2(2a+ \\ x\sin(a)\sin^4(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((-104*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(315*b)
- 64*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(315*b) - 107*sin(2
*a + 2*b*x)**4*cos(a + b*x)/(315*b) - 16*sin(2*a + 2*b*x)**2*cos(a + b*x)*
cos(2*a + 2*b*x)**2/(21*b) - 128*cos(a + b*x)*cos(2*a + 2*b*x)**4/(315*b),
 Ne(b, 0)), (x*sin(a)*sin(2*a)**4, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx =$$
$$-\frac{35\cos(9\,bx+9\,a)-45\cos(7\,bx+7\,a)-252\cos(5\,bx+5\,a)+420\cos(3\,bx+3\,a)+1890\cos(bx+a)}{5040\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
-1/5040*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
+ 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx =$$
$$-\frac{35\cos(9\,bx+9\,a)-45\cos(7\,bx+7\,a)-252\cos(5\,bx+5\,a)+420\cos(3\,bx+3\,a)+1890\cos(bx+a)}{5040\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/5040*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
+ 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

3.4.9 Mupad [B] (verification not implemented)

Time = 19.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\left(35\cos(a+b\,x)^9-90\cos(a+b\,x)^7+63\cos(a+b\,x)^5\right)}{315\,b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^4,x)
```

output
```
-(16*(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9))/(315*b)
```

3.5 $\int \sin(a+bx)\sin^3(2a+2bx)\,dx$

3.5.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}$$

output
```
8/5*sin(b*x+a)^5/b-8/7*sin(b*x+a)^7/b
```

3.5.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx=\frac{4(9+5\cos(2(a+bx)))\sin^5(a+bx)}{35b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(4*(9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(35*b)
```

3.5.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^3dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^3\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{8\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{8\int \left(\sin^4(a+bx)-\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8\left(\frac{1}{5}\sin^5(a+bx)-\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*(Sin[a + b*x]^5/5 - Sin[a + b*x]^7/7))/b
```

3.5.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.5.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

method	result
default	$\frac{3\sin(xb+a)}{8b}-\frac{\sin(3xb+3a)}{8b}-\frac{\sin(5xb+5a)}{40b}+\frac{\sin(7xb+7a)}{56b}$
risch	$\frac{3\sin(xb+a)}{8b}-\frac{\sin(3xb+3a)}{8b}-\frac{\sin(5xb+5a)}{40b}+\frac{\sin(7xb+7a)}{56b}$
parallelrisch	$\frac{\left(-16\tan(xb+a)^{5}-40\tan(xb+a)^{3}-16\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}+\left(32\tan(xb+a)^{6}+80\tan(xb+a)^{4}-80\tan(xb+a)^{2}-32\right)\tan(\cdots}{35b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)\left(1+\tan(xb+a)^{2}\right)^{3}}$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b-1/40/b*sin(5*b*x+5*a)+1/56/b*sin(7*b
*x+7*a)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(5\cos\left(bx+a\right)^6-8\cos\left(bx+a\right)^4+\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{35\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
8/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x +
a)/b
```

3.5.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(26) = 52.

Time = 0.78 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.06

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \begin{cases} -\frac{22\sin\left(a+bx\right)\sin^2\left(2a+2bx\right)\cos\left(2a+2bx\right)}{35b} - \frac{16\sin\left(a+bx\right)\cos^3\left(2a+2bx\right)}{35b} + \frac{9\sin^3\left(2a+2bx\right)\cos\left(a+bx\right)}{35b} + \frac{8\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos}{35b} \\ x\sin\left(a\right)\sin^3\left(2a\right) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-22*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(35*b) -
16*sin(a + b*x)*cos(2*a + 2*b*x)**3/(35*b) + 9*sin(2*a + 2*b*x)**3*cos(a +
 b*x)/(35*b) + 8*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b),
 Ne(b, 0)), (x*sin(a)*sin(2*a)**3, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \frac{5\,\sin\left(7\,bx+7\,a\right)-7\,\sin\left(5\,bx+5\,a\right)-35\,\sin\left(3\,bx+3\,a\right)+105\,\sin\left(bx+a\right)}{280\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/280*(5*sin(7*b*x + 7*a) - 7*sin(5*b*x + 5*a) - 35*sin(3*b*x + 3*a) + 105
*sin(b*x + a))/b
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(5\,\sin\left(bx+a\right)^7-7\,\sin\left(bx+a\right)^5\right)}{35\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-8/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(7\sin\left(a+b\,x\right)^5-5\sin\left(a+b\,x\right)^7\right)}{35\,b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^3,x)
```

output
```
(8*(7*sin(a + b*x)^5 - 5*sin(a + b*x)^7))/(35*b)
```

3.6 $\int \sin(a+bx)\sin^2(2a+2bx)\,dx$

3.6.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx=-\frac{4\cos^3(a+bx)}{3b}+\frac{4\cos^5(a+bx)}{5b}$$

output
```
-4/3*cos(b*x+a)^3/b+4/5*cos(b*x+a)^5/b
```

3.6.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx=\frac{2\cos^3(a+bx)(-7+3\cos(2(a+bx)))}{15b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(2*Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(15*b)
```

3.6.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{4776}$$

$$4\int \cos^2(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^2\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{4\int \cos^2(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{4\int \left(\cos^2(a+bx)-\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\left(\frac{1}{3}\cos^3(a+bx)-\frac{1}{5}\cos^5(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*(Cos[a + b*x]^3/3 - Cos[a + b*x]^5/5))/b
```

3.6.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.6.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

method	result	size
default	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(3xb+3a)}{12b}+\frac{\cos(5xb+5a)}{20b}$	41
risch	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(3xb+3a)}{12b}+\frac{\cos(5xb+5a)}{20b}$	41
parallelrisch	$\frac{\frac{4\left(4\tan(xb+a)^4+7\tan(xb+a)^2+4\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{15}+\frac{16\left(\tan(xb+a)^3-\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{15}+\frac{4\tan(xb+a)^2}{15}}{b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan(xb+a)^2\right)^2}$	104
norman	$\frac{-\frac{16}{15b}-\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)}{15b}+\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)^3}{15b}-\frac{16\tan(xb+a)^4}{15b}-\frac{28\tan(xb+a)^2}{15b}-\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan(xb+a)^2}{15b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan(xb+a)^2\right)^2}$	127

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = \frac{4\left(3\cos(bx+a)^5 - 5\cos(bx+a)^3\right)}{15\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
4/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(26) = 52.

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.97

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{4\sin(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{15b} - \frac{7\sin^2(2a+2bx)\cos(a+bx)}{15b} - \frac{8\cos(a+bx)\cos^2(2a+2bx)}{15b} & \text{for } b \neq 0 \\ x\sin(a)\sin^2(2a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((-4*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(15*b) - 7*si
n(2*a + 2*b*x)**2*cos(a + b*x)/(15*b) - 8*cos(a + b*x)*cos(2*a + 2*b*x)**2
/(15*b), Ne(b, 0)), (x*sin(a)*sin(2*a)**2, True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = \frac{3\cos(5bx+5a)-5\cos(3bx+3a)-30\cos(bx+a)}{60b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/60*(3*cos(5*b*x + 5*a) - 5*cos(3*b*x + 3*a) - 30*cos(b*x + a))/b
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = \frac{3\cos(5bx+5a)-5\cos(3bx+3a)-30\cos(bx+a)}{60b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/60*(3*cos(5*b*x + 5*a) - 5*cos(3*b*x + 3*a) - 30*cos(b*x + a))/b
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(5\cos(a+b\,x)^3-3\cos(a+b\,x)^5\right)}{15\,b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^2,x)
```

output
```
-(4*(5*cos(a + b*x)^3 - 3*cos(a + b*x)^5))/(15*b)
```

3.7 $\int \sin(a+bx)\sin(2a+2bx)\,dx$

3.7.1 Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \sin(a+bx)\sin(2a+2bx)\,dx=\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

output
```
1/2*sin(b*x+a)/b-1/6*sin(3*b*x+3*a)/b
```

3.7.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int \sin(a+bx)\sin(2a+2bx)\,dx=\frac{2\sin^3(a+bx)}{3b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
(2*Sin[a + b*x]^3)/(3*b)
```

3.7.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)dx$$

$$\Big\downarrow \text{4770}$$

$$\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
Sin[a + b*x]/(2*b) - Sin[3*a + 3*b*x]/(6*b)
```

3.7.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{-\sin(3xb+3a)+3\sin(xb+a)}{6b}$	26
default	$\frac{\sin(xb+a)}{2b}-\frac{\sin(3xb+3a)}{6b}$	27
risch	$\frac{\sin(xb+a)}{2b}-\frac{\sin(3xb+3a)}{6b}$	27
norman	$\frac{-\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{3b}+\frac{2\tan(xb+a)}{3b}+\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)^2}{3b}-\frac{2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan(xb+a)}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan(xb+a)^2\right)}$	99

input
```
int(sin(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-sin(3*b*x+3*a)+3*sin(b*x+a))/b
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = -\frac{2\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \begin{cases} -\frac{2\sin\left(a+bx\right)\cos\left(2a+2bx\right)}{3b}+\frac{\sin\left(2a+2bx\right)\cos\left(a+bx\right)}{3b} & \text{for } b\neq 0 \\ x\sin\left(a\right)\sin\left(2a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-2*sin(a + b*x)*cos(2*a + 2*b*x)/(3*b) + sin(2*a + 2*b*x)*cos(a
 + b*x)/(3*b), Ne(b, 0)), (x*sin(a)*sin(2*a), True))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = -\frac{\sin(3\,bx+3\,a)}{6\,b} + \frac{\sin(bx+a)}{2\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/6*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin(bx+a)^3}{3\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
2/3*sin(b*x + a)^3/b
```

3.7.9 Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \begin{cases} 2\,x\,\left(\cos(a)-\cos(a)^3\right) & \text{if } b=0 \\ \frac{3\,\sin(a+b\,x)-\sin(3\,a+3\,b\,x)}{6\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x),x)
```

output
```
piecewise(b == 0, 2*x*(cos(a) - cos(a)^3), b ~= 0, (3*sin(a + b*x) - sin(3
*a + 3*b*x))/(6*b))
```

3.7. $\int \sin(a+bx)\sin(2a+2bx)\,dx$

3.8 $\int \csc(2a+2bx)\sin(a+bx)\,dx$

3.8.1 Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \csc(2a+2bx)\sin(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}$$

output
```
1/2*arctanh(sin(b*x+a))/b
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \csc(2a+2bx)\sin(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}$$

input
```
Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x],x]
```

output
```
ArcTanh[Sin[a + b*x]]/(2*b)
```

3.8.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{2}\int \sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(a+bx))}{2b}$$

input
```
Int[Csc[2*a + 2*b*x]*Sin[a + b*x],x]
```

output
```
ArcTanh[Sin[a + b*x]]/(2*b)
```

3.8.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.8.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{\ln(\sec(xb+a)+\tan(xb+a))}{2b}$	20
risch	$-\frac{\ln(e^{i(xb+a)}-i)}{2b}+\frac{\ln(i+e^{i(xb+a)})}{2b}$	38

input
```
int(csc(2*b*x+2*a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \frac{\log(\sin(bx+a)+1)-\log(-\sin(bx+a)+1)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a),x)
```

output
```
Timed out
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 8.21

$$\int \csc(2a+2bx)\sin(a+bx)\,dx$$
$$= -\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)}{4b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
+ 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^
2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) +
 sin(a)^2))/b
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \frac{\log(\sin(bx+a)+1) - \log(-\sin(bx+a)+1)}{4b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

3.8.9 Mupad [B] (verification not implemented)

Time = 19.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \frac{\operatorname{atanh}(\sin\left(a+b\,x\right))}{2\,b}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x),x)
```

output
```
atanh(sin(a + b*x))/(2*b)
```

3.9 $\int \csc^2(2a+2bx)\sin(a+bx)\,dx$

3.9.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{4b}+\frac{\sec(a+bx)}{4b}$$

output
```
-1/4*arctanh(cos(b*x+a))/b+1/4*sec(b*x+a)/b
```

3.9.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx=-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{4b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{4b}+\frac{\sec(a+bx)}{4b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
```

output
```
-1/4*Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/(4*b) + Sec[a + b*x]/
(4*b)
```

3.9.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4776, 3042, 3102, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \csc(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sec(a+bx)-\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(a+bx)-\text{arctanh}(\sec(a+bx))}{4b}$$

input
```
Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
```

output
```
(-ArcTanh[Sec[a + b*x]] + Sec[a + b*x])/(4*b)
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.9.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\frac{1}{\cos(xb+a)}+\ln(\csc(xb+a)-\cot(xb+a))}{4b}$	31
risch	$\frac{e^{i(xb+a)}}{2b\left(e^{2i(xb+a)}+1\right)}+\frac{\ln\left(e^{i(xb+a)}-1\right)}{4b}-\frac{\ln\left(e^{i(xb+a)}+1\right)}{4b}$	63

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx$$
$$= -\frac{\cos(bx+a)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-2}{8\,b\cos(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos
(b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**2*sin(b*x+a),x)
```

output
```
Timed out
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(24) = 48$.

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.43

$$\int \csc^2(2a + 2bx) \sin(a + bx)\, dx$$
$$= \frac{4 \cos(2\,bx + 2\,a)\cos(bx + a) - \left(\cos(2\,bx + 2\,a)^2 + \sin(2\,bx + 2\,a)^2 + 2\,\cos(2\,bx + 2\,a) + 1\right)\log\left(\cos(bx\right.}{}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/8*(4*cos(2*b*x + 2*a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a
)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(2*b*x
 + 2*a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x
 + 2*a)^2 + 2*b*cos(2*b*x + 2*a) + b)
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \csc^2(2a + 2bx) \sin(a + bx)\, dx = \frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1} + \log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{8\,b}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
1/8*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + log(-(cos(b*x + a) -
1)/(cos(b*x + a) + 1)))/b
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx = \frac{1}{4\,b\,\cos\left(a+b\,x\right)} - \frac{\mathrm{atanh}(\cos\left(a+b\,x\right))}{4\,b}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^2,x)
```

output
```
1/(4*b*cos(a + b*x)) - atanh(cos(a + b*x))/(4*b)
```

3.10 $\int \csc^3(2a+2bx)\sin(a+bx)\,dx$

3.10.1 Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx=\frac{3\text{arctanh}(\sin(a+bx))}{16b}-\frac{3\csc(a+bx)}{16b}+\frac{\csc(a+bx)\sec^2(a+bx)}{16b}$$

output
```
3/16*arctanh(sin(b*x+a))/b-3/16*csc(b*x+a)/b+1/16*csc(b*x+a)*sec(b*x+a)^2/
b
```

3.10.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx=-\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\sin^2(a+bx)\right)}{8b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b
```

3.10.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 4776, 3042, 3101, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc^2(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^2\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^4(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)-\csc(a+bx)\right)}{8b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\csc(a+bx))-\csc(a+bx))}{8b}$$

input
```
Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*((-3*(ArcTanh[Csc[a + b*x]] - Csc[a + b*x]))/2 + Csc[a + b*x]^3/(2*(1
 - Csc[a + b*x]^2)))/b
```

3.10.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.10.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\frac{1}{2\sin(xb+a)\cos(xb+a)^2}-\frac{3}{2\sin(xb+a)}+\frac{3\ln(\sec(xb+a)+\tan(xb+a))}{2}}{8b}$	51
risch	$-\frac{i\left(3\,e^{5i(xb+a)}+2\,e^{3i(xb+a)}+3\,e^{i(xb+a)}\right)}{8b\left(e^{2i(xb+a)}+1\right)^2\left(e^{2i(xb+a)}-1\right)}+\frac{3\ln\left(i+e^{i(xb+a)}\right)}{16b}-\frac{3\ln\left(e^{i(xb+a)}-i\right)}{16b}$	104

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(1/2/sin(b*x+a)/cos(b*x+a)^2-3/2/sin(b*x+a)+3/2*ln(sec(b*x+a)+tan(b*
x+a)))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx$$
$$=\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)\sin(bx+a)-3\cos(bx+a)^2\log(-\sin(bx+a)+1)\sin(bx+a)-\ldots}{32\,b\cos(bx+a)^2\sin(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/32*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)
^2*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x
+ a)^2*sin(b*x + a))
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**3*sin(b*x+a),x)
```

output
```
Timed out
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(43) = 86.

Time = 0.36 (sec) , antiderivative size = 808, normalized size of antiderivative = 16.49

$$\int \csc^3(2a + 2bx)\sin(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/32*(4*(3*sin(5*b*x + 5*a) + 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b
*x + 6*a) - 12*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 4*
(2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 3*(2*(cos(4*b*x +
 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + cos(6*b*x + 6*a)^2 - 2*(c
os(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + cos(2*b*x + 2
*a)^2 + 2*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b
*x + 6*a)^2 + sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)
^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b
*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x + 5*a)
 + 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) + 12*(cos(4*b*x +
 4*a) - cos(2*b*x + 2*a) - 1)*sin(5*b*x + 5*a) - 4*(2*cos(3*b*x + 3*a) + 3
*cos(b*x + a))*sin(4*b*x + 4*a) - 8*(cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a
) + 8*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + 12*cos(b*x + a)*sin(2*b*x + 2*a)
 - 12*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*cos(6*b*x + 6*a)
^2 + b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 +
b*sin(4*b*x + 4*a)^2 - 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x
 + 2*a)^2 + 2*(b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*
a) - 2*(b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a)...
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \csc^3(2a + 2bx)\sin(a + bx)\,dx$$

$$= -\frac{\frac{2\left(3\sin(bx+a)^2-2\right)}{\sin(bx+a)^3-\sin(bx+a)} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(-\sin\left(bx+a\right)+1\right)}{32\,b}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/32*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(si
n(b*x + a) + 1) + 3*log(-sin(b*x + a) + 1))/b
```

3.10.9 Mupad [B] (verification not implemented)

Time = 19.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx = \frac{3\,\mathrm{atanh}(\sin(a+b\,x))}{16\,b}+\frac{\frac{3\sin(a+b\,x)^2}{16}-\frac{1}{8}}{b\,\left(\sin(a+b\,x)-\sin(a+b\,x)^3\right)}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^3,x)
```

output
```
(3*atanh(sin(a + b*x)))/(16*b) + ((3*sin(a + b*x)^2)/16 - 1/8)/(b*(sin(a +
 b*x) - sin(a + b*x)^3))
```

3.11 $\int \csc^4(2a+2bx)\sin(a+bx)\,dx$

3.11.1 Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx=-\frac{5\text{arctanh}(\cos(a+bx))}{32b}+\frac{5\sec(a+bx)}{32b}+\frac{5\sec^3(a+bx)}{96b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b}$$

output
```
-5/32*arctanh(cos(b*x+a))/b+5/32*sec(b*x+a)/b+5/96*sec(b*x+a)^3/b-1/32*csc
(b*x+a)^2*sec(b*x+a)^3/b
```

3.11.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(66) = 132.

Time = 0.66 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.11

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx=\frac{\csc^8(a+bx)\left(22-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15\cos\right.}{}$$

input
```
Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

output
```
(Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos[4
*(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/
2]] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Log[
Sin[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a + b*
x]*(-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(24*b*(Cs
c[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4776, 3042, 3102, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^3(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^3\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)}{16b}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\left(\operatorname{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)}{16b}$$

input
```
Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

output
```
(Sec[a + b*x]^5/(2*(1 - Sec[a + b*x]^2)) - (5*(ArcTanh[Sec[a + b*x]] - Sec
[a + b*x] - Sec[a + b*x]^3/3))/2)/(16*b)
```

3.11.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.11.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\frac{1}{3\sin(xb+a)^2\cos(xb+a)^3}-\frac{5}{6\sin(xb+a)^2\cos(xb+a)}+\frac{5}{2\cos(xb+a)}+\frac{5\ln(\csc(xb+a)-\cot(xb+a))}{2}}{16b}$	71
risch	$\frac{15\,e^{9i(xb+a)}+20\,e^{7i(xb+a)}-22\,e^{5i(xb+a)}+20\,e^{3i(xb+a)}+15\,e^{i(xb+a)}}{48b\left(e^{2i(xb+a)}+1\right)^3\left(e^{2i(xb+a)}-1\right)^2}-\frac{5\ln\left(e^{i(xb+a)}+1\right)}{32b}+\frac{5\ln\left(e^{i(xb+a)}-1\right)}{32b}$	123

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(1/3/sin(b*x+a)^2/cos(b*x+a)^3-5/6/sin(b*x+a)^2/cos(b*x+a)+5/2/cos(
b*x+a)+5/2*ln(csc(b*x+a)-cot(b*x+a)))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.70

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx$$
$$= \frac{30\cos(bx+a)^4-20\cos(bx+a)^2-15\left(\cos(bx+a)^5-\cos(bx+a)^3\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+15\,(\mathrm{co}}{192\left(b\cos(bx+a)^5-b\cos(bx+a)^3\right)}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/192*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*
x + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^
3)*log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

3.11.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**4*sin(b*x+a),x)
```

output
```
Timed out
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2174 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 2174, normalized size of antiderivative = 32.94

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/192*(4*(15*cos(9*b*x + 9*a) + 20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a)
+ 20*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) + 60*(cos(8*b*
x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)
*cos(9*b*x + 9*a) + 4*(20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) + 20*cos(
3*b*x + 3*a) + 15*cos(b*x + a))*cos(8*b*x + 8*a) - 80*(2*cos(6*b*x + 6*a)
+ 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(7*b*x + 7*a) + 8*(22*cos(
5*b*x + 5*a) - 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 8
8*(2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) - 40*(4*cos
(3*b*x + 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 80*(cos(2*b*x + 2*a) +
1)*cos(3*b*x + 3*a) + 60*cos(2*b*x + 2*a)*cos(b*x + a) - 15*(2*(cos(8*b*x
+ 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*c
os(10*b*x + 10*a) + cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) + 2*cos(4
*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2
+ 4*(2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 4*cos(6
*b*x + 6*a)^2 - 4*(cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*x +
4*a)^2 + cos(2*b*x + 2*a)^2 + 2*(sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2
*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(10*b*x + 10
*a)^2 - 2*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin
(8*b*x + 8*a) + sin(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)^2 - 4...
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(58) = 116$.

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.42

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx =$$

$$-\frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{9(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}-30\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right.}{384\,b}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/384*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1
)/(cos(b*x + a) - 1) + 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + 9*(cos(b*x + a) - 1)^2/(cos(b*x + a)
 + 1)^2 + 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 - 30*log(-(cos(
b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

3.11.9 Mupad [B] (verification not implemented)

Time = 20.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \frac{-\frac{5\cos(a+b\,x)^4}{32}+\frac{5\cos(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\cos\left(a+b\,x\right)^3-\cos\left(a+b\,x\right)^5\right)}-\frac{5\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{32\,b}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^4,x)
```

output
```
((5*cos(a + b*x)^2)/48 - (5*cos(a + b*x)^4)/32 + 1/48)/(b*(cos(a + b*x)^3
- cos(a + b*x)^5)) - (5*atanh(cos(a + b*x)))/(32*b)
```

3.12 $\int \csc^5(2a+2bx)\sin(a+bx)\,dx$

3.12.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx=\frac{35\text{arctanh}(\sin(a+bx))}{256b}-\frac{35\csc(a+bx)}{256b}-\frac{35\csc^3(a+bx)}{768b}+\frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{128b}$$

output
```
35/256*arctanh(sin(b*x+a))/b-35/256*csc(b*x+a)/b-35/768*csc(b*x+a)^3/b+7/2
56*csc(b*x+a)^3*sec(b*x+a)^2/b+1/128*csc(b*x+a)^3*sec(b*x+a)^4/b
```

3.12.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.35

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx=-\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\sin^2(a+bx)\right)}{96b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```

output
```
-1/96*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/b
```

3.12.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4776, 3042, 3101, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{32}\int \csc^4(a+bx)\sec^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^4\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^8(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{32b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^8(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{32b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{7}{4}\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\csc(a+bx))-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

input
```
Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```

output
```
-1/32*(-1/4*Csc[a + b*x]^7/(1 - Csc[a + b*x]^2)^2 + (7*(Csc[a + b*x]^5/(2*
(1 - Csc[a + b*x]^2)) - (5*(ArcTanh[Csc[a + b*x]] - Csc[a + b*x] - Csc[a +
 b*x]^3/3))/2))/4)/b
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.12.4 Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

method	result
default	$\frac{\frac{1}{4\sin(xb+a)^3\cos(xb+a)^4}-\frac{7}{12\sin(xb+a)^3\cos(xb+a)^2}+\frac{35}{24\sin(xb+a)\cos(xb+a)^2}-\frac{35}{8\sin(xb+a)}+\frac{35\ln(\sec(xb+a)+\tan(xb+a))}{8}}{32b}$
risch	$-\frac{i\left(105\,e^{13i(xb+a)}+70\,e^{11i(xb+a)}-329\,e^{9i(xb+a)}-204\,e^{7i(xb+a)}-329\,e^{5i(xb+a)}+70\,e^{3i(xb+a)}+105\,e^{i(xb+a)}\right)}{384b\left(e^{2i(xb+a)}+1\right)^4\left(e^{2i(xb+a)}-1\right)^3}-\frac{35\ln\left(e^{i(xb+a)}-i\right)}{256b}$

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(1/4/sin(b*x+a)^3/cos(b*x+a)^4-7/12/sin(b*x+a)^3/cos(b*x+a)^2+35/24
/sin(b*x+a)/cos(b*x+a)^2-35/8/sin(b*x+a)+35/8*ln(sec(b*x+a)+tan(b*x+a)))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = -\frac{210\cos(bx+a)^6-280\cos(bx+a)^4-105\left(\cos(bx+a)^6-\cos(bx+a)^4\right)\log(\sin(bx+a)+1)\sin(bx}{1536\left(b\cos(bx+a)^6-b\right.}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/1536*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - c
os(b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 -
cos(b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 +
12)/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

3.12.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**5*sin(b*x+a),x)
```

output
```
Timed out
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3088 vs. $2(79) = 158$.

Time = 0.44 (sec) , antiderivative size = 3088, normalized size of antiderivative = 34.70

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/1536*(4*(105*sin(13*b*x + 13*a) + 70*sin(11*b*x + 11*a) - 329*sin(9*b*x
+ 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a)
 + 105*sin(b*x + a))*cos(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) - 3*sin(
10*b*x + 10*a) - 3*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4
*a) - sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) - 32
9*sin(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(
3*b*x + 3*a) + 105*sin(b*x + a))*cos(12*b*x + 12*a) + 280*(3*sin(10*b*x +
10*a) + 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + sin
(2*b*x + 2*a))*cos(11*b*x + 11*a) + 12*(329*sin(9*b*x + 9*a) + 204*sin(7*b
*x + 7*a) + 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))
*cos(10*b*x + 10*a) - 1316*(3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*si
n(4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 12*(204*sin(7*b*x +
7*a) + 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(
8*b*x + 8*a) + 816*(3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x +
2*a))*cos(7*b*x + 7*a) - 84*(47*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) - 1
5*sin(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*sin(4*b*x + 4*a) - sin(2*b*x +
2*a))*cos(5*b*x + 5*a) + 420*(2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b
*x + 4*a) - 105*(2*(cos(12*b*x + 12*a) - 3*cos(10*b*x + 10*a) - 3*cos(8*b*
x + 8*a) + 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)
*cos(14*b*x + 14*a) + cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a) + ...
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = -\frac{\frac{6\left(11\sin(bx+a)^3-13\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+\frac{16\left(9\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-105\log\left(\sin\left(bx+a\right)+1\right)+105\log\left(-\sin\left(bx+a\right)+1\right)}{1536\,b}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/1536*(6*(11*sin(b*x + a)^3 - 13*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 +
16*(9*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 105*log(sin(b*x + a) + 1) + 105
*log(-sin(b*x + a) + 1))/b
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \frac{35\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{256\,b} - \frac{\frac{35\sin(a+b\,x)^6}{256}-\frac{175\sin(a+b\,x)^4}{768}+\frac{7\sin(a+b\,x)^2}{96}+\frac{1}{96}}{b\,\left(\sin\left(a+b\,x\right)^7-2\sin\left(a+b\,x\right)^5+\sin\left(a+b\,x\right)^3\right)}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^5,x)
```

output
```
(35*atanh(sin(a + b*x)))/(256*b) - ((7*sin(a + b*x)^2)/96 - (175*sin(a + b
*x)^4)/768 + (35*sin(a + b*x)^6)/256 + 1/96)/(b*(sin(a + b*x)^3 - 2*sin(a
+ b*x)^5 + sin(a + b*x)^7))
```

3.13 $\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$

3.13.1 Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx=\frac{4\sin^8(a+bx)}{b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{8\sin^{12}(a+bx)}{3b}$$

output
```
4*sin(b*x+a)^8/b-32/5*sin(b*x+a)^10/b+8/3*sin(b*x+a)^12/b
```

3.13.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$$
$$=\frac{-600\cos(2(a+bx))+75\cos(4(a+bx))+100\cos(6(a+bx))-30\cos(8(a+bx))-12\cos(10(a+bx))+}{3840b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] + 100*Cos[6*(a + b*x)] - 30*C
os[8*(a + b*x)] - 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/(3840*b)
```

3.13.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4776, 3042, 3044, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(a+bx)^2\sin(2a+2bx)^5dx$$
$$\Big\downarrow \text{4776}$$
$$32\int \cos^5(a+bx)\sin^7(a+bx)dx$$
$$\Big\downarrow \text{3042}$$
$$32\int \cos(a+bx)^5\sin(a+bx)^7dx$$
$$\Big\downarrow \text{3044}$$
$$\frac{32\int \sin^7(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$
$$\Big\downarrow \text{243}$$
$$\frac{16\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin^2(a+bx)}{b}$$
$$\Big\downarrow \text{49}$$
$$\frac{16\int \left(\sin^{10}(a+bx)-2\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin^2(a+bx)}{b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{16\left(\frac{1}{6}\sin^{12}(a+bx)-\frac{2}{5}\sin^{10}(a+bx)+\frac{1}{4}\sin^8(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(16*(Sin[a + b*x]^8/4 - (2*Sin[a + b*x]^10)/5 + Sin[a + b*x]^12/6))/b
```

3.13.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.13.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

method	result	size
parallelrisch	$\frac{-600\cos(2xb+2a)+462+5\cos(12xb+12a)-12\cos(10xb+10a)-30\cos(8xb+8a)+100\cos(6xb+6a)+75\cos(4xb+4a)}{3840b}$	74
default	$-\frac{5\cos(2xb+2a)}{32b}+\frac{5\cos(4xb+4a)}{256b}+\frac{5\cos(6xb+6a)}{192b}-\frac{\cos(8xb+8a)}{128b}-\frac{\cos(10xb+10a)}{320b}+\frac{\cos(12xb+12a)}{768b}$	86
risch	$-\frac{5\cos(2xb+2a)}{32b}+\frac{5\cos(4xb+4a)}{256b}+\frac{5\cos(6xb+6a)}{192b}-\frac{\cos(8xb+8a)}{128b}-\frac{\cos(10xb+10a)}{320b}+\frac{\cos(12xb+12a)}{768b}$	86

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3840*(-600*cos(2*b*x+2*a)+462+5*cos(12*b*x+12*a)-12*cos(10*b*x+10*a)-30*
cos(8*b*x+8*a)+100*cos(6*b*x+6*a)+75*cos(4*b*x+4*a))/b
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\left(10\cos(bx+a)^{12}-36\cos(bx+a)^{10}+45\cos(bx+a)^{8}-20\cos(bx+a)^{6}\right)}{15\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
4/15*(10*cos(b*x + a)^12 - 36*cos(b*x + a)^10 + 45*cos(b*x + a)^8 - 20*cos
(b*x + a)^6)/b
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(37) = 74.

Time = 11.20 (sec) , antiderivative size = 593, normalized size of antiderivative = 13.48

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = \begin{cases} \frac{5x\sin^2(a+bx)\sin^5(2a+2bx)}{32} + \frac{5x\sin^2(a+bx)\sin^3(2a+2bx)\cos^2(2a+2bx)}{16} + \frac{5x\sin^2(a+bx)\sin(2a+2bx)\cos^4(2a+2bx)}{32} + \frac{5x\sin(a+bx}{} \\ x\sin^2(a)\sin^5(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((5*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**5/32 + 5*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/16 + 5*x*sin(a + b*x)**2*sin(2*a
 + 2*b*x)*cos(2*a + 2*b*x)**4/32 + 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**4*co
s(a + b*x)*cos(2*a + 2*b*x)/16 + 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(
a + b*x)*cos(2*a + 2*b*x)**3/8 + 5*x*sin(a + b*x)*cos(a + b*x)*cos(2*a + 2
*b*x)**5/16 - 5*x*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/32 - 5*x*sin(2*a + 2
*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - 5*x*sin(2*a + 2*b*x)*cos
(a + b*x)**2*cos(2*a + 2*b*x)**4/32 - 65*sin(a + b*x)**2*sin(2*a + 2*b*x)*
*4*cos(2*a + 2*b*x)/(128*b) - 2*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(2*
a + 2*b*x)**3/(3*b) - 167*sin(a + b*x)**2*cos(2*a + 2*b*x)**5/(640*b) + 11
*sin(a + b*x)*sin(2*a + 2*b*x)**5*cos(a + b*x)/(64*b) + sin(a + b*x)*sin(2
*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(4*b) + 19*sin(a + b*x)*si
n(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(192*b) + sin(2*a + 2*b*x)
**4*cos(a + b*x)**2*cos(2*a + 2*b*x)/(128*b) - 11*cos(a + b*x)**2*cos(2*a
+ 2*b*x)**5/(1920*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**5, True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{5\,\cos(12\,bx+12\,a)-12\,\cos(10\,bx+10\,a)-30\,\cos(8\,bx+8\,a)+100\,\cos(6\,bx+6\,a)+75\,\cos(4\,bx+4}{3840\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/3840*(5*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
 + 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) - 600*cos(2*b*x + 2*a))/b
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\left(10\,\sin(bx+a)^{12}-24\,\sin(bx+a)^{10}+15\,\sin(bx+a)^{8}\right)}{15\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
4/15*(10*sin(b*x + a)^12 - 24*sin(b*x + a)^10 + 15*sin(b*x + a)^8)/b
```

3.13.9 Mupad [B] (verification not implemented)

Time = 19.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$$
$$= -\frac{-\frac{8\cos(a+b\,x)^{12}}{3}+\frac{48\cos(a+b\,x)^{10}}{5}-12\cos\left(a+b\,x\right)^8+\frac{16\cos(a+b\,x)^6}{3}}{b}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^5,x)
```

output
```
-((16*cos(a + b*x)^6)/3 - 12*cos(a + b*x)^8 + (48*cos(a + b*x)^10)/5 - (8*
cos(a + b*x)^12)/3)/b
```

3.14 $\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$

3.14.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3x}{16}-\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b}-\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b}-\frac{\sin^5(2a+2bx)}{20b}$$

output
```
3/16*x-3/32*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/16*cos(2*b*x+2*a)*sin(2*b*x+
2*a)^3/b-1/20*sin(2*b*x+2*a)^5/b
```

3.14.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{120bx-20\sin(2(a+bx))-40\sin(4(a+bx))+10\sin(6(a+bx))+5\sin(8(a+bx))-2\sin(10(a+bx))}{640b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
(120*b*x - 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] + 10*Sin[6*(a + b*x)]
 + 5*Sin[8*(a + b*x)] - 2*Sin[10*(a + b*x)])/(640*b)
```

3.14.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 4774, 3042, 3044, 15, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{4774}$$

$$\frac{1}{2}\int \sin^4(2a+2bx)dx-\frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx-\frac{1}{2}\int \cos(2a+2bx)\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx-\frac{\int \sin^4(2a+2bx)d\sin(2a+2bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx-\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin^2(2a+2bx)dx-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)-\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin(2a+2bx)^2dx-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)-\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)-\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)-\frac{\sin^5(2a+2bx)}{20b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
-1/20*Sin[2*a + 2*b*x]^5/b + (-1/8*(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^3)/b
 + (3*(x/2 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b)))/4)/2
```

3.14.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4774
```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Simp[1/2   Int[(g*Sin[c + d*x])^p, x], x] - Simp[1/2   Int[Cos[c + d*
x]*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d
, 0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

3.14.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{120xb-20\sin(2xb+2a)-40\sin(4xb+4a)+10\sin(6xb+6a)+5\sin(8xb+8a)-2\sin(10xb+10a)}{640b}$	66
default	$\frac{3x}{16}-\frac{\sin(2xb+2a)}{32b}-\frac{\sin(4xb+4a)}{16b}+\frac{\sin(6xb+6a)}{64b}+\frac{\sin(8xb+8a)}{128b}-\frac{\sin(10xb+10a)}{320b}$	75
risch	$\frac{3x}{16}-\frac{\sin(2xb+2a)}{32b}-\frac{\sin(4xb+4a)}{16b}+\frac{\sin(6xb+6a)}{64b}+\frac{\sin(8xb+8a)}{128b}-\frac{\sin(10xb+10a)}{320b}$	75

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/640*(120*x*b-20*sin(2*b*x+2*a)-40*sin(4*b*x+4*a)+10*sin(6*b*x+6*a)+5*sin
(8*b*x+8*a)-2*sin(10*b*x+10*a))/b
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\frac{15\,bx-\left(128\,\cos\left(bx+a\right)^9-336\,\cos\left(bx+a\right)^7+248\,\cos\left(bx+a\right)^5-10\,\cos\left(bx+a\right)^3-15\,\cos\left(bx+a\right)\right)}{80\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/80*(15*b*x - (128*cos(b*x + a)^9 - 336*cos(b*x + a)^7 + 248*cos(b*x + a)
^5 - 10*cos(b*x + a)^3 - 15*cos(b*x + a))*sin(b*x + a))/b
```

3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(70) = 140.

Time = 4.81 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.71

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\begin{cases}\frac{3x\sin^2(a+bx)\sin^4(2a+2bx)}{16}+\frac{3x\sin^2(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{8}+\frac{3x\sin^2(a+bx)\cos^4(2a+2bx)}{16}+\frac{3x\sin^4(2a+2bx)\cos^2(a+\\ x\sin^2(a)\sin^4(2a)\end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**4/16 + 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*sin(a + b*x)**2*cos(2*a
+ 2*b*x)**4/16 + 3*x*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/16 + 3*x*sin(2*a
+ 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*cos(a + b*x)**2*co
s(2*a + 2*b*x)**4/16 - 57*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(2*a + 2*
b*x)/(160*b) - 109*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(4
80*b) - sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)/(10*b) - 2*sin(a + b
*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(5*b) - 4*sin(a +
 b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(15*b) + 7*sin(2*a + 2*b*x)**3*cos(
a + b*x)**2*cos(2*a + 2*b*x)/(160*b) + 19*sin(2*a + 2*b*x)*cos(a + b*x)**2
*cos(2*a + 2*b*x)**3/(480*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**4, True))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{120\,bx - 2\,\sin(10\,bx + 10\,a) + 5\,\sin(8\,bx + 8\,a) + 10\,\sin(6\,bx + 6\,a) - 40\,\sin(4\,bx + 4\,a) - 20\,\sin(2\,bx}{640\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/640*(120*b*x - 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) + 10*sin(6*b*x
+ 6*a) - 40*sin(4*b*x + 4*a) - 20*sin(2*b*x + 2*a))/b
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{120\,bx + 120\,a - 2\,\sin(10\,bx + 10\,a) + 5\,\sin(8\,bx + 8\,a) + 10\,\sin(6\,bx + 6\,a) - 40\,\sin(4\,bx + 4\,a) - 20\,\mathrm{s}}{640\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/640*(120*b*x + 120*a - 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) + 10*si
n(6*b*x + 6*a) - 40*sin(4*b*x + 4*a) - 20*sin(2*b*x + 2*a))/b
```

3.14.9 Mupad [B] (verification not implemented)

Time = 21.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.45

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3\,x}{16} - \frac{-\frac{3\tan(a+b\,x)^9}{16}-\frac{7\tan(a+b\,x)^7}{8}+\frac{8\tan(a+b\,x)^5}{5}+\frac{7\tan(a+b\,x)^3}{8}+\frac{3\tan(a+b\,x)}{16}}{b\,\left(\tan\left(a+b\,x\right)^{10}+5\tan\left(a+b\,x\right)^8+10\tan\left(a+b\,x\right)^6+10\tan\left(a+b\,x\right)^4+5\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^4,x)
```

output
```
(3*x)/16 - ((3*tan(a + b*x))/16 + (7*tan(a + b*x)^3)/8 + (8*tan(a + b*x)^5
)/5 - (7*tan(a + b*x)^7)/8 - (3*tan(a + b*x)^9)/16)/(b*(5*tan(a + b*x)^2 +
 10*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^1
0 + 1))
```

3.15 $\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$

3.15.1 Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{4\sin^6(a+bx)}{3b} - \frac{\sin^8(a+bx)}{b}$$

output
```
4/3*sin(b*x+a)^6/b-sin(b*x+a)^8/b
```

3.15.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{-72\cos(2(a+bx)) + 12\cos(4(a+bx)) + 8\cos(6(a+bx)) - 3\cos(8(a+bx))}{384b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8
*(a + b*x)])/(384*b)
```

3.15.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^3dx$$

$$\Big\downarrow \; 4776$$

$$8\int \cos^3(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$8\int \cos(a+bx)^3\sin(a+bx)^5dx$$

$$\Big\downarrow \; 3044$$

$$\frac{8\int \sin^5(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$\frac{8\int \left(\sin^5(a+bx)-\sin^7(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{8\left(\frac{1}{6}\sin^6(a+bx)-\frac{1}{8}\sin^8(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*(Sin[a + b*x]^6/6 - Sin[a + b*x]^8/8))/b
```

3.15.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.15.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

method	result	size
parallelrisch	$\frac{-72\cos(2xb+2a)-3\cos(8xb+8a)+8\cos(6xb+6a)+12\cos(4xb+4a)+71}{384b}$	52
default	$-\frac{3\cos(2xb+2a)}{16b}+\frac{\cos(4xb+4a)}{32b}+\frac{\cos(6xb+6a)}{48b}-\frac{\cos(8xb+8a)}{128b}$	58
risch	$-\frac{3\cos(2xb+2a)}{16b}+\frac{\cos(4xb+4a)}{32b}+\frac{\cos(6xb+6a)}{48b}-\frac{\cos(8xb+8a)}{128b}$	58

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/384*(-72*cos(2*b*x+2*a)-3*cos(8*b*x+8*a)+8*cos(6*b*x+6*a)+12*cos(4*b*x+4
*a)+71)/b
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{3\cos(bx+a)^8 - 8\cos(bx+a)^6 + 6\cos(bx+a)^4}{3b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(22) = 44.

Time = 2.01 (sec) , antiderivative size = 359, normalized size of antiderivative = 12.38

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16} + \frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16} + \frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8} + \frac{3x\,\mathrm{s}}{} \\ x\sin^2(a)\sin^3(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 + 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 + 3*x*sin(a + b*x)*sin(2*a + 2*b
*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 + 3*x*sin(a + b*x)*cos(a + b*x)*cos
(2*a + 2*b*x)**3/8 - 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 - 3*x*sin(
2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - sin(a + b*x)**2*sin(
2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(2*b) - 31*sin(a + b*x)**2*cos(2*a + 2*b*
x)**3/(96*b) + 3*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16*b) + si
n(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b) - cos(a
 + b*x)**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**3
, True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{3\cos(8bx+8a)-8\cos(6bx+6a)-12\cos(4bx+4a)+72\cos(2bx+2a)}{384b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
-1/384*(3*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) + 72
*cos(2*b*x + 2*a))/b
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{3\sin(bx+a)^8-4\sin(bx+a)^6}{3b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-1/3*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

3.15.9 Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{\cos(a+bx)^4\left(\cos(a+bx)^4-\frac{8\cos(a+bx)^2}{3}+2\right)}{b}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^3,x)
```

output
```
-(cos(a + b*x)^4*(cos(a + b*x)^4 - (8*cos(a + b*x)^2)/3 + 2))/b
```

3.16 $\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx$

3.16.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx=\frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}-\frac{\sin^3(2a+2bx)}{12b}$$

output
```
1/4*x-1/8*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/12*sin(2*b*x+2*a)^3/b
```

3.16.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx=\frac{12bx-3\sin(2(a+bx))-3\sin(4(a+bx))+\sin(6(a+bx))}{48b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
(12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(48*
b)
```

3.16.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4774, 3042, 3044, 15, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{4774}$$

$$\frac{1}{2}\int \sin^2(2a+2bx)dx-\frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx-\frac{1}{2}\int \cos(2a+2bx)\sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx-\frac{\int \sin^2(2a+2bx)d\sin(2a+2bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx-\frac{\sin^3(2a+2bx)}{12b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)}{12b}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{x}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)}{12b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
-1/12*Sin[2*a + 2*b*x]^3/b + (x/2 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4
*b))/2
```

3.16.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4774
```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Simp[1/2   Int[(g*Sin[c + d*x])^p, x], x] - Simp[1/2   Int[Cos[c + d*
x]*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d
, 0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$\frac{12xb-3\sin(2xb+2a)-3\sin(4xb+4a)+\sin(6xb+6a)}{48b}$	42
default	$\frac{x}{4}-\frac{\sin(2xb+2a)}{16b}-\frac{\sin(4xb+4a)}{16b}+\frac{\sin(6xb+6a)}{48b}$	47
risch	$\frac{x}{4}-\frac{\sin(2xb+2a)}{16b}-\frac{\sin(4xb+4a)}{16b}+\frac{\sin(6xb+6a)}{48b}$	47

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(12*x*b-3*sin(2*b*x+2*a)-3*sin(4*b*x+4*a)+sin(6*b*x+6*a))/b
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{3\,bx+\left(8\,\cos\left(bx+a\right)^5-14\,\cos\left(bx+a\right)^3+3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{12\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(
b*x + a))/b
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(41) = 82.

Time = 0.83 (sec) , antiderivative size = 231, normalized size of antiderivative = 4.71

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \begin{cases} \frac{x\sin^2(a+bx)\sin^2(2a+2bx)}{4}+\frac{x\sin^2(a+bx)\cos^2(2a+2bx)}{4}+\frac{x\sin^2(2a+2bx)\cos^2(a+bx)}{4}+\frac{x\cos^2(a+bx)\cos^2(2a+2bx)}{4}-\frac{7\sin^2(a+b}{} \\ x\sin^2(a)\sin^2(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos
(2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b
*x)**2*cos(2*a + 2*b*x)**2/4 - 7*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a
+ 2*b*x)/(24*b) - sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) - si
n(a + b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) + sin(2*a + 2*b*x)*cos(a
 + b*x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**2, T
rue))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{12\,bx + \sin\left(6\,bx + 6\,a\right) - 3\,\sin\left(4\,bx + 4\,a\right) - 3\,\sin\left(2\,bx + 2\,a\right)}{48\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/48*(12*b*x + sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))
/b
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{12\,bx + 12\,a + \sin\left(6\,bx + 6\,a\right) - 3\,\sin\left(4\,bx + 4\,a\right) - 3\,\sin\left(2\,bx + 2\,a\right)}{48\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/48*(12*b*x + 12*a + sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 3*sin(2*b*x
+ 2*a))/b
```

3.16.9 Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{x}{4} - \frac{\frac{\sin(2\,a+2\,b\,x)}{16} + \frac{\sin(4\,a+4\,b\,x)}{16} - \frac{\sin(6\,a+6\,b\,x)}{48}}{b}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^2,x)
```

output
```
x/4 - (sin(2*a + 2*b*x)/16 + sin(4*a + 4*b*x)/16 - sin(6*a + 6*b*x)/48)/b
```

3.17 $\int \sin^2(a+bx)\sin(2a+2bx)\,dx$

3.17.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\sin^4(a+bx)}{2b}$$

output
```
1/2*sin(b*x+a)^4/b
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\sin^4(a+bx)}{2b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
Sin[a + b*x]^4/(2*b)
```

3.17.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4776, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)dx$$

$$\Big\downarrow \text{4776}$$

$$2\int \cos(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{2\int \sin^3(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^4(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
Sin[a + b*x]^4/(2*b)
```

3.17.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

method	result
default	$-\frac{\cos(2xb+2a)}{4b}+\frac{\cos(4xb+4a)}{16b}$
risch	$-\frac{\cos(2xb+2a)}{4b}+\frac{\cos(4xb+4a)}{16b}$
parallelrisch	$\frac{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}\tan(xb+a)xb+\left(2\tan(xb+a)^{2}xb-2xb-2\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}+\left(6\tan(xb+a)xb+4\tan(xb+a)^{2}-4\right)\tan(}{2b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{2}\left(1+\tan(xb+a)^{2}\right)}$
norman	$\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}\tan(xb+a)^{2}+\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)}{b}-x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}-\frac{x\tan(xb+a)}{2}-x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)^{2}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(2*b*x+2*a)/b+1/16*cos(4*b*x+4*a)/b
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\cos(bx+a)^4 - 2\cos(bx+a)^2}{2b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(10) = 20.

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 8.73

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx$$
$$= \begin{cases} \frac{x\sin^2(a+bx)\sin(2a+2bx)}{4} + \frac{x\sin(a+bx)\cos(a+bx)\cos(2a+2bx)}{2} - \frac{x\sin(2a+2bx)\cos^2(a+bx)}{4} - \frac{\sin^2(a+bx)\cos(2a+2bx)}{2b} + \frac{\sin(a+ \\ x\sin^2(a)\sin(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a),x)
```

output
```
Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 + x*sin(a + b*x)*cos(a + b
*x)*cos(2*a + 2*b*x)/2 - x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - sin(a + b*
x)**2*cos(2*a + 2*b*x)/(2*b) + sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)/
(4*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a), True))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\cos(4\,bx+4\,a)-4\,\cos(2\,bx+2\,a)}{16\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/16*(cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a))/b
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\sin(bx+a)^4}{2\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
1/2*sin(b*x + a)^4/b
```

3.17.9 Mupad [B] (verification not implemented)

Time = 19.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\sin(a+b\,x)^4}{2\,b}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x),x)
```

output
```
sin(a + b*x)^4/(2*b)
```

3.18 $\int \csc(2a+2bx)\sin^2(a+bx)\,dx$

3.18.1 Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{2b}$$

output
```
-1/2*ln(cos(b*x+a))/b
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{2b}$$

input
```
Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/2*Log[Cos[a + b*x]]/b
```

3.18.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4776, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \ 4776$$

$$\frac{1}{2}\int \tan(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \tan(a+bx)dx$$

$$\Big\downarrow \ 3956$$

$$-\frac{\log(\cos(a+bx))}{2b}$$

input
```
Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/2*Log[Cos[a + b*x]]/b
```

3.18.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.18.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\ln(\cos(xb+a))}{2b}$	13
risch	$\frac{ix}{2}+\frac{ia}{b}-\frac{\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{2b}$	30

input
```
int(csc(2*b*x+2*a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cos(b*x+a))/b
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\log\left(-\cos\left(bx+a\right)\right)}{2\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*log(-cos(b*x + a))/b
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.93

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\log\left(\cos\left(2\,bx\right)^2 + 2\,\cos\left(2\,bx\right)\cos\left(2\,a\right) + \cos\left(2\,a\right)^2 + \sin\left(2\,bx\right)^2 - 2\,\sin\left(2\,bx\right)\sin\left(2\,a\right) + \sin\left(2\,a\right)^2\right)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2
- 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)/b
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\log\left(-\sin\left(bx+a\right)^2+1\right)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*log(-sin(b*x + a)^2 + 1)/b
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\ln\left(\cos\left(a+b\,x\right)\right)}{2\,b}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x),x)
```

output
```
-log(cos(a + b*x))/(2*b)
```

3.19 $\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx$

3.19.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx=\frac{\tan(a+bx)}{4b}$$

output
```
1/4*tan(b*x+a)/b
```

3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx=\frac{\tan(a+bx)}{4b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]
```

output
```
Tan[a + b*x]/(4*b)
```

3.19.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{ 4776}$$

$$\frac{1}{4}\int \sec^2(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}\int \csc\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\int 1d(-\tan(a+bx))}{4b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\tan(a+bx)}{4b}$$

input
```
Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]
```

output
```
Tan[a + b*x]/(4*b)
```

3.19.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.19.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\tan(xb+a)}{4b}$	12
risch	$\frac{i}{2b\left(\mathrm{e}^{2i(xb+a)}+1\right)}$	20

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(b*x+a)/b
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\sin(bx+a)}{4\,b\cos(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*sin(b*x + a)/(b*cos(b*x + a))
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\sin(2\,bx+2\,a)}{2\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2+2\,b\cos(2\,bx+2\,a)+b\right)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*co
s(2*b*x + 2*a) + b)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(bx+a)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*tan(b*x + a)/b
```

3.19.9 Mupad [B] (verification not implemented)

Time = 20.46 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(a+b\,x)}{4\,b}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^2,x)
```

output
```
tan(a + b*x)/(4*b)
```

3.20 $\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx$

3.20.1 Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx=\frac{\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}$$

output
```
1/8*ln(tan(b*x+a))/b+1/16*tan(b*x+a)^2/b
```

3.20.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx=\frac{1}{8}\left(-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}\right)$$

input
```
Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(-(Log[Cos[a + b*x]]/b) + Log[Sin[a + b*x]]/b + Sec[a + b*x]^2/(2*b))/8
```

3.20.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int(\cot(a+bx)+\tan(a+bx))d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\tan^2(a+bx)+\log(\tan(a+bx))}{8b}$$

input
```
Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(Log[Tan[a + b*x]] + Tan[a + b*x]^2/2)/(8*b)
```

3.20.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.20.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result
default	$\frac{\frac{1}{2\cos(xb+a)^2}+\ln(\tan(xb+a))}{8b}$
risch	$\frac{e^{2i(xb+a)}}{4b\left(e^{2i(xb+a)}+1\right)^2}-\frac{\ln\left(e^{2i(xb+a)}+1\right)}{8b}+\frac{\ln\left(e^{2i(xb+a)}-1\right)}{8b}$
parallelrisch	$\frac{-\csc(xb+a)^2\left(\sec(xb+a)^2-2\right)\cos(2xb+2a)-2\csc(xb+a)\sec(xb+a)\sin(2xb+2a)+16\ln\left(\tan(xb+a)^{\frac{1}{4}}\right)+\csc(xb+a)^2\left(\sec(xb+\right.}{32b}$

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\cos(bx+a)^2\log\left(\cos(bx+a)^2\right)-\cos(bx+a)^2\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{16\,b\cos(bx+a)^2}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*
x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

3.20.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 641, normalized size of antiderivative = 21.37

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx = \frac{4\cos(4bx+4a)\cos(2bx+2a)+8\cos(2bx+2a)^2-\left(2\left(2\cos(2bx+2a)+1\right)\cos(4bx+4a)+\cos(\right.}{}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + 8*cos(2*b*x + 2*a)^2 - (2*(2*c
os(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x +
 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin
(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*
cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)
 + (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*c
os(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos
(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (2*
(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b
*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4
*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*
cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b
*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*
sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*
b*x + 2*a) + b)*cos(4*b*x + 4*a) + 4*b*cos(2*b*x + 2*a) + b)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2-1} + \log\left(-\sin(bx+a)^2+1\right) - 2\log\left(\left|\sin(bx+a)\right|\right)}{16\,b}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/16*(1/(sin(b*x + a)^2 - 1) + log(-sin(b*x + a)^2 + 1) - 2*log(abs(sin(b
*x + a))))/b
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx = \frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{16}-\frac{\ln(\cos(a+b\,x))}{8}+\frac{1}{16\cos(a+b\,x)^2}}{b}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^3,x)
```

output
```
(log(sin(a + b*x)^2)/16 - log(cos(a + b*x))/8 + 1/(16*cos(a + b*x)^2))/b
```

3.21 $\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx$

3.21.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\cot(a+bx)}{16b}+\frac{\tan(a+bx)}{8b}+\frac{\tan^3(a+bx)}{48b}$$

output
```
-1/16*cot(b*x+a)/b+1/8*tan(b*x+a)/b+1/48*tan(b*x+a)^3/b
```

3.21.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\cot(a+bx)}{16b}+\frac{5\tan(a+bx)}{48b}+\frac{\sec^2(a+bx)\tan(a+bx)}{48b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

output
```
-1/16*Cot[a + b*x]/b + (5*Tan[a + b*x])/(48*b) + (Sec[a + b*x]^2*Tan[a + b
*x])/(48*b)
```

3.21.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^2(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^2\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^2(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^2(a+bx)+\tan^2(a+bx)+2\right)d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\tan^3(a+bx)+2\tan(a+bx)-\cot(a+bx)}{16b}$$

input
```
Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

output
```
(-Cot[a + b*x] + 2*Tan[a + b*x] + Tan[a + b*x]^3/3)/(16*b)
```

3.21.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.21.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.83 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
risch	$-\frac{i\left(2\,{\mathrm e}^{2 i \left(x b +a \right)}+1\right)}{3 b \left({\mathrm e}^{2 i \left(x b +a \right)}+1\right)^{3} \left({\mathrm e}^{2 i \left(x b +a \right)}-1\right)}$	46
default	$\frac{\frac{1}{3 \sin \left(x b +a \right) \cos \left(x b +a \right)^{3}}+\frac{4}{3 \sin \left(x b +a \right) \cos \left(x b +a \right)}-\frac{8 \cot \left(x b +a \right)}{3}}{16 b}$	51

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*(2*exp(2*I*(b*x+a))+1)/b/(exp(2*I*(b*x+a))+1)^3/(exp(2*I*(b*x+a))-1
)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = -\frac{8\cos(bx+a)^4-4\cos(bx+a)^2-1}{48\,b\cos(bx+a)^3\sin(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/48*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x
+ a))
```

3.21.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**2,x)
```

output
```
Timed out
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(36) = 72.

Time = 0.23 (sec) , antiderivative size = 308, normalized size of antiderivative = 7.33

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx =$$

$$-\frac{(}{3\left(b\cos(8\,bx+8\,a)^2+4\,b\cos(6\,bx+6\,a)^2+4\,b\cos(2\,bx+2\,a)^2+b\sin(8\,bx+8\,a)^2+4\,b\sin(6\,bx+6}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*((2*cos(2*b*x + 2*a) + 1)*sin(8*b*x + 8*a) + 2*(2*cos(2*b*x + 2*a) +
1)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) - 4*cos(6*b*x +
6*a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*
b*cos(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b
*sin(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(6
*b*x + 6*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*
x + 2*a) + b)*cos(6*b*x + 6*a) + 4*b*cos(2*b*x + 2*a) + 4*(b*sin(6*b*x + 6
*a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(bx+a)^3 - \frac{3}{\tan(bx+a)} + 6\,\tan(bx+a)}{48\,b}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/48*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

3.21.9 Mupad [B] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(a+b\,x)^4 + 6\tan(a+b\,x)^2 - 3}{48\,b\tan(a+b\,x)}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^4,x)
```

output
```
(6*tan(a + b*x)^2 + tan(a + b*x)^4 - 3)/(48*b*tan(a + b*x))
```

3.22 $\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$

3.22.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{3\tan^2(a+bx)}{64b}+\frac{\tan^4(a+bx)}{128b}$$

output
```
-1/64*cot(b*x+a)^2/b+3/32*ln(tan(b*x+a))/b+3/64*tan(b*x+a)^2/b+1/128*tan(b
*x+a)^4/b
```

3.22.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = -\frac{2\csc^2(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-4\sec^2(a+bx)-\sec^4(a+bx)}{128b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

output
```
-1/128*(2*Csc[a + b*x]^2 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 4
*Sec[a + b*x]^2 - Sec[a + b*x]^4)/b
```

3.22.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \ 4776$$

$$\frac{1}{32}\int \csc^3(a+bx)\sec^5(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{32}\int \csc(a+bx)^3\sec(a+bx)^5dx$$

$$\Big\downarrow \ 3100$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{32b}$$

$$\Big\downarrow \ 243$$

$$\frac{\int \cot^2(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(\cot^2(a+bx)+3\cot(a+bx)+\tan^2(a+bx)+3\right)d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{2}\tan^4(a+bx)+3\tan^2(a+bx)-\cot(a+bx)+3\log\left(\tan^2(a+bx)\right)}{64b}$$

input
```
Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

output
```
(-Cot[a + b*x] + 3*Log[Tan[a + b*x]^2] + 3*Tan[a + b*x]^2 + Tan[a + b*x]^4
/2)/(64*b)
```

3.22.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.22.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{1}{4 \sin(xb+a)^{2} \cos(xb+a)^{4}}+\frac{3}{4 \sin(xb+a)^{2} \cos(xb+a)^{2}}-\frac{3}{2 \sin(xb+a)^{2}}+3 \ln(\tan(xb+a))}{32 b}$
risch	$\frac{3 \,\mathrm{e}^{10 i (xb+a)}+6 \,\mathrm{e}^{8 i (xb+a)}-2 \,\mathrm{e}^{6 i (xb+a)}+6 \,\mathrm{e}^{4 i (xb+a)}+3 \,\mathrm{e}^{2 i (xb+a)}}{16 b \left(\mathrm{e}^{2 i (xb+a)}+1\right)^{4} \left(\mathrm{e}^{2 i (xb+a)}-1\right)^{2}}-\frac{3 \ln\left(\mathrm{e}^{2 i (xb+a)}+1\right)}{32 b}+\frac{3 \ln\left(\mathrm{e}^{2 i (xb+a)}-1\right)}{32 b}$
parallelrisch	$\left(\left(-3 \sec(xb+a)^{4}-10 \sec(xb+a)^{2}+48\right) \csc(xb+a)^{4}-32 \csc(xb+a)^{2} \cot(xb+a)^{2}\right) \cos(2xb+2a)+\left(\left(-2 \sec(xb+a)^{3}-32 \sec(xb+\right.\right.$

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(1/4/sin(b*x+a)^2/cos(b*x+a)^4+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/si
n(b*x+a)^2+3*ln(tan(b*x+a)))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(52) = 104.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.87

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$$
$$= \frac{6 \cos\left(bx+a\right)^{4} - 3 \cos\left(bx+a\right)^{2} - 6 \left(\cos\left(bx+a\right)^{6} - \cos\left(bx+a\right)^{4}\right) \log\left(\cos\left(bx+a\right)^{2}\right) + 6 \left(\cos\left(bx+a\right)^{6}\right.}{128 \left(b \cos\left(bx+a\right)^{6} - b \cos\left(bx+a\right)^{4}\right)}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/128*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x +
 a)^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*
cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

3.22.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**2,x)
```

output
```
Timed out
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3164 vs. $2(52) = 104$.

Time = 0.29 (sec) , antiderivative size = 3164, normalized size of antiderivative = 52.73

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/64*(4*(3*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) +
6*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b
*x + 8*a) - 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) + 12*cos(2*b*x + 2*a)
 + 3)*cos(10*b*x + 10*a) + 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a
) + 12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) - 6)*cos(8*b*x + 8*a) - 24*co
s(8*b*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) + 8*cos(2*b*x + 2*a) + 1)*cos(6*
b*x + 6*a) + 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) + 2)*cos(4*b*x
 + 4*a) - 24*cos(4*b*x + 4*a)^2 + 24*cos(2*b*x + 2*a)^2 - 3*(2*(2*cos(10*b
*x + 10*a) - cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 2*
cos(2*b*x + 2*a) + 1)*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 - 4*(cos(8
*b*x + 8*a) + 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) -
 1)*cos(10*b*x + 10*a) + 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) +
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x +
8*a)^2 + 8*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) +
16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(
4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b*
x + 8*a) - 4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x
 + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*si
n(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2...
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$$
$$= -\frac{\frac{6\,\sin(bx+a)^4-9\,\sin(bx+a)^2+2}{\left(\sin(bx+a)^2-1\right)^2\sin(bx+a)^2}+6\,\log\left(-\sin\left(bx+a\right)^2+1\right)-12\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{128\,b}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/128*((6*sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/((sin(b*x + a)^2 - 1)^2*
sin(b*x + a)^2) + 6*log(-sin(b*x + a)^2 + 1) - 12*log(abs(sin(b*x + a))))/
b
```

3.22.9 Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = \frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{64\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{32\,b}$$
$$+\frac{-\frac{3\cos(a+b\,x)^4}{64}+\frac{3\cos(a+b\,x)^2}{128}+\frac{1}{128}}{b\,\left(\cos\left(a+b\,x\right)^4-\cos\left(a+b\,x\right)^6\right)}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^5,x)
```

output
```
(3*log(sin(a + b*x)^2))/(64*b) - (3*log(cos(a + b*x)))/(32*b) + ((3*cos(a
+ b*x)^2)/128 - (3*cos(a + b*x)^4)/64 + 1/128)/(b*(cos(a + b*x)^4 - cos(a
+ b*x)^6))
```

3.23 $\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$

3.23.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx=\frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^{13}(a+bx)}{13b}$$

output
```
32/9*sin(b*x+a)^9/b-64/11*sin(b*x+a)^11/b+32/13*sin(b*x+a)^13/b
```

3.23.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx =\frac{4(505+540\cos(2(a+bx))+99\cos(4(a+bx)))\sin^9(a+bx)}{1287b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*(505 + 540*Cos[2*(a + b*x)] + 99*Cos[4*(a + b*x)])*Sin[a + b*x]^9)/(128
7*b)
```

3.23.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^5dx$$

$$\Big\downarrow \; 4776$$

$$32\int \cos^5(a+bx)\sin^8(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$32\int \cos(a+bx)^5\sin(a+bx)^8dx$$

$$\Big\downarrow \; 3044$$

$$\frac{32\int \sin^8(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$\frac{32\int \left(\sin^{12}(a+bx)-2\sin^{10}(a+bx)+\sin^8(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{32\left(\frac{1}{13}\sin^{13}(a+bx)-\frac{2}{11}\sin^{11}(a+bx)+\frac{1}{9}\sin^9(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*(Sin[a + b*x]^9/9 - (2*Sin[a + b*x]^11)/11 + Sin[a + b*x]^13/13))/b
```

3.23.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.

Time = 9.74 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

method	result
default	$\frac{5\sin(xb+a)}{32b} - \frac{25\sin(3xb+3a)}{384b} - \frac{\sin(5xb+5a)}{128b} + \frac{\sin(7xb+7a)}{64b} - \frac{\sin(9xb+9a)}{576b} - \frac{3\sin(11xb+11a)}{1408b} + \frac{\sin(13xb+13a)}{1664b}$
risch	$\frac{5\sin(xb+a)}{32b} - \frac{25\sin(3xb+3a)}{384b} - \frac{\sin(5xb+5a)}{128b} + \frac{\sin(7xb+7a)}{64b} - \frac{\sin(9xb+9a)}{576b} - \frac{3\sin(11xb+11a)}{1408b} + \frac{\sin(13xb+13a)}{1664b}$
parallelrisch	$\left(512\tan(xb+a)^7+1216\tan(xb+a)^5+512\tan(xb+a)^3\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6+\left(-3072\tan(xb+a)^8-4992\tan(xb+a)^6+4992\tan(xb+$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
5/32*sin(b*x+a)/b-25/384*sin(3*b*x+3*a)/b-1/128/b*sin(5*b*x+5*a)+1/64/b*si
n(7*b*x+7*a)-1/576/b*sin(9*b*x+9*a)-3/1408/b*sin(11*b*x+11*a)+1/1664/b*sin
(13*b*x+13*a)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{32\left(99\cos\left(bx+a\right)^{12}-360\cos\left(bx+a\right)^{10}+458\cos\left(bx+a\right)^{8}-212\cos\left(bx+a\right)^{6}+3\cos\left(bx+a\right)^{4}+4\cos\right.}{1287\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
32/1287*(99*cos(b*x + a)^12 - 360*cos(b*x + a)^10 + 458*cos(b*x + a)^8 - 2
12*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 8)*sin(b*x + a)/
b
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(39) = 78$.

Time = 25.24 (sec) , antiderivative size = 447, normalized size of antiderivative = 9.72

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{1366\sin^3(a+bx)\sin^4(2a+2bx)\cos(2a+2bx)}{3003b} - \frac{4960\sin^3(a+bx)\sin^2(2a+2bx)\cos^3(2a+2bx)}{9009b} - \frac{256\sin^3(a+bx)\cos^5(2a+2bx)}{1287b} - \frac{271}{} \\ x\sin^3(a)\sin^5(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-1366*sin(a + b*x)**3*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)/(300
3*b) - 4960*sin(a + b*x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**3/(9009*
b) - 256*sin(a + b*x)**3*cos(2*a + 2*b*x)**5/(1287*b) - 271*sin(a + b*x)**
2*sin(2*a + 2*b*x)**5*cos(a + b*x)/(3003*b) - 48*sin(a + b*x)**2*sin(2*a +
 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(143*b) - 640*sin(a + b*x)**2*
sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(3003*b) - 1388*sin(a +
b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)**2*cos(2*a + 2*b*x)/(3003*b) - 2944*
sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**3/(3003
*b) - 512*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**5/(1001*b) + 2234
*sin(2*a + 2*b*x)**5*cos(a + b*x)**3/(9009*b) + 4544*sin(2*a + 2*b*x)**3*c
os(a + b*x)**3*cos(2*a + 2*b*x)**2/(9009*b) + 256*sin(2*a + 2*b*x)*cos(a +
 b*x)**3*cos(2*a + 2*b*x)**4/(1001*b), Ne(b, 0)), (x*sin(a)**3*sin(2*a)**5
, True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{99\sin(13\,bx+13\,a)-351\sin(11\,bx+11\,a)-286\sin(9\,bx+9\,a)+2574\sin(7\,bx+7\,a)-1287\sin(5\,b}{164736\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/164736*(99*sin(13*b*x + 13*a) - 351*sin(11*b*x + 11*a) - 286*sin(9*b*x +
 9*a) + 2574*sin(7*b*x + 7*a) - 1287*sin(5*b*x + 5*a) - 10725*sin(3*b*x +
3*a) + 25740*sin(b*x + a))/b
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{32\left(99\sin(bx+a)^{13}-234\sin(bx+a)^{11}+143\sin(bx+a)^{9}\right)}{1287\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
32/1287*(99*sin(b*x + a)^13 - 234*sin(b*x + a)^11 + 143*sin(b*x + a)^9)/b
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(99\sin\left(a+b\,x\right)^{13}-234\sin\left(a+b\,x\right)^{11}+143\sin\left(a+b\,x\right)^{9}\right)}{1287\,b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^5,x)
```

output
```
(32*(143*sin(a + b*x)^9 - 234*sin(a + b*x)^11 + 99*sin(a + b*x)^13))/(1287
*b)
```

3.24 $\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$

3.24.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\cos^5(a+bx)}{5b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{3b}+\frac{16\cos^{11}(a+bx)}{11b}$$

output
```
-16/5*cos(b*x+a)^5/b+48/7*cos(b*x+a)^7/b-16/3*cos(b*x+a)^9/b+16/11*cos(b*x
+a)^11/b
```

3.24.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{\cos^5(a+bx)(-3042+3335\cos(2(a+bx))-910\cos(4(a+bx))+105\cos(6(a+bx)))}{2310b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(Cos[a + b*x]^5*(-3042 + 3335*Cos[2*(a + b*x)] - 910*Cos[4*(a + b*x)] + 10
5*Cos[6*(a + b*x)]))/(2310*b)
```

3.24.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{4776}$$

$$16\int \cos^4(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^4\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{16\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{16\int \left(-\cos^{10}(a+bx)+3\cos^8(a+bx)-3\cos^6(a+bx)+\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(-\frac{1}{11}\cos^{11}(a+bx)+\frac{1}{3}\cos^9(a+bx)-\frac{3}{7}\cos^7(a+bx)+\frac{1}{5}\cos^5(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*(Cos[a + b*x]^5/5 - (3*Cos[a + b*x]^7)/7 + Cos[a + b*x]^9/3 - Cos[a +
 b*x]^11/11))/b
```

3.24.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.24.4 Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\frac{-32768-16170\cos(xb+a)-165\cos(7xb+7a)+2541\cos(5xb+5a)+105\cos(11xb+11a)-385\cos(9xb+9a)-2310\cos(3xb+3a)}{73920b}$
default	$-\frac{7\cos(xb+a)}{32b}-\frac{\cos(3xb+3a)}{32b}+\frac{11\cos(5xb+5a)}{320b}-\frac{\cos(7xb+7a)}{448b}-\frac{\cos(9xb+9a)}{192b}+\frac{\cos(11xb+11a)}{704b}$
risch	$-\frac{7\cos(xb+a)}{32b}-\frac{\cos(3xb+3a)}{32b}+\frac{11\cos(5xb+5a)}{320b}-\frac{\cos(7xb+7a)}{448b}-\frac{\cos(9xb+9a)}{192b}+\frac{\cos(11xb+11a)}{704b}$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/73920*(-32768-16170*cos(b*x+a)-165*cos(7*b*x+7*a)+2541*cos(5*b*x+5*a)+10
5*cos(11*b*x+11*a)-385*cos(9*b*x+9*a)-2310*cos(3*b*x+3*a))/b
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\left(105\cos(bx+a)^{11}-385\cos(bx+a)^9+495\cos(bx+a)^7-231\cos(bx+a)^5\right)}{1155\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
16/1155*(105*cos(b*x + a)^11 - 385*cos(b*x + a)^9 + 495*cos(b*x + a)^7 - 2
31*cos(b*x + a)^5)/b
```

3.24.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(53) = 106.

Time = 11.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 6.00

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = \begin{cases} -\frac{472\sin^3(a+bx)\sin^3(2a+2bx)\cos(2a+2bx)}{1155b} - \frac{64\sin^3(a+bx)\sin(2a+2bx)\cos^3(2a+2bx)}{231b} - \frac{211\sin^2(a+bx)\sin^4(2a+2bx)\cos(a+bx)}{1155b} - \\ x\sin^3(a)\sin^4(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((-472*sin(a + b*x)**3*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(1155
*b) - 64*sin(a + b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(231*b) - 21
1*sin(a + b*x)**2*sin(2*a + 2*b*x)**4*cos(a + b*x)/(1155*b) - 304*sin(a +
b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(385*b) - 128
*sin(a + b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**4/(231*b) + 272*sin(a + b*
x)*sin(2*a + 2*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)/(1155*b) + 256*sin
(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**3/(1155*b) -
46*sin(2*a + 2*b*x)**4*cos(a + b*x)**3/(165*b) - 192*sin(2*a + 2*b*x)**2*c
os(a + b*x)**3*cos(2*a + 2*b*x)**2/(385*b) - 256*cos(a + b*x)**3*cos(2*a +
 2*b*x)**4/(1155*b), Ne(b, 0)), (x*sin(a)**3*sin(2*a)**4, True))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{105\,\cos(11\,bx+11\,a)-385\,\cos(9\,bx+9\,a)-165\,\cos(7\,bx+7\,a)+2541\,\cos(5\,bx+5\,a)-2310\,\cos(3\,b}{73920\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/73920*(105*cos(11*b*x + 11*a) - 385*cos(9*b*x + 9*a) - 165*cos(7*b*x + 7
*a) + 2541*cos(5*b*x + 5*a) - 2310*cos(3*b*x + 3*a) - 16170*cos(b*x + a))/
b
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{105\,\cos(11\,bx+11\,a)-385\,\cos(9\,bx+9\,a)-165\,\cos(7\,bx+7\,a)+2541\,\cos(5\,bx+5\,a)-2310\,\cos(3\,b}{73920\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/73920*(105*cos(11*b*x + 11*a) - 385*cos(9*b*x + 9*a) - 165*cos(7*b*x + 7
*a) + 2541*cos(5*b*x + 5*a) - 2310*cos(3*b*x + 3*a) - 16170*cos(b*x + a))/
b
```

3.24.9 Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{-\frac{16\cos(a+b\,x)^{11}}{11}+\frac{16\cos(a+b\,x)^9}{3}-\frac{48\cos(a+b\,x)^7}{7}+\frac{16\cos(a+b\,x)^5}{5}}{b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^4,x)
```

output
```
-((16*cos(a + b*x)^5)/5 - (48*cos(a + b*x)^7)/7 + (16*cos(a + b*x)^9)/3 -
(16*cos(a + b*x)^11)/11)/b
```

3.24. $\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$

3.25 $\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx$

3.25.1 Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}$$

output
```
8/7*sin(b*x+a)^7/b-8/9*sin(b*x+a)^9/b
```

3.25.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx=\frac{4(11+7\cos(2(a+bx)))\sin^7(a+bx)}{63b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(4*(11 + 7*Cos[2*(a + b*x)])*Sin[a + b*x]^7)/(63*b)
```

3.25.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^3dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^3\sin(a+bx)^6dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{8\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{8\int \left(\sin^6(a+bx)-\sin^8(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8\left(\frac{1}{7}\sin^7(a+bx)-\frac{1}{9}\sin^9(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*(Sin[a + b*x]^7/7 - Sin[a + b*x]^9/9))/b
```

3.25.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.25.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

method	result
default	$\frac{3\sin(xb+a)}{16b}-\frac{\sin(3xb+3a)}{12b}+\frac{3\sin(7xb+7a)}{224b}-\frac{\sin(9xb+9a)}{288b}$
risch	$\frac{3\sin(xb+a)}{16b}-\frac{\sin(3xb+3a)}{12b}+\frac{3\sin(7xb+7a)}{224b}-\frac{\sin(9xb+9a)}{288b}$
parallelrisch	$\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{6}\tan(xb+a)^{3}+\left(-96\tan(xb+a)^{4}+96\tan(xb+a)^{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{5}+\left(192\tan(xb+a)^{5}-720\tan(xb+a)^{3}+192\tan\right.}{}$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/16*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b+3/224/b*sin(7*b*x+7*a)-1/288/b*sin
(9*b*x+9*a)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(7\cos(bx+a)^8-19\cos(bx+a)^6+15\cos(bx+a)^4-\cos(bx+a)^2-2\right)\sin(bx+a)}{63\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-8/63*(7*cos(b*x + a)^8 - 19*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - cos(b*x
+ a)^2 - 2)*sin(b*x + a)/b
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(26) = 52.

Time = 4.83 (sec) , antiderivative size = 284, normalized size of antiderivative = 9.16

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = \begin{cases} -\frac{46\sin^3(a+bx)\sin^2(2a+2bx)\cos(2a+2bx)}{105b} - \frac{16\sin^3(a+bx)\cos^3(2a+2bx)}{63b} - \frac{13\sin^2(a+bx)\sin^3(2a+2bx)\cos(a+bx)}{105b} - \frac{8\sin^2(a+bx)}{} \\ x\sin^3(a)\sin^3(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-46*sin(a + b*x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(105*b
) - 16*sin(a + b*x)**3*cos(2*a + 2*b*x)**3/(63*b) - 13*sin(a + b*x)**2*sin
(2*a + 2*b*x)**3*cos(a + b*x)/(105*b) - 8*sin(a + b*x)**2*sin(2*a + 2*b*x)
*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) - 4*sin(a + b*x)*sin(2*a + 2*b*x)
**2*cos(a + b*x)**2*cos(2*a + 2*b*x)/(7*b) - 64*sin(a + b*x)*cos(a + b*x)*
*2*cos(2*a + 2*b*x)**3/(105*b) + 94*sin(2*a + 2*b*x)**3*cos(a + b*x)**3/(3
15*b) + 32*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), N
e(b, 0)), (x*sin(a)**3*sin(2*a)**3, True))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{7\sin(9bx+9a)-27\sin(7bx+7a)+168\sin(3bx+3a)-378\sin(bx+a)}{2016b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
-1/2016*(7*sin(9*b*x + 9*a) - 27*sin(7*b*x + 7*a) + 168*sin(3*b*x + 3*a) -
 378*sin(b*x + a))/b
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(7\sin(bx+a)^9-9\sin(bx+a)^7\right)}{63b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-8/63*(7*sin(b*x + a)^9 - 9*sin(b*x + a)^7)/b
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(9\sin(a+bx)^7-7\sin(a+bx)^9\right)}{63b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^3,x)
```

output
```
(8*(9*sin(a + b*x)^7 - 7*sin(a + b*x)^9))/(63*b)
```

3.26 $\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$

3.26.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos^3(a+bx)}{3b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^7(a+bx)}{7b}$$

output
```
-4/3*cos(b*x+a)^3/b+8/5*cos(b*x+a)^5/b-4/7*cos(b*x+a)^7/b
```

3.26.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{\cos^3(a+bx)(-157+108\cos(2(a+bx))-15\cos(4(a+bx)))}{210b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(210*
b)
```

3.26.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{4776}$$

$$4\int \cos^2(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^2\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{4\int \cos^2(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{4\int \left(\cos^6(a+bx)-2\cos^4(a+bx)+\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4(\frac{1}{7}\cos^7(a+bx)-\frac{2}{5}\cos^5(a+bx)+\frac{1}{3}\cos^3(a+bx))}{b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*(Cos[a + b*x]^3/3 - (2*Cos[a + b*x]^5)/5 + Cos[a + b*x]^7/7))/b
```

3.26.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.26.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

method	result
default	$-\frac{5\cos(xb+a)}{16b}-\frac{\cos(3xb+3a)}{48b}+\frac{3\cos(5xb+5a)}{80b}-\frac{\cos(7xb+7a)}{112b}$
risch	$-\frac{5\cos(xb+a)}{16b}-\frac{\cos(3xb+3a)}{48b}+\frac{3\cos(5xb+5a)}{80b}-\frac{\cos(7xb+7a)}{112b}$
parallelrisch	$\frac{24+\left(64\tan(xb+a)^4+176\tan(xb+a)^2+88\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6-192\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5\tan(xb+a)^3+\left(384\tan(xb+a)^4-48\tan(xb+a)^2+\right.}{105b\left(1+\tan\left(\frac{a}{2}\right.\right.}$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/16*cos(b*x+a)/b-1/48*cos(3*b*x+3*a)/b+3/80*cos(5*b*x+5*a)/b-1/112*cos(7
*b*x+7*a)/b
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(15\cos(bx+a)^7 - 42\cos(bx+a)^5 + 35\cos(bx+a)^3\right)}{105\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-4/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(39) = 78.

Time = 2.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.39

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{12\sin^3(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{35b} - \frac{11\sin^2(a+bx)\sin^2(2a+2bx)\cos(a+bx)}{35b} - \frac{24\sin^2(a+bx)\cos(a+bx)\cos^2(2a+2bx)}{35b} + \frac{8\sin \\ x\sin^3(a)\sin^2(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((-12*sin(a + b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(35*b) -
11*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)/(35*b) - 24*sin(a + b*
x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) + 8*sin(a + b*x)*sin(2*a + 2
*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)/(35*b) - 38*sin(2*a + 2*b*x)**2*cos
(a + b*x)**3/(105*b) - 32*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), Ne(
b, 0)), (x*sin(a)**3*sin(2*a)**2, True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{15\cos(7\,bx+7\,a)-63\cos(5\,bx+5\,a)+35\cos(3\,bx+3\,a)+525\cos(bx+a)}{1680\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/1680*(15*cos(7*b*x + 7*a) - 63*cos(5*b*x + 5*a) + 35*cos(3*b*x + 3*a) +
 525*cos(b*x + a))/b
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{15\cos(7\,bx+7\,a)-63\cos(5\,bx+5\,a)+35\cos(3\,bx+3\,a)+525\cos(bx+a)}{1680\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/1680*(15*cos(7*b*x + 7*a) - 63*cos(5*b*x + 5*a) + 35*cos(3*b*x + 3*a) +
 525*cos(b*x + a))/b
```

3.26.9 Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(15\cos\left(a+b\,x\right)^7-42\cos\left(a+b\,x\right)^5+35\cos\left(a+b\,x\right)^3\right)}{105\,b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

output
```
-(4*(35*cos(a + b*x)^3 - 42*cos(a + b*x)^5 + 15*cos(a + b*x)^7))/(105*b)
```

3.27 $\int \sin^3(a+bx)\sin(2a+2bx)\,dx$

3.27.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx=\frac{2\sin^5(a+bx)}{5b}$$

output
```
2/5*sin(b*x+a)^5/b
```

3.27.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx=\frac{2\sin^5(a+bx)}{5b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(2*Sin[a + b*x]^5)/(5*b)
```

3.27.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4776, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)dx$$

$$\Big\downarrow \text{4776}$$

$$2\int \cos(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{2\int \sin^4(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\sin^5(a+bx)}{5b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(2*Sin[a + b*x]^5)/(5*b)
```

3.27.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.27.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.

Time = 0.76 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

method	result	size
default	$\frac{\sin(xb+a)}{4b} - \frac{\sin(3xb+3a)}{8b} + \frac{\sin(5xb+5a)}{40b}$	41
risch	$\frac{\sin(xb+a)}{4b} - \frac{\sin(3xb+3a)}{8b} + \frac{\sin(5xb+5a)}{40b}$	41
parallelrisch	$\frac{-\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{5}+\frac{8\tan(xb+a)}{5}+\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)^2}{5}-8\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan(xb+a)}{b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^3\left(1+\tan(xb+a)^2\right)}$	89

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b+1/40/b*sin(5*b*x+5*a)
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = \frac{2\left(\cos\left(bx+a\right)^4 - 2\,\cos\left(bx+a\right)^2 + 1\right)\sin\left(bx+a\right)}{5\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

3.27.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(12) = 24$.

Time = 0.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 7.80

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{2\sin^3(a+bx)\cos(2a+2bx)}{5b} - \frac{\sin^2(a+bx)\sin(2a+2bx)\cos(a+bx)}{5b} - \frac{4\sin(a+bx)\cos^2(a+bx)\cos(2a+2bx)}{5b} + \frac{2\sin(2a+2bx)\cos^3(a+b}{5b} \\ x\sin^3(a)\sin(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-2*sin(a + b*x)**3*cos(2*a + 2*b*x)/(5*b) - sin(a + b*x)**2*sin
(2*a + 2*b*x)*cos(a + b*x)/(5*b) - 4*sin(a + b*x)*cos(a + b*x)**2*cos(2*a
+ 2*b*x)/(5*b) + 2*sin(2*a + 2*b*x)*cos(a + b*x)**3/(5*b), Ne(b, 0)), (x*s
in(a)**3*sin(2*a), True))
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = \frac{\sin(5\,bx+5\,a) - 5\,\sin(3\,bx+3\,a) + 10\,\sin(bx+a)}{40\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/40*(sin(5*b*x + 5*a) - 5*sin(3*b*x + 3*a) + 10*sin(b*x + a))/b
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin(bx+a)^5}{5\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
2/5*sin(b*x + a)^5/b
```

3.27.9 Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = \frac{2\sin(a+b\,x)^5}{5\,b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x),x)
```

output
```
(2*sin(a + b*x)^5)/(5*b)
```

3.28 $\int \csc(2a+2bx)\sin^3(a+bx)\,dx$

3.28.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{\sin(a+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+a))/b-1/2*sin(b*x+a)/b
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx=\frac{1}{2}\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}\right)$$

input
```
Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

output
```
(ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b)/2
```

3.28.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{ 4776}$$

$$\frac{1}{2}\int \sin(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \sin(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{ 3072}$$

$$\frac{\int \frac{\sin^2(a+bx)}{1-\sin^2(a+bx)}d\sin(a+bx)}{2b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\int \frac{1}{1-\sin^2(a+bx)}d\sin(a+bx)-\sin(a+bx)}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}(\sin(a+bx))-\sin(a+bx)}{2b}$$

input
```
Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

output
```
(ArcTanh[Sin[a + b*x]] - Sin[a + b*x])/(2*b)
```

3.28.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.28.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{-\sin(xb+a)+\ln(\sec(xb+a)+\tan(xb+a))}{2b}$	29
risch	$\frac{i\,\mathrm{e}^{i(xb+a)}}{4b}-\frac{i\,\mathrm{e}^{-i(xb+a)}}{4b}-\frac{\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{2b}+\frac{\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{2b}$	68

input
```
int(csc(2*b*x+2*a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \frac{\log(\sin(bx+a)+1)-\log(-\sin(bx+a)+1)-2\sin(bx+a)}{4b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(24) = 48.

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.43

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)+2\sin(bx+a)}{4b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
 + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)
^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)) + 2*sin(b*x + a))/b
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)-2\,\sin\left(bx+a\right)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\frac{\sin(a+b\,x)}{2}-\frac{\mathrm{atanh}(\sin(a+b\,x))}{2}}{b}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x),x)
```

output
```
-(sin(a + b*x)/2 - atanh(sin(a + b*x))/2)/b
```

3.29 $\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx$

3.29.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx=\frac{\sec(a+bx)}{4b}$$

output
```
1/4*sec(b*x+a)/b
```

3.29.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx=\frac{\sec(a+bx)}{4b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

output
```
Sec[a + b*x]/(4*b)
```

3.29.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \sec(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sec(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int 1d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sec(a+bx)}{4b}$$

input
```
Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

output
```
Sec[a + b*x]/(4*b)
```

3.29.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.29.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{1}{4\cos(xb+a)b}$	14
risch	$\frac{e^{i(xb+a)}}{2b\left(e^{2i(xb+a)}+1\right)}$	28

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/cos(b*x+a)/b
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{4\,b\cos(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4/(b*cos(b*x + a))
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

3.29.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 6.38

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \frac{\cos(2\,bx+2\,a)\cos(bx+a)+\sin(2\,bx+2\,a)\sin(bx+a)+\cos(bx+a)}{2\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2+2\,b\cos(2\,bx+2\,a)+b\right)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b
*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*
a) + b)
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.15

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{2\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1))
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{4\,b\,\cos\left(a+b\,x\right)}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^2,x)
```

output
```
1/(4*b*cos(a + b*x))
```

3.30 $\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx$

3.30.1 Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{16b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}$$

output
```
1/16*arctanh(sin(b*x+a))/b+1/16*sec(b*x+a)*tan(b*x+a)/b
```

3.30.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx=\frac{1}{8}\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}\right)$$

input
```
Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))/8
```

3.30.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \sec(a+bx)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{8}\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)$$

input
```
Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))/8
```

3.30.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.30.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\frac{\sec(xb+a)\tan(xb+a)}{2}+\frac{\ln(\sec(xb+a)+\tan(xb+a))}{2}}{8b}$	37
risch	$-\frac{i\left(e^{3i(xb+a)}-e^{i(xb+a)}\right)}{8b\left(e^{2i(xb+a)}+1\right)^{2}}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{16b}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{16b}$	78

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(1/2*sec(b*x+a)*tan(b*x+a)+1/2*ln(sec(b*x+a)+tan(b*x+a)))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx = \frac{\cos(bx+a)^2\log(\sin(bx+a)+1)-\cos(bx+a)^2\log(-\sin(bx+a)+1)+2\sin(bx+a)}{32\,b\cos(bx+a)^2}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
 a) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(30) = 60.

Time = 0.34 (sec) , antiderivative size = 480, normalized size of antiderivative = 14.12

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx = \frac{4(\sin(3bx+3a)-\sin(bx+a))\cos(4bx+4a)-(2(2\cos(2bx+2a)+1)\cos(4bx+4a)+\cos(4bx-\cdots}{\cdots}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/32*(4*(sin(3*b*x + 3*a) - sin(b*x + a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b
*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^
2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*c
os(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)
^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a
)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(cos(3*b*x + 3*a) - cos(b*x
 + a))*sin(4*b*x + 4*a) + 4*(2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) - 8*
cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + 8*cos(b*x + a)*sin(2*b*x + 2*a) - 8*co
s(2*b*x + 2*a)*sin(b*x + a) - 4*sin(b*x + a))/(b*cos(4*b*x + 4*a)^2 + 4*b*
cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*b*x + 2*a) + b)*cos(4*b*x
+ 4*a) + 4*b*cos(2*b*x + 2*a) + b)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} - \log(\sin(bx+a)+1) + \log(-\sin(bx+a)+1)}{32\,b}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/32*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(-
sin(b*x + a) + 1))/b
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx = \frac{\mathrm{atanh}(\sin(a+b\,x))}{16\,b} - \frac{\sin(a+b\,x)}{16\,b\,\left(\sin(a+b\,x)^2-1\right)}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^3,x)
```

output
```
atanh(sin(a + b*x))/(16*b) - sin(a + b*x)/(16*b*(sin(a + b*x)^2 - 1))
```

3.30. $\int \csc^3(2a + 2bx)\sin^3(a + bx)\, dx$

3.31 $\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx$

3.31.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{16b}+\frac{\sec(a+bx)}{16b}+\frac{\sec^3(a+bx)}{48b}$$

output
```
-1/16*arctanh(cos(b*x+a))/b+1/16*sec(b*x+a)/b+1/48*sec(b*x+a)^3/b
```

3.31.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx=\frac{1}{16}\left(-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}\right)$$

input
```
Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

output
```
(-(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + S
ec[a + b*x]^3/(3*b))/16
```

3.31.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 3102, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\text{arctanh}(\sec(a+bx))+\frac{1}{3}\sec^3(a+bx)+\sec(a+bx)}{16b}$$

input
```
Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

output
```
(-ArcTanh[Sec[a + b*x]] + Sec[a + b*x] + Sec[a + b*x]^3/3)/(16*b)
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.31.4 Maple [A] (verified)

Time = 6.73 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\frac{1}{3\cos(xb+a)^3}+\frac{1}{\cos(xb+a)}+\ln(\csc(xb+a)-\cot(xb+a))}{16b}$	41
risch	$\frac{3\,e^{5i(xb+a)}+10\,e^{3i(xb+a)}+3\,e^{i(xb+a)}}{24b\left(e^{2i(xb+a)}+1\right)^3}+\frac{\ln\left(e^{i(xb+a)}-1\right)}{16b}-\frac{\ln\left(e^{i(xb+a)}+1\right)}{16b}$	88

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(1/3/cos(b*x+a)^3+1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = -\frac{3\,\cos(bx+a)^3\log\left(\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}\right)-3\,\cos(bx+a)^3\log\left(-\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}\right)-6\,\cos(bx+a)^2 -}{96\,b\cos(bx+a)^3}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/96*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log
(-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

3.31.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**3,x)
```

output
```
Timed out
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 987, normalized size of antiderivative = 22.95

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/96*(4*(3*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*
b*x + 6*a) + 12*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x +
5*a) + 12*(10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 40*(3*
cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) + 36*cos(2*b*x + 2*a)*cos(b*x + a)
- 3*(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + co
s(6*b*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 9*cos(4*b
*x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a
))*sin(6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 + 18*sin(4
*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) +
 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x
)*sin(a) + sin(a)^2) + 3*(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*
cos(6*b*x + 6*a) + cos(6*b*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b
*x + 4*a) + 9*cos(4*b*x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4
*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*
x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 +
 6*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b*x + 5*a) + 10*sin
(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x + 6*a) + 36*(sin(4*b*x + 4*a) +
sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 12*(10*sin(3*b*x + 3*a) + 3*sin(b*x +
 a))*sin(4*b*x + 4*a) + 120*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 36*sin(...
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(37) = 74.

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = \frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}+3\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{96\,b}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/96*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 +
3*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = \frac{\frac{\cos(a+b\,x)^2}{16}+\frac{1}{48}}{b\cos\left(a+b\,x\right)^3} - \frac{\operatorname{atanh}(\cos\left(a+b\,x\right))}{16\,b}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^4,x)
```

output
```
(cos(a + b*x)^2/16 + 1/48)/(b*cos(a + b*x)^3) - atanh(cos(a + b*x))/(16*b)
```

3.32 $\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx$

3.32.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \frac{15\text{arctanh}(\sin(a+bx))}{256b} - \frac{15\csc(a+bx)}{256b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc(a+bx)\sec^4(a+bx)}{128b}$$

output
```
15/256*arctanh(sin(b*x+a))/b-15/256*csc(b*x+a)/b+5/256*csc(b*x+a)*sec(b*x+
a)^2/b+1/128*csc(b*x+a)*sec(b*x+a)^4/b
```

3.32.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\sin^2(a+bx)\right)}{32b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
```

output
```
-1/32*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/b
```

3.32.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 4776, 3042, 3101, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{32}\int \csc^2(a+bx)\sec^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^2\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{32b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{32b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{5}{4}\int \frac{\csc^4(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)-\csc(a+bx)\right)\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\csc(a+bx))-\csc(a+bx))\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

input
```
Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
```

output
```
-1/32*(-1/4*Csc[a + b*x]^5/(1 - Csc[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Csc[a
 + b*x]] - Csc[a + b*x]))/2 + Csc[a + b*x]^3/(2*(1 - Csc[a + b*x]^2))))/4)
/b
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.32.4 Maple [A] (verified)

Time = 13.98 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\frac{1}{4\sin(xb+a)\cos(xb+a)^4}+\frac{5}{8\sin(xb+a)\cos(xb+a)^2}-\frac{15}{8\sin(xb+a)}+\frac{15\ln(\sec(xb+a)+\tan(xb+a))}{8}}{32b}$	69
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(xb+a)}+40\,\mathrm{e}^{7i(xb+a)}+18\,\mathrm{e}^{5i(xb+a)}+40\,\mathrm{e}^{3i(xb+a)}+15\,\mathrm{e}^{i(xb+a)}\right)}{128b\left(\mathrm{e}^{2i(xb+a)}+1\right)^4\left(\mathrm{e}^{2i(xb+a)}-1\right)}-\frac{15\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{256b}+\frac{15\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{256b}$	126

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(1/4/sin(b*x+a)/cos(b*x+a)^4+5/8/sin(b*x+a)/cos(b*x+a)^2-15/8/sin(b
*x+a)+15/8*ln(sec(b*x+a)+tan(b*x+a)))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx$$
$$=\frac{15\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a)-15\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a)}{512\,b\cos(bx+a)^4\sin(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/512*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x +
 a)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x
 + a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

3.32.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**3,x)
```

output
```
Timed out
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1805 vs. $2(62) = 124$.

Time = 0.39 (sec) , antiderivative size = 1805, normalized size of antiderivative = 25.79

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/512*(4*(15*sin(9*b*x + 9*a) + 40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a)
+ 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) - 60*(3*sin(8*
b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))
*cos(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a) + 40*sin
(3*b*x + 3*a) + 15*sin(b*x + a))*cos(8*b*x + 8*a) - 160*(2*sin(6*b*x + 6*a
) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(18*sin(
5*b*x + 5*a) + 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(6*b*x + 6*a) + 7
2*(2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(8*sin(3
*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 15*(2*(3*cos(8*b*x + 8*a)
 + 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) - 1)*cos(1
0*b*x + 10*a) + cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x
 + 4*a) - 3*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + 9*cos(8*b*x + 8*a)^2
- 4*(2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + 4*cos
(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*
x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) + 2*sin(6*b*x +
6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(1
0*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x
 + 2*a))*sin(8*b*x + 8*a) + 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) +
 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x
 + 4*a)^2 + 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2...
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\frac{2\left(7\sin(bx+a)^3-9\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+\frac{16}{\sin(bx+a)}-15\log\left(\sin\left(bx+a\right)+1\right)+15\log\left(-\sin\left(bx+a\right)+1\right)}{512\,b}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/512*(2*(7*sin(b*x + a)^3 - 9*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16/
sin(b*x + a) - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

3.32.9 Mupad [B] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \frac{15\,\mathrm{atanh}(\sin(a+b\,x))}{256\,b} - \frac{\frac{15\sin(a+b\,x)^4}{256} - \frac{25\sin(a+b\,x)^2}{256} + \frac{1}{32}}{b\,\left(\sin(a+b\,x)^5 - 2\sin(a+b\,x)^3 + \sin(a+b\,x)\right)}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^5,x)
```

output
```
(15*atanh(sin(a + b*x)))/(256*b) - ((15*sin(a + b*x)^4)/256 - (25*sin(a +
b*x)^2)/256 + 1/32)/(b*(sin(a + b*x) - 2*sin(a + b*x)^3 + sin(a + b*x)^5))
```

3.33 $\int \csc(a+bx)\sin^8(2a+2bx)\,dx$

3.33.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = -\frac{256\cos^9(a+bx)}{9b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{256\cos^{15}(a+bx)}{15b}$$

output
```
-256/9*cos(b*x+a)^9/b+768/11*cos(b*x+a)^11/b-768/13*cos(b*x+a)^13/b+256/15
*cos(b*x+a)^15/b
```

3.33.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = -\frac{35\cos(a+bx)}{64b}-\frac{35\cos(3(a+bx))}{192b}+\frac{21\cos(5(a+bx))}{320b}+\frac{3\cos(7(a+bx))}{64b}-\frac{7\cos(9(a+bx))}{576b}-\frac{7\cos(11(a+bx))}{704b}+\frac{\cos(13(a+bx))}{832b}+\frac{\cos(15(a+bx))}{960b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

output
```
(-35*Cos[a + b*x])/(64*b) - (35*Cos[3*(a + b*x)])/(192*b) + (21*Cos[5*(a +
 b*x)])/(320*b) + (3*Cos[7*(a + b*x)])/(64*b) - (7*Cos[9*(a + b*x)])/(576*
b) - (7*Cos[11*(a + b*x)])/(704*b) + Cos[13*(a + b*x)]/(832*b) + Cos[15*(a
 + b*x)]/(960*b)
```

3.33.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^8}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$256\int \cos^8(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$256\int \cos(a+bx)^8\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{256\int \cos^8(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{256\int \left(-\cos^{14}(a+bx)+3\cos^{12}(a+bx)-3\cos^{10}(a+bx)+\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{256\left(-\frac{1}{15}\cos^{15}(a+bx)+\frac{3}{13}\cos^{13}(a+bx)-\frac{3}{11}\cos^{11}(a+bx)+\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

output
```
(-256*(Cos[a + b*x]^9/9 - (3*Cos[a + b*x]^11)/11 + (3*Cos[a + b*x]^13)/13
- Cos[a + b*x]^15/15))/b
```

3.33.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.33.4 Maple [A] (verified)

Time = 19.92 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
default	$\frac{\frac{256 \cos(xb+a)^{15}}{15}-\frac{768 \cos(xb+a)^{13}}{13}+\frac{768 \cos(xb+a)^{11}}{11}-\frac{256 \cos(xb+a)^{9}}{9}}{b}$
risch	$-\frac{35 \cos(xb+a)}{64b}+\frac{\cos(15xb+15a)}{960b}+\frac{\cos(13xb+13a)}{832b}-\frac{7 \cos(11xb+11a)}{704b}-\frac{7 \cos(9xb+9a)}{576b}+\frac{3 \cos(7xb+7a)}{64b}+\frac{21 \cos(5xb+5}{320b}$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

output
```
256/b*(1/15*cos(b*x+a)^15-3/13*cos(b*x+a)^13+3/11*cos(b*x+a)^11-1/9*cos(b*
x+a)^9)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = \frac{256\left(429\cos\left(bx+a\right)^{15}-1485\cos\left(bx+a\right)^{13}+1755\cos\left(bx+a\right)^{11}-715\cos\left(bx+a\right)^{9}\right)}{6435\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

output
```
256/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^1
1 - 715*cos(b*x + a)^9)/b
```

3.33.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**8,x)
```

output
```
Timed out
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = \frac{429\cos\left(15\,bx+15\,a\right)+495\cos\left(13\,bx+13\,a\right)-4095\cos\left(11\,bx+11\,a\right)-5005\cos\left(9\,bx+9\,a\right)+19305}{411840\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

output
```
1/411840*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*
x + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b
*x + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(53) = 106$.

Time = 0.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.43

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = -\frac{8192\left(\frac{15(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{105(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{455(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{5070(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + \frac{30030(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} + \frac{70070}{(\mathrm{co}}\right.}{6435}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

output
```
-8192/6435*(15*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 105*(cos(b*x + a) -
 1)^2/(cos(b*x + a) + 1)^2 + 455*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3
 + 5070*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 30030*(cos(b*x + a) -
1)^5/(cos(b*x + a) + 1)^5 + 70070*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^
6 + 115830*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 109395*(cos(b*x + a
) - 1)^8/(cos(b*x + a) + 1)^8 + 75075*(cos(b*x + a) - 1)^9/(cos(b*x + a) +
 1)^9 + 27027*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 6435*(cos(b*x
+ a) - 1)^11/(cos(b*x + a) + 1)^11 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x +
a) + 1) - 1)^15)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 20.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = -\frac{-\frac{256\cos(a+b\,x)^{15}}{15} + \frac{768\cos(a+b\,x)^{13}}{13} - \frac{768\cos(a+b\,x)^{11}}{11} + \frac{256\cos(a+b\,x)^9}{9}}{b}$$

input
```
int(sin(2*a + 2*b*x)^8/sin(a + b*x),x)
```

output
```
-((256*cos(a + b*x)^9)/9 - (768*cos(a + b*x)^11)/11 + (768*cos(a + b*x)^13
)/13 - (256*cos(a + b*x)^15)/15)/b
```

3.33. $\int \csc(a+bx)\sin^8(2a+2bx)\,dx$

3.34 $\int \csc(a+bx)\sin^7(2a+2bx)\,dx$

3.34.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = \frac{128\sin^7(a+bx)}{7b} - \frac{128\sin^9(a+bx)}{3b} + \frac{384\sin^{11}(a+bx)}{11b} - \frac{128\sin^{13}(a+bx)}{13b}$$

output
```
128/7*sin(b*x+a)^7/b-128/3*sin(b*x+a)^9/b+384/11*sin(b*x+a)^11/b-128/13*si
n(b*x+a)^13/b
```

3.34.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = \frac{128\sin^7(a+bx)}{7b} - \frac{128\sin^9(a+bx)}{3b} + \frac{384\sin^{11}(a+bx)}{11b} - \frac{128\sin^{13}(a+bx)}{13b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*Sin[a + b*x]^7)/(7*b) - (128*Sin[a + b*x]^9)/(3*b) + (384*Sin[a + b*x
]^11)/(11*b) - (128*Sin[a + b*x]^13)/(13*b)
```

3.34.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^7}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$128\int \cos^7(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$128\int \cos(a+bx)^7\sin(a+bx)^6dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{128\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{128\int \left(-\sin^{12}(a+bx)+3\sin^{10}(a+bx)-3\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{128\left(-\frac{1}{13}\sin^{13}(a+bx)+\frac{3}{11}\sin^{11}(a+bx)-\frac{1}{3}\sin^9(a+bx)+\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*(Sin[a + b*x]^7/7 - Sin[a + b*x]^9/3 + (3*Sin[a + b*x]^11)/11 - Sin[a
 + b*x]^13/13))/b
```

3.34.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.34.4 Maple [A] (verified)

Time = 8.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result	siz
default	$-\frac{128\left(\frac{\sin(xb+a)^{13}}{13}-\frac{3\sin(xb+a)^{11}}{11}+\frac{\sin(xb+a)^{9}}{3}-\frac{\sin(xb+a)^{7}}{7}\right)}{b}$	47
risch	$\frac{5\sin(xb+a)}{8b}-\frac{\sin(13xb+13a)}{416b}-\frac{\sin(11xb+11a)}{352b}+\frac{\sin(9xb+9a)}{48b}+\frac{3\sin(7xb+7a)}{112b}-\frac{3\sin(5xb+5a)}{32b}-\frac{5\sin(3xb+3a)}{32b}$	97

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
-128/b*(1/13*sin(b*x+a)^13-3/11*sin(b*x+a)^11+1/3*sin(b*x+a)^9-1/7*sin(b*x
+a)^7)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx =$$
$$-\frac{128\left(231\cos(bx+a)^{12}-567\cos(bx+a)^{10}+371\cos(bx+a)^{8}-5\cos(bx+a)^{6}-6\cos(bx+a)^{4}-\right.}{3003\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
-128/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8
- 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a
)/b
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**7,x)
```

output
```
Timed out
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx =$$
$$-\frac{231\sin(13\,bx+13\,a)+273\sin(11\,bx+11\,a)-2002\sin(9\,bx+9\,a)-2574\sin(7\,bx+7\,a)+9009\,\mathrm{si}}{96096\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
-1/96096*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x
 + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x
+ 3*a) - 60060*sin(b*x + a))/b
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = -\frac{128\left(231\sin\left(bx+a\right)^{13}-819\sin\left(bx+a\right)^{11}+1001\sin\left(bx+a\right)^{9}-429\sin\left(bx+a\right)^{7}\right)}{3003\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
-128/3003*(231*sin(b*x + a)^13 - 819*sin(b*x + a)^11 + 1001*sin(b*x + a)^9
 - 429*sin(b*x + a)^7)/b
```

3.34.9 Mupad [B] (verification not implemented)

Time = 19.97 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = \frac{-\frac{128\sin(a+b\,x)^{13}}{13}+\frac{384\sin(a+b\,x)^{11}}{11}-\frac{128\sin(a+b\,x)^{9}}{3}+\frac{128\sin(a+b\,x)^{7}}{7}}{b}$$

input
```
int(sin(2*a + 2*b*x)^7/sin(a + b*x),x)
```

output
```
((128*sin(a + b*x)^7)/7 - (128*sin(a + b*x)^9)/3 + (384*sin(a + b*x)^11)/1
1 - (128*sin(a + b*x)^13)/13)/b
```

3.35 $\int \csc(a+bx)\sin^6(2a+2bx)\,dx$

3.35.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx=-\frac{64\cos^7(a+bx)}{7b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^{11}(a+bx)}{11b}$$

output
```
-64/7*cos(b*x+a)^7/b+128/9*cos(b*x+a)^9/b-64/11*cos(b*x+a)^11/b
```

3.35.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx=-\frac{5\cos(a+bx)}{8b}-\frac{5\cos(3(a+bx))}{24b}+\frac{\cos(5(a+bx))}{16b}+\frac{5\cos(7(a+bx))}{112b}-\frac{\cos(9(a+bx))}{144b}-\frac{\cos(11(a+bx))}{176b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(-5*Cos[a + b*x])/(8*b) - (5*Cos[3*(a + b*x)])/(24*b) + Cos[5*(a + b*x)]/(
16*b) + (5*Cos[7*(a + b*x)])/(112*b) - Cos[9*(a + b*x)]/(144*b) - Cos[11*(
a + b*x)]/(176*b)
```

3.35.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2a+2bx)^6}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{ 4776}$$

$$64\int \cos^6(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$64\int \cos(a+bx)^6\sin(a+bx)^5dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{64\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{64\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{64\left(\frac{1}{11}\cos^{11}(a+bx)-\frac{2}{9}\cos^9(a+bx)+\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(-64*(Cos[a + b*x]^7/7 - (2*Cos[a + b*x]^9)/9 + Cos[a + b*x]^11/11))/b
```

3.35.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.35.4 Maple [A] (verified)

Time = 6.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{64\left(\frac{\cos(xb+a)^{11}}{11}-\frac{2\cos(xb+a)^{9}}{9}+\frac{\cos(xb+a)^{7}}{7}\right)}{b}$	37
risch	$-\frac{5\cos(xb+a)}{8b}-\frac{\cos(11xb+11a)}{176b}-\frac{\cos(9xb+9a)}{144b}+\frac{5\cos(7xb+7a)}{112b}+\frac{\cos(5xb+5a)}{16b}-\frac{5\cos(3xb+3a)}{24b}$	83

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
-64/b*(1/11*cos(b*x+a)^11-2/9*cos(b*x+a)^9+1/7*cos(b*x+a)^7)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\left(63\cos\left(bx+a\right)^{11}-154\cos\left(bx+a\right)^{9}+99\cos\left(bx+a\right)^{7}\right)}{693\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
-64/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**6,x)
```

output
```
Timed out
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = -\frac{63\cos\left(11\,bx+11\,a\right)+77\cos\left(9\,bx+9\,a\right)-495\cos\left(7\,bx+7\,a\right)-693\cos\left(5\,bx+5\,a\right)+2310\cos\left(3\,bx\right.}{11088\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
-1/11088*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*
a) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(40) = 80$.

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.43

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx =$$

$$-\frac{1024\left(\frac{11(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{55(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{297(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{1485(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-\frac{2079(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-\frac{2541\,(\mathrm{co}}{(\mathrm{cos}(}\right.}{693\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{11}}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
-1024/693*(11*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 55*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 - 297*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 -
 1485*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 2079*(cos(b*x + a) - 1)^
5/(cos(b*x + a) + 1)^5 - 2541*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 -
1155*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 462*(cos(b*x + a) - 1)^8/
(cos(b*x + a) + 1)^8 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^1
1)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\left(63\cos\left(a+b\,x\right)^{11}-154\cos\left(a+b\,x\right)^9+99\cos\left(a+b\,x\right)^7\right)}{693\,b}$$

input
```
int(sin(2*a + 2*b*x)^6/sin(a + b*x),x)
```

output
```
-(64*(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11))/(693*b
)
```

3.36 $\int \csc(a+bx)\sin^5(2a+2bx)\,dx$

3.36.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\sin^5(a+bx)}{5b} - \frac{64\sin^7(a+bx)}{7b} + \frac{32\sin^9(a+bx)}{9b}$$

output
```
32/5*sin(b*x+a)^5/b-64/7*sin(b*x+a)^7/b+32/9*sin(b*x+a)^9/b
```

3.36.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\sin^5(a+bx)}{5b} - \frac{64\sin^7(a+bx)}{7b} + \frac{32\sin^9(a+bx)}{9b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*Sin[a + b*x]^5)/(5*b) - (64*Sin[a + b*x]^7)/(7*b) + (32*Sin[a + b*x]^9
)/(9*b)
```

3.36.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^5}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{32\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{32\int \left(\sin^8(a+bx)-2\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{32\left(\frac{1}{9}\sin^9(a+bx)-\frac{2}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*(Sin[a + b*x]^5/5 - (2*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/9))/b
```

3.36.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.36.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\frac{32 \sin(xb+a)^9}{9}-\frac{64 \sin(xb+a)^7}{7}+\frac{32 \sin(xb+a)^5}{5}}{b}$	37
risch	$\frac{3 \sin(xb+a)}{4b}+\frac{\sin(9xb+9a)}{72b}+\frac{\sin(7xb+7a)}{56b}-\frac{\sin(5xb+5a)}{10b}-\frac{\sin(3xb+3a)}{6b}$	69

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
32/b*(1/9*sin(b*x+a)^9-2/7*sin(b*x+a)^7+1/5*sin(b*x+a)^5)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(35\cos\left(bx+a\right)^8-50\cos\left(bx+a\right)^6+3\cos\left(bx+a\right)^4+4\cos\left(bx+a\right)^2+8\right)\sin\left(bx+a\right)}{315\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
32/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b
*x + a)^2 + 8)*sin(b*x + a)/b
```

3.36.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{35\sin\left(9\,bx+9\,a\right)+45\sin\left(7\,bx+7\,a\right)-252\sin\left(5\,bx+5\,a\right)-420\sin\left(3\,bx+3\,a\right)+1890\sin\left(bx+a\right)}{2520\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/2520*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
 420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(35\sin\left(bx+a\right)^9-90\sin\left(bx+a\right)^7+63\sin\left(bx+a\right)^5\right)}{315\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
32/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(35\sin\left(a+b\,x\right)^9-90\sin\left(a+b\,x\right)^7+63\sin\left(a+b\,x\right)^5\right)}{315\,b}$$

input
```
int(sin(2*a + 2*b*x)^5/sin(a + b*x),x)
```

output
```
(32*(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9))/(315*b)
```

3.37 $\int \csc(a+bx)\sin^4(2a+2bx)\,dx$

3.37.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\cos^5(a+bx)}{5b}+\frac{16\cos^7(a+bx)}{7b}$$

output
```
-16/5*cos(b*x+a)^5/b+16/7*cos(b*x+a)^7/b
```

3.37.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = -\frac{3\cos(a+bx)}{4b}-\frac{\cos(3(a+bx))}{4b}+\frac{\cos(5(a+bx))}{20b}+\frac{\cos(7(a+bx))}{28b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-3*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(4*b) + Cos[5*(a + b*x)]/(20*b)
 + Cos[7*(a + b*x)]/(28*b)
```

3.37.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^4}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$16\int \cos^4(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^4\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{16\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{16\int \left(\cos^4(a+bx)-\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(\frac{1}{5}\cos^5(a+bx)-\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*(Cos[a + b*x]^5/5 - Cos[a + b*x]^7/7))/b
```

3.37.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.37.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{16\cos(xb+a)^7}{7}-\frac{16\cos(xb+a)^5}{5}}{b}$	27
risch	$-\frac{3\cos(xb+a)}{4b}+\frac{\cos(7xb+7a)}{28b}+\frac{\cos(5xb+5a)}{20b}-\frac{\cos(3xb+3a)}{4b}$	55

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
16/b*(1/7*cos(b*x+a)^7-1/5*cos(b*x+a)^5)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\left(5\cos\left(bx+a\right)^7-7\cos\left(bx+a\right)^5\right)}{35\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
16/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

3.37.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = \frac{5\cos\left(7\,bx+7\,a\right)+7\cos\left(5\,bx+5\,a\right)-35\cos\left(3\,bx+3\,a\right)-105\cos\left(bx+a\right)}{140\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/140*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105
*cos(b*x + a))/b
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.45

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = -\frac{64\left(\frac{7(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{14(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{70(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{35(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{35(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-1\right)}{35\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^7}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-64/35*(7*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 14*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 70*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 35*(
cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 35*(cos(b*x + a) - 1)^5/(cos(b*
x + a) + 1)^5 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^7)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\left(7\cos\left(a+b\,x\right)^5-5\cos\left(a+b\,x\right)^7\right)}{35\,b}$$

input
```
int(sin(2*a + 2*b*x)^4/sin(a + b*x),x)
```

output
```
-(16*(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7))/(35*b)
```

3.38 $\int \csc(a+bx)\sin^3(2a+2bx)\,dx$

3.38.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\sin^3(a+bx)}{3b} - \frac{8\sin^5(a+bx)}{5b}$$

output
```
8/3*sin(b*x+a)^3/b-8/5*sin(b*x+a)^5/b
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\sin^3(a+bx)}{3b} - \frac{8\sin^5(a+bx)}{5b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b)
```

3.38.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^3}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^3\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{8\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{8\int \left(\sin^2(a+bx)-\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8\left(\frac{1}{3}\sin^3(a+bx)-\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*(Sin[a + b*x]^3/3 - Sin[a + b*x]^5/5))/b
```

3.38.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.38.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{8\sin(xb+a)^5}{5}+\frac{8\sin(xb+a)^3}{3}}{b}$	27
risch	$\frac{\sin(xb+a)}{b}-\frac{\sin(5xb+5a)}{10b}-\frac{\sin(3xb+3a)}{6b}$	40

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
8/b*(-1/5*sin(b*x+a)^5+1/3*sin(b*x+a)^3)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(3\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2-2\right)\sin\left(bx+a\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-8/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

3.38.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = -\frac{3\sin\left(5\,bx+5\,a\right)+5\sin\left(3\,bx+3\,a\right)-30\sin\left(bx+a\right)}{30\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
-1/30*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(3\,\sin\left(bx+a\right)^5-5\,\sin\left(bx+a\right)^3\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-8/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(5\sin\left(a+b\,x\right)^3-3\sin\left(a+b\,x\right)^5\right)}{15\,b}$$

input
```
int(sin(2*a + 2*b*x)^3/sin(a + b*x),x)
```

output
```
(8*(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5))/(15*b)
```

3.39 $\int \csc(a+bx)\sin^2(2a+2bx)\,dx$

3.39.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx=-\frac{4\cos^3(a+bx)}{3b}$$

output
```
-4/3*cos(b*x+a)^3/b
```

3.39.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx=-\frac{4\cos^3(a+bx)}{3b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*Cos[a + b*x]^3)/(3*b)
```

3.39.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4776, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^2}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$4\int \cos^2(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^2\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{4\int \cos^2(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{4\cos^3(a+bx)}{3b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*Cos[a + b*x]^3)/(3*b)
```

3.39.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.39.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{4 \cos(xb+a)^{3}}{3b}$	14
risch	$-\frac{\cos(xb+a)}{b}-\frac{\cos(3xb+3a)}{3b}$	27

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3*cos(b*x+a)^3/b
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\,\cos\left(bx+a\right)^3}{3\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-4/3*cos(b*x + a)^3/b
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15030 vs. 2(14) = 28.

Time = 93.15 (sec) , antiderivative size = 104225, normalized size of antiderivative = 6948.33

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**2,x)
```

output
```
16*Piecewise((0, Eq(a, 0) & Eq(b, 0)), (-cos(b*x)**3/(3*b), Eq(a, 0)), (0,
 Eq(b, 0)), (12*log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**6/(3*b*
tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**
8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36*b*
tan(a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*tan(a/2)
**6 + 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)**4 + 54
*b*tan(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*tan(b*x
/2)**6 + 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)**2 +
 12*b*tan(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*x/2)
**2 + 3*b) + 36*log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**4/(3*b*
tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**
8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36*b*
tan(a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*tan(a/2)
**6 + 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)**4 + 54
*b*tan(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*tan(b*x
/2)**6 + 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)**2 +
 12*b*tan(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*x/2)
**2 + 3*b) + 36*log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**2/(3*b*
tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**
8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36...
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = -\frac{\cos(3\,bx+3\,a)+3\,\cos(bx+a)}{3\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/3*(cos(3*b*x + 3*a) + 3*cos(b*x + a))/b
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.47

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = \frac{8\left(\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
8/3*(3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 1)/(b*((cos(b*x + a) -
1)/(cos(b*x + a) + 1) - 1)^3)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos\left(a+b\,x\right)^3}{3\,b}$$

input
```
int(sin(2*a + 2*b*x)^2/sin(a + b*x),x)
```

output
```
-(4*cos(a + b*x)^3)/(3*b)
```

3.40 $\int \csc(a+bx)\sin(2a+2bx)\,dx$

3.40.1 Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \csc(a+bx)\sin(2a+2bx)\,dx=\frac{2\sin(a+bx)}{b}$$

output
```
2*sin(b*x+a)/b
```

3.40.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \csc(a+bx)\sin(2a+2bx)\,dx=2\left(\frac{\cos(bx)\sin(a)}{b}+\frac{\cos(a)\sin(bx)}{b}\right)$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
2*((Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b)
```

3.40.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(2a+2bx)}{\sin(a+bx)}dx$$

$$\Big\downarrow \; 4776$$

$$2\int \cos(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$2\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 3117$$

$$\frac{2\sin(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
(2*Sin[a + b*x])/b
```

3.40.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.40.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{2\sin(xb+a)}{b}$	12
risch	$\frac{2\sin(xb+a)}{b}$	12

input
```
int(csc(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
2*sin(b*x+a)/b
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin\left(bx+a\right)}{b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*sin(b*x + a)/b
```

3.40.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(8) = 16$.

Time = 11.57 (sec) , antiderivative size = 3636, normalized size of antiderivative = 330.55

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
4*Piecewise((x, Eq(a, 0) & Eq(b, 0)), (sin(b*x)/b, Eq(a, 0)), (0, Eq(b, 0)
), (2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**4*
tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/
2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3/(b
*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2
 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(a/2) + tan(b*x/2))*t
an(a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*t
an(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(
tan(a/2) + tan(b*x/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)*
*4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b
) - 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**
4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(
a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**
3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2
)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(a
/2))*tan(a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 +
 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*t
an(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2
)**2 + b) - 2*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*t...
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin\left(bx+a\right)}{b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
2*sin(b*x + a)/b
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \frac{2\sin(bx+a)}{b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
2*sin(b*x + a)/b
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \frac{2\sin(a+b\,x)}{b}$$

input
```
int(sin(2*a + 2*b*x)/sin(a + b*x),x)
```

output
```
(2*sin(a + b*x))/b
```

3.41 $\int \csc(a+bx)\csc(2a+2bx)\,dx$

3.41.1 Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \csc(a+bx)\csc(2a+2bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+a))/b-1/2*csc(b*x+a)/b
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \csc(a+bx)\csc(2a+2bx)\,dx=-\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{2b}$$

input
```
Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
```

3.41.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 4776, 3042, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{2}\int \csc^2(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc(a+bx)^2\sec(a+bx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\csc(a+bx)-\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\csc(a+bx)-\text{arctanh}(\csc(a+bx))}{2b}$$

input
```
Int[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*(-ArcTanh[Csc[a + b*x]] + Csc[a + b*x])/b
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.41.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\frac{1}{\sin(xb+a)}+\ln(\sec(xb+a)+\tan(xb+a))}{2b}$	31
risch	$-\frac{i e^{i(xb+a)}}{b\left(e^{2i(xb+a)}-1\right)}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{2b}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{2b}$	66

input
```
int(csc(b*x+a)*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = \frac{\log(\sin(bx+a)+1)\sin(bx+a) - \log(-\sin(bx+a)+1)\sin(bx+a) - 2}{4\,b\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/4*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
 a) - 2)/(b*sin(b*x + a))
```

3.41.6 Sympy [F]

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = \int \csc(a+bx)\csc(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a),x)
```

output
```
Integral(csc(a + b*x)*csc(2*a + 2*b*x), x)
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(24) = 48.

Time = 0.33 (sec) , antiderivative size = 233, normalized size of antiderivative = 8.32

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = -\frac{\left(\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,bx+2\,a\right)^2-2\,\cos\left(2\,bx+2\,a\right)+1\right)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\sin(}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\sin(}\right.}{4\left(b\cos\left(2\,bx+2\,a\right)^2+b\sin\left(\right.\right.}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/4*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)
^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*
cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a
)^2)) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a)
+ 4*sin(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2
*b*x + 2*a) + b)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = -\frac{\frac{2}{\sin(bx+a)}-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(-\sin\left(bx+a\right)+1\right)}{4\,b}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/4*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

3.41.9 Mupad [B] (verification not implemented)

Time = 19.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = \frac{\operatorname{atanh}(\sin{(a+b\,x)})}{2\,b} - \frac{1}{2\,b\,\sin{(a+b\,x)}}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)),x)
```

output
```
atanh(sin(a + b*x))/(2*b) - 1/(2*b*sin(a + b*x))
```

3.42 $\int \csc(a+bx)\csc^2(2a+2bx)\,dx$

3.42.1 Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = -\frac{3\text{arctanh}(\cos(a+bx))}{8b}+\frac{3\sec(a+bx)}{8b} - \frac{\csc^2(a+bx)\sec(a+bx)}{8b}$$

output
```
-3/8*arctanh(cos(b*x+a))/b+3/8*sec(b*x+a)/b-1/8*csc(b*x+a)^2*sec(b*x+a)/b
```

3.42.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. $2(49) = 98$.

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.92

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \frac{\csc^4(a+bx)\left(2-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3(a\right.}{8b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\right.}$$

input
```
Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

output
```
(Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a
+ b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] +
 Cos[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(
8*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

3.42.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4776, 3042, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \ 4776$$

$$\frac{1}{4}\int \csc^3(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\int \csc(a+bx)^3\sec(a+bx)^2dx$$

$$\Big\downarrow \ 3102$$

$$\frac{\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \ 252$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)}{4b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\sec(a+bx))-\sec(a+bx))}{4b}$$

input
```
Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

output
```
((-3*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Se
c[a + b*x]^2)))/(4*b)
```

3.42.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.42.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{-\frac{1}{2\sin(xb+a)^2\cos(xb+a)}+\frac{3}{2\cos(xb+a)}+\frac{3\ln(\csc(xb+a)-\cot(xb+a))}{2}}{4b}$	53
risch	$\frac{3\,\mathrm{e}^{5i(xb+a)}-2\,\mathrm{e}^{3i(xb+a)}+3\,\mathrm{e}^{i(xb+a)}}{4b\left(\mathrm{e}^{2i(xb+a)}-1\right)^2\left(\mathrm{e}^{2i(xb+a)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{8b}-\frac{3\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{8b}$	101

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(-1/2/sin(b*x+a)^2/cos(b*x+a)+3/2/cos(b*x+a)+3/2*ln(csc(b*x+a)-cot(b
*x+a)))
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.96

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx$$
$$= \frac{6\,\cos\left(bx+a\right)^2-3\left(\cos\left(bx+a\right)^3-\cos\left(bx+a\right)\right)\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)+3\left(\cos\left(bx+a\right)^3-\cos\left(bx+\right.\right.}{16\left(b\cos\left(bx+a\right)^3-b\cos\left(bx+a\right)\right)}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/16*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
 + a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1
/2) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

3.42.6 Sympy [F]

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \int \csc(a+bx)\csc^2(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)**2,x)
```

output
```
Integral(csc(a + b*x)*csc(2*a + 2*b*x)**2, x)
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(43) = 86$.

Time = 0.24 (sec) , antiderivative size = 974, normalized size of antiderivative = 19.88

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/16*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b
*x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a)
+ 4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x
+ 2*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(co
s(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)
^2 - 2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(
2*b*x + 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a)
- sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*co
s(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*
(2*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x
+ 6*a)^2 - 2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- cos(2*b*x + 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x +
 6*a) - sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2
- 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2
) + 4*(3*sin(5*b*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x
 + 6*a) - 12*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2
*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*
sin(2*b*x + 2*a) - 12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/...
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(43) = 86.

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.80

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \frac{\frac{\frac{14(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+6\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{32\,b}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/32*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x
 + a) - 1)^2/(cos(b*x + a) + 1)^2) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1)
 + 6*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

3.42.9 Mupad [B] (verification not implemented)

Time = 19.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = -\frac{3\,\mathrm{atanh}(\cos(a+b\,x))}{8\,b}-\frac{\frac{3\cos(a+b\,x)^2}{8}-\frac{1}{4}}{b\,\left(\cos(a+b\,x)-\cos(a+b\,x)^3\right)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^2),x)
```

output
```
- (3*atanh(cos(a + b*x)))/(8*b) - ((3*cos(a + b*x)^2)/8 - 1/4)/(b*(cos(a +
 b*x) - cos(a + b*x)^3))
```

3.43 $\int \csc(a+bx)\csc^3(2a+2bx)\,dx$

3.43.1 Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx=\frac{5\text{arctanh}(\sin(a+bx))}{16b}-\frac{5\csc(a+bx)}{16b}-\frac{5\csc^3(a+bx)}{48b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{16b}$$

output
```
5/16*arctanh(sin(b*x+a))/b-5/16*csc(b*x+a)/b-5/48*csc(b*x+a)^3/b+1/16*csc(
b*x+a)^3*sec(b*x+a)^2/b
```

3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.47

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx=-\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\sin^2(a+bx)\right)}{24b}$$

input
```
Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/24*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

3.43.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 4776, 3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc^4(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^4\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\csc(a+bx))-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)}{8b}$$

input
```
Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/8*(Csc[a + b*x]^5/(2*(1 - Csc[a + b*x]^2)) - (5*(ArcTanh[Csc[a + b*x]]
- Csc[a + b*x] - Csc[a + b*x]^3/3))/2)/b
```

3.43.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.43.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{-\frac{1}{3\sin(xb+a)^3\cos(xb+a)^2}+\frac{5}{6\sin(xb+a)\cos(xb+a)^2}-\frac{5}{2\sin(xb+a)}+\frac{5\ln(\sec(xb+a)+\tan(xb+a))}{2}}{8b}$	69
risch	$-\frac{i\left(15\,e^{9i(xb+a)}-20\,e^{7i(xb+a)}-22\,e^{5i(xb+a)}-20\,e^{3i(xb+a)}+15\,e^{i(xb+a)}\right)}{24b\left(e^{2i(xb+a)}-1\right)^3\left(e^{2i(xb+a)}+1\right)^2}-\frac{5\ln\left(e^{i(xb+a)}-i\right)}{16b}+\frac{5\ln\left(i+e^{i(xb+a)}\right)}{16b}$	126

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)^2+5/6/sin(b*x+a)/cos(b*x+a)^2-5/2/sin(
b*x+a)+5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(58) = 116.

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.97

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx =$$
$$-\frac{30\cos(bx+a)^4-15\left(\cos(bx+a)^4-\cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a)+15\left(\cos(bx+\right.}{96\left(b\cos(bx+a)^4-b\cos(bx+a)^2\right)}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/96*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*
x + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b
*x + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*
cos(b*x + a)^2)*sin(b*x + a))
```

3.43.6 Sympy [F]

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx = \int \csc(a+bx)\csc^3(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)**3,x)
```

output
```
Integral(csc(a + b*x)*csc(2*a + 2*b*x)**3, x)
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(58) = 116$.

Time = 0.36 (sec) , antiderivative size = 1780, normalized size of antiderivative = 26.97

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/96*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a) -
 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*x
 + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(
9*b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*x
 + 3*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*s
in(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x + 5
*a) + 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin(
4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a)
 - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b*
x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) -
 cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2
*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x +
4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*(
cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) -
 sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6*
b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin
(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*
a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b*...
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\frac{6\,\sin(bx+a)}{\sin(bx+a)^2-1} + \frac{4\left(6\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 15\,\log\left(\sin\left(bx+a\right)+1\right) + 15\,\log\left(-\sin\left(bx+a\right)+1\right)}{96\,b}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-1/96*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(
b*x + a)^3 - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

3.43.9 Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx = \frac{5\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{16\,b} - \frac{-\frac{5\sin(a+b\,x)^4}{16} + \frac{5\sin(a+b\,x)^2}{24} + \frac{1}{24}}{b\,\left(\sin\left(a+b\,x\right)^3 - \sin\left(a+b\,x\right)^5\right)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^3),x)
```

output
```
(5*atanh(sin(a + b*x)))/(16*b) - ((5*sin(a + b*x)^2)/24 - (5*sin(a + b*x)^
4)/16 + 1/24)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

3.44 $\int \csc(a+bx)\csc^4(2a+2bx)\,dx$

3.44.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = -\frac{35\text{arctanh}(\cos(a+bx))}{128b}+\frac{35\sec(a+bx)}{128b}+\frac{35\sec^3(a+bx)}{384b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b}$$

output
```
-35/128*arctanh(cos(b*x+a))/b+35/128*sec(b*x+a)/b+35/384*sec(b*x+a)^3/b-7/
128*csc(b*x+a)^2*sec(b*x+a)^3/b-1/64*csc(b*x+a)^4*sec(b*x+a)^3/b
```

3.44.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(89) = 178.

Time = 0.72 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.01

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\csc^{10}(a+bx)\left(-204+658\cos(2(a+bx))-228\cos(3(a+bx))+140\cos(4(a+bx))-76\cos(5(a+bx$$

input
```
Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/384*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)
] + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76
*Cos[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5
*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)
/2]] + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b
*x)/2]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x
)]*Log[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

3.44.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4776, 3042, 3102, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^5(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^5\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{16b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{16b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{16b}$$

input
```
Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-1/4*Sec[a + b*x]^7/(1 - Sec[a + b*x]^2)^2 + (7*(Sec[a + b*x]^5/(2*(1 - S
ec[a + b*x]^2)) - (5*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x] - Sec[a + b*x]^
3/3))/2))/4)/(16*b)
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.44.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

method	result
default	$\frac{-\frac{1}{4\sin(xb+a)^4\cos(xb+a)^3}+\frac{7}{12\sin(xb+a)^2\cos(xb+a)^3}-\frac{35}{24\sin(xb+a)^2\cos(xb+a)}+\frac{35}{8\cos(xb+a)}+\frac{35\ln(\csc(xb+a)-\cot(xb+a))}{8}}{16b}$
risch	$\frac{105\,e^{13i(xb+a)}-70\,e^{11i(xb+a)}-329\,e^{9i(xb+a)}+204\,e^{7i(xb+a)}-329\,e^{5i(xb+a)}-70\,e^{3i(xb+a)}+105\,e^{i(xb+a)}}{192b\left(e^{2i(xb+a)}-1\right)^4\left(e^{2i(xb+a)}+1\right)^3}+\frac{35\ln\left(e^{i(xb+a)}-1\right)}{128b}-35$

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^3+7/12/sin(b*x+a)^2/cos(b*x+a)^3-35/2
4/sin(b*x+a)^2/cos(b*x+a)+35/8/cos(b*x+a)+35/8*ln(csc(b*x+a)-cot(b*x+a)))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx$$
$$=\frac{210\cos(bx+a)^6-350\cos(bx+a)^4+112\cos(bx+a)^2-105\left(\cos(bx+a)^7-2\cos(bx+a)^5+\cos(b\right.}{768\left(b\cos(bx+a)^7-\right.}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/768*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*
(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a)
+ 1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2
*cos(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(
b*x + a)^3)
```

3.44.6 Sympy [F]

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \int \csc(a+bx)\csc^4(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)**4,x)
```

output
```
Integral(csc(a + b*x)*csc(2*a + 2*b*x)**4, x)
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3846 vs. $2(79) = 158$.

Time = 0.35 (sec) , antiderivative size = 3846, normalized size of antiderivative = 43.21

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/768*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x +
 9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a)
+ 105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(1
0*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*
a) + cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a) +
 329*cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*c
os(3*b*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*x
 + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) +
cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204*
cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b*
x + a))*cos(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a)
 - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*c
os(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*x
 + a))*cos(8*b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - c
os(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(3
*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a)
 + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*co
s(b*x + a))*cos(4*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)
 - 420*cos(2*b*x + 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos(
10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x ...
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(79) = 158$.

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.31

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx$$

$$= \frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{6(\cos(bx+a}{(\cos(bx+a)}\right.}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}}{3072\,b}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/3072*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 210*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 -
 72*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*
x + a) + 1)^2 + 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6*(cos(b*x
+ a) - 1)^2/(cos(b*x + a) + 1)^2 + 5)/((cos(b*x + a) - 1)/(cos(b*x + a) +
1) + 1)^3 + 420*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

3.44.9 Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \frac{\frac{35\cos(a+b\,x)^6}{128}-\frac{175\cos(a+b\,x)^4}{384}+\frac{7\cos(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)} - \frac{35\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{128\,b}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^4),x)
```

output
```
((7*cos(a + b*x)^2)/48 - (175*cos(a + b*x)^4)/384 + (35*cos(a + b*x)^6)/12
8 + 1/48)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*a
tanh(cos(a + b*x)))/(128*b)
```

3.45 $\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$

3.45.1 Optimal result

Integrand size = 20, antiderivative size = 155

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx=\frac{5x}{8}+\frac{5\cos(a+bx)\sin(a+bx)}{8b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{12b}+\frac{\cos^5(a+bx)\sin(a+bx)}{3b}+\frac{2\cos^7(a+bx)\sin(a+bx)}{7b}-\frac{16\cos^9(a+bx)\sin(a+bx)}{7b}-\frac{160\cos^9(a+bx)\sin^3(a+bx)}{21b}-\frac{128\cos^9(a+bx)\sin^5(a+bx)}{7b}$$

output
```
5/8*x+5/8*cos(b*x+a)*sin(b*x+a)/b+5/12*cos(b*x+a)^3*sin(b*x+a)/b+1/3*cos(b
*x+a)^5*sin(b*x+a)/b+2/7*cos(b*x+a)^7*sin(b*x+a)/b-16/7*cos(b*x+a)^9*sin(b
*x+a)/b-160/21*cos(b*x+a)^9*sin(b*x+a)^3/b-128/7*cos(b*x+a)^9*sin(b*x+a)^5
/b
```

3.45.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.55

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \frac{840a+840bx+105\sin(2(a+bx))-315\sin(4(a+bx))-63\sin(6(a+bx))+63\sin(8(a+bx))+21\sin(}{1344b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

output
```
(840*a + 840*b*x + 105*Sin[2*(a + b*x)] - 315*Sin[4*(a + b*x)] - 63*Sin[6*
(a + b*x)] + 63*Sin[8*(a + b*x)] + 21*Sin[10*(a + b*x)] - 7*Sin[12*(a + b*
x)] - 3*Sin[14*(a + b*x)])/(1344*b)
```

3.45.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.850$, Rules used = {3042, 4776, 3042, 3048, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(2a+2bx)^8}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \ 4776$$

$$256\int \cos^8(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$256\int \cos(a+bx)^8\sin(a+bx)^6dx$$

$$\Big\downarrow \ 3048$$

$$256\left(\frac{5}{14}\int \cos^8(a+bx)\sin^4(a+bx)dx-\frac{\sin^5(a+bx)\cos^9(a+bx)}{14b}\right)$$

$\downarrow$ 3042

$$256\left(\frac{5}{14}\int \cos(a+bx)^8\sin(a+bx)^4dx-\frac{\sin^5(a+bx)\cos^9(a+bx)}{14b}\right)$$

$\downarrow$ 3048

$$256\left(\frac{5}{14}\left(\frac{1}{4}\int \cos^8(a+bx)\sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos^9(a+bx)}{12b}\right)-\frac{\sin^5(a+bx)\cos^9(a+bx)}{14b}\right)$$

$\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\int \cos(a+bx)^8\sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos^9(a+bx)}{12b}\right)-\frac{\sin^5(a+bx)\cos^9(a+bx)}{14b}\right)$$

$\downarrow$ 3048

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\int \cos^8(a+bx)dx-\frac{\sin(a+bx)\cos^9(a+bx)}{10b}\right)-\frac{\sin^3(a+bx)\cos^9(a+bx)}{12b}\right)-\frac{\sin^5(a+bx)\cos^9}{14b}\right.$$

$\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\int \sin\left(a+bx+\frac{\pi}{2}\right)^8dx-\frac{\sin(a+bx)\cos^9(a+bx)}{10b}\right)-\frac{\sin^3(a+bx)\cos^9(a+bx)}{12b}\right)-\frac{\sin^5(a+b}{}\right.$$

$\downarrow$ 3115

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\int \cos^6(a+bx)dx+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin(a+bx)\cos^9(a+bx)}{10b}\right)-\frac{\sin^3(a+bx)\mathrm{co}}{12b}\right.\right.$$

$\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\int \sin\left(a+bx+\frac{\pi}{2}\right)^6dx+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin(a+bx)\cos^9(a+bx)}{10b}\right)-\frac{\sin^3(a+}{}\right.\right.$$

$\downarrow$ 3115

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(a+bx)dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin(a+bx)}{1}\right.\right.\right.$$

$\downarrow$ 3042

3.45. $\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin(a$$

$\Big\downarrow$ 3115

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+$$

$\Big\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\mathrm{si}$$

$\Big\downarrow$ 3115

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+}{6b}$$

$\Big\downarrow$ 24

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{7}{8}\left(\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin}{}\right.\right.\right.\right.\right.\right.$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

output
```
256*(-1/14*(Cos[a + b*x]^9*Sin[a + b*x]^5)/b + (5*(-1/12*(Cos[a + b*x]^9*S
in[a + b*x]^3)/b + (-1/10*(Cos[a + b*x]^9*Sin[a + b*x])/b + ((Cos[a + b*x]
^7*Sin[a + b*x])/(8*b) + (7*((Cos[a + b*x]^5*Sin[a + b*x])/(6*b) + (5*((Co
s[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(
2*b)))/4))/6))/8)/10)/4))/14)
```

3.45.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.45.4 Maple [A] (verified)

Time = 46.86 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

method	result
risch	$\frac{5x}{8}-\frac{\sin(14xb+14a)}{448b}-\frac{\sin(12xb+12a)}{192b}+\frac{\sin(10xb+10a)}{64b}+\frac{3\sin(8xb+8a)}{64b}-\frac{3\sin(6xb+6a)}{64b}-\frac{15\sin(4xb+4a)}{64b}+\frac{5\sin(2xb}{64b}$
default	$\frac{-\frac{128\sin(xb+a)^5\cos(xb+a)^9}{7}-\frac{160\sin(xb+a)^3\cos(xb+a)^9}{21}-\frac{16\sin(xb+a)\cos(xb+a)^9}{7}+\frac{2\left(\cos(xb+a)^7+\frac{7\cos(xb+a)^5}{6}+\frac{35\cos(xb+a)^3}{24}+\frac{35\cos(xb+}{16}\right.}{7}}{b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

output
```
5/8*x-1/448/b*sin(14*b*x+14*a)-1/192/b*sin(12*b*x+12*a)+1/64/b*sin(10*b*x+
10*a)+3/64/b*sin(8*b*x+8*a)-3/64/b*sin(6*b*x+6*a)-15/64/b*sin(4*b*x+4*a)+5
/64*sin(2*b*x+2*a)/b
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$$
$$= \frac{105\,bx - \left(3072\,\cos\left(bx+a\right)^{13} - 7424\,\cos\left(bx+a\right)^{11} + 4736\,\cos\left(bx+a\right)^{9} - 48\,\cos\left(bx+a\right)^{7} - 56\,\cos\left(bx\right.}{168\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

output
```
1/168*(105*b*x - (3072*cos(b*x + a)^13 - 7424*cos(b*x + a)^11 + 4736*cos(b
*x + a)^9 - 48*cos(b*x + a)^7 - 56*cos(b*x + a)^5 - 70*cos(b*x + a)^3 - 10
5*cos(b*x + a))*sin(b*x + a))/b
```

3.45.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**8,x)
```

output
```
Timed out
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$$
$$= \frac{840\,bx - 3\,\sin\left(14\,bx+14\,a\right) - 7\,\sin\left(12\,bx+12\,a\right) + 21\,\sin\left(10\,bx+10\,a\right) + 63\,\sin\left(8\,bx+8\,a\right) - 63\,\sin\left(\right.}{1344\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

output
```
1/1344*(840*b*x - 3*sin(14*b*x + 14*a) - 7*sin(12*b*x + 12*a) + 21*sin(10*
b*x + 10*a) + 63*sin(8*b*x + 8*a) - 63*sin(6*b*x + 6*a) - 315*sin(4*b*x +
4*a) + 105*sin(2*b*x + 2*a))/b
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.61

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \frac{105\,bx + 105\,a + \frac{105\,\tan(bx+a)^{13} + 700\,\tan(bx+a)^{11} + 1981\,\tan(bx+a)^{9} + 3072\,\tan(bx+a)^{7} - 1981\,\tan(bx+a)^{5} - 700\,\tan(bx+a)^{3} - 105\,\tan}{\left(\tan(bx+a)^2+1\right)^7}}{168\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

output
```
1/168*(105*b*x + 105*a + (105*tan(b*x + a)^13 + 700*tan(b*x + a)^11 + 1981
*tan(b*x + a)^9 + 3072*tan(b*x + a)^7 - 1981*tan(b*x + a)^5 - 700*tan(b*x
+ a)^3 - 105*tan(b*x + a))/(tan(b*x + a)^2 + 1)^7)/b
```

3.45.9 Mupad [B] (verification not implemented)

Time = 21.91 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \frac{5\,x}{8} + \frac{\frac{5\tan(a+b\,x)^{13}}{8} + \frac{25\tan(a+b\,x)^{11}}{6} + \frac{283\tan(a+b\,x)^{9}}{24} + \frac{128\tan(a+b\,x)^{7}}{7} - \frac{283\tan(a+b\,x)^{5}}{24} - \frac{25\tan(a+b}{6}}{b\,\left(\tan\left(a+b\,x\right)^{14} + 7\tan\left(a+b\,x\right)^{12} + 21\tan\left(a+b\,x\right)^{10} + 35\tan\left(a+b\,x\right)^{8} + 35\tan\left(a+b\,x\right)^{6} + 21\tan\right.}$$

input
```
int(sin(2*a + 2*b*x)^8/sin(a + b*x)^2,x)
```

output
```
(5*x)/8 + ((128*tan(a + b*x)^7)/7 - (25*tan(a + b*x)^3)/6 - (283*tan(a + b
*x)^5)/24 - (5*tan(a + b*x))/8 + (283*tan(a + b*x)^9)/24 + (25*tan(a + b*x
)^11)/6 + (5*tan(a + b*x)^13)/8)/(b*(7*tan(a + b*x)^2 + 21*tan(a + b*x)^4
+ 35*tan(a + b*x)^6 + 35*tan(a + b*x)^8 + 21*tan(a + b*x)^10 + 7*tan(a + b
*x)^12 + tan(a + b*x)^14 + 1))
```

3.46 $\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx$

3.46.1 Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx=-\frac{16\cos^8(a+bx)}{b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{32\cos^{12}(a+bx)}{3b}$$

output
```
-16*cos(b*x+a)^8/b+128/5*cos(b*x+a)^10/b-32/3*cos(b*x+a)^12/b
```

3.46.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx=\frac{64\sin^6(a+bx)}{3b}-\frac{48\sin^8(a+bx)}{b}+\frac{192\sin^{10}(a+bx)}{5b}-\frac{32\sin^{12}(a+bx)}{3b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

output
```
(64*Sin[a + b*x]^6)/(3*b) - (48*Sin[a + b*x]^8)/b + (192*Sin[a + b*x]^10)/
(5*b) - (32*Sin[a + b*x]^12)/(3*b)
```

3.46.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4776, 3042, 3045, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^7}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$128\int \cos^7(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$128\int \cos(a+bx)^7\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{128\int \cos^7(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{64\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos^2(a+bx)}{b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{64\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos^2(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{64\left(\frac{1}{6}\cos^{12}(a+bx)-\frac{2}{5}\cos^{10}(a+bx)+\frac{1}{4}\cos^8(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

output
```
(-64*(Cos[a + b*x]^8/4 - (2*Cos[a + b*x]^10)/5 + Cos[a + b*x]^12/6))/b
```

3.46.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.46.4 Maple [A] (verified)

Time = 28.90 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{128\left(\frac{\sin(xb+a)^{12}}{12}-\frac{3\sin(xb+a)^{10}}{10}+\frac{3\sin(xb+a)^{8}}{8}-\frac{\sin(xb+a)^{6}}{6}\right)}{b}$	47
risch	$-\frac{\cos(12xb+12a)}{192b}-\frac{\cos(10xb+10a)}{80b}+\frac{\cos(8xb+8a)}{32b}+\frac{5\cos(6xb+6a)}{48b}-\frac{5\cos(4xb+4a)}{64b}-\frac{5\cos(2xb+2a)}{8b}$	86

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
-128/b*(1/12*sin(b*x+a)^12-3/10*sin(b*x+a)^10+3/8*sin(b*x+a)^8-1/6*sin(b*x
+a)^6)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = -\frac{16\left(10\cos(bx+a)^{12}-24\cos(bx+a)^{10}+15\cos(bx+a)^{8}\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
-16/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**7,x)
```

output
```
Timed out
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = -\frac{5\cos(12\,bx+12\,a)+12\cos(10\,bx+10\,a)-30\cos(8\,bx+8\,a)-100\cos(6\,bx+6\,a)+75\cos(4\,bx+}{960\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
-1/960*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
 - 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = -\frac{16\left(10\sin(bx+a)^{12}-36\sin(bx+a)^{10}+45\sin(bx+a)^8-20\sin(bx+a)^6\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
-16/15*(10*sin(b*x + a)^12 - 36*sin(b*x + a)^10 + 45*sin(b*x + a)^8 - 20*s
in(b*x + a)^6)/b
```

3.46.9 Mupad [B] (verification not implemented)

Time = 19.91 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = -\frac{16\cos(a+b\,x)^8\left(10\cos(a+b\,x)^4-24\cos(a+b\,x)^2+15\right)}{15\,b}$$

input
```
int(sin(2*a + 2*b*x)^7/sin(a + b*x)^2,x)
```

output
```
-(16*cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(15*b)
```

3.47 $\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx$

3.47.1 Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{3x}{4}+\frac{3\cos(a+bx)\sin(a+bx)}{4b}+\frac{\cos^3(a+bx)\sin(a+bx)}{2b}+\frac{2\cos^5(a+bx)\sin(a+bx)}{5b}-\frac{12\cos^7(a+bx)\sin(a+bx)}{5b}-\frac{32\cos^7(a+bx)\sin^3(a+bx)}{5b}$$

output
```
3/4*x+3/4*cos(b*x+a)*sin(b*x+a)/b+1/2*cos(b*x+a)^3*sin(b*x+a)/b+2/5*cos(b*
x+a)^5*sin(b*x+a)/b-12/5*cos(b*x+a)^7*sin(b*x+a)/b-32/5*cos(b*x+a)^7*sin(b
*x+a)^3/b
```

3.47.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{120bx+20\sin(2(a+bx))-40\sin(4(a+bx))-10\sin(6(a+bx))+5\sin(8(a+bx))+2\sin(10(a+bx))}{160b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

output
```
(120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
 + 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(160*b)
```

3.47.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {3042, 4776, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^6}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$64\int \cos^6(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^6\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$64\left(\frac{3}{10}\int \cos^6(a+bx)\sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3042}$$

$$64\left(\frac{3}{10}\int \cos(a+bx)^6\sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3048}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\int \cos^6(a+bx)dx-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3042}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\int \sin\left(a+bx+\frac{\pi}{2}\right)^6dx-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \; 3115$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cos^4(a+bx)dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a\dots}{10b}\right.$$

$$\Big\downarrow \; 3042$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\,\mathrm{c}\dots}{10\dots}\right.$$

$$\Big\downarrow \; 3115$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(\dots}{8b}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx\dots}{}\right.\right.$$

$$\Big\downarrow \; 3115$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\dots\right.\right.$$

$$\Big\downarrow \; 24$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)-\frac{\sin\dots}{}\right.\right.$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

output
```
64*(-1/10*(Cos[a + b*x]^7*Sin[a + b*x]^3)/b + (3*(-1/8*(Cos[a + b*x]^7*Sin
[a + b*x])/b + ((Cos[a + b*x]^5*Sin[a + b*x])/(6*b) + (5*((Cos[a + b*x]^3*
Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/6)
/8))/10)
```

3.47. $\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx$

3.47.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.47.4 Maple [A] (verified)

Time = 15.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{3x}{4}+\frac{\sin(10xb+10a)}{80b}+\frac{\sin(8xb+8a)}{32b}-\frac{\sin(6xb+6a)}{16b}-\frac{\sin(4xb+4a)}{4b}+\frac{\sin(2xb+2a)}{8b}$	75
default	$\frac{-\frac{32\sin(xb+a)^{3}\cos(xb+a)^{7}}{5}-\frac{12\cos(xb+a)^{7}\sin(xb+a)}{5}+\frac{2\left(\cos(xb+a)^{5}+\frac{5\cos(xb+a)^{3}}{4}+\frac{15\cos(xb+a)}{8}\right)\sin(xb+a)}{5}+\frac{3xb}{4}+\frac{3a}{4}}{b}$	83

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
3/4*x+1/80/b*sin(10*b*x+10*a)+1/32/b*sin(8*b*x+8*a)-1/16/b*sin(6*b*x+6*a)-
1/4/b*sin(4*b*x+4*a)+1/8*sin(2*b*x+2*a)/b
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{15\,bx + \left(128\,\cos\left(bx+a\right)^9 - 176\,\cos\left(bx+a\right)^7 + 8\,\cos\left(bx+a\right)^5 + 10\,\cos\left(bx+a\right)^3 + 15\,\cos\left(bx+a\right)\right)\sin}{20\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
1/20*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**6,x)
```

output
```
Timed out
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{120\,bx + 2\,\sin\left(10\,bx+10\,a\right) + 5\,\sin\left(8\,bx+8\,a\right) - 10\,\sin\left(6\,bx+6\,a\right) - 40\,\sin\left(4\,bx+4\,a\right) + 20\,\sin\left(2\,bx\right.}{160\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
1/160*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x
+ 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{15\,bx + 15\,a + \frac{15\,\tan(bx+a)^9 + 70\,\tan(bx+a)^7 + 128\,\tan(bx+a)^5 - 70\,\tan(bx+a)^3 - 15\,\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^5}}{20\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
1/20*(15*b*x + 15*a + (15*tan(b*x + a)^9 + 70*tan(b*x + a)^7 + 128*tan(b*x
 + a)^5 - 70*tan(b*x + a)^3 - 15*tan(b*x + a))/(tan(b*x + a)^2 + 1)^5)/b
```

3.47.9 Mupad [B] (verification not implemented)

Time = 22.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{3\,x}{4} + \frac{\frac{3\tan(a+b\,x)^9}{4} + \frac{7\tan(a+b\,x)^7}{2} + \frac{32\tan(a+b\,x)^5}{5} - \frac{7\tan(a+b\,x)^3}{2} - \frac{3\tan(a+b\,x)}{4}}{b\,\left(\tan\left(a+b\,x\right)^{10} + 5\tan\left(a+b\,x\right)^8 + 10\tan\left(a+b\,x\right)^6 + 10\tan\left(a+b\,x\right)^4 + 5\tan\left(a+b\,x\right)^2 + 1\right)}$$

input
```
int(sin(2*a + 2*b*x)^6/sin(a + b*x)^2,x)
```

output
```
(3*x)/4 + ((32*tan(a + b*x)^5)/5 - (7*tan(a + b*x)^3)/2 - (3*tan(a + b*x))
/4 + (7*tan(a + b*x)^7)/2 + (3*tan(a + b*x)^9)/4)/(b*(5*tan(a + b*x)^2 + 1
0*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^10
+ 1))
```

3.48 $\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx$

3.48.1 Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx=-\frac{16\cos^6(a+bx)}{3b}+\frac{4\cos^8(a+bx)}{b}$$

output
```
-16/3*cos(b*x+a)^6/b+4*cos(b*x+a)^8/b
```

3.48.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{96b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8
*(a + b*x)])/(96*b)
```

3.48.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^5}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{32\int \cos^5(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{32\int \left(\cos^5(a+bx)-\cos^7(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{32\left(\frac{1}{6}\cos^6(a+bx)-\frac{1}{8}\cos^8(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-32*(Cos[a + b*x]^6/6 - Cos[a + b*x]^8/8))/b
```

3.48.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.48.4 Maple [A] (verified)

Time = 8.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{4 \cos(xb+a)^8 - \frac{16 \cos(xb+a)^6}{3}}{b}$	27
risch	$\frac{\cos(8xb+8a)}{32b} + \frac{\cos(6xb+6a)}{12b} - \frac{\cos(4xb+4a)}{8b} - \frac{3\cos(2xb+2a)}{4b}$	58

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
32/b*(1/8*cos(b*x+a)^8-1/6*cos(b*x+a)^6)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\left(3\cos(bx+a)^8-4\cos(bx+a)^6\right)}{3b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
4/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{3\cos(8bx+8a)+8\cos(6bx+6a)-12\cos(4bx+4a)-72\cos(2bx+2a)}{96b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/96*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*c
os(2*b*x + 2*a))/b
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\left(3\sin\left(bx+a\right)^8-8\sin\left(bx+a\right)^6+6\sin\left(bx+a\right)^4\right)}{3b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
4/3*(3*sin(b*x + a)^8 - 8*sin(b*x + a)^6 + 6*sin(b*x + a)^4)/b
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\cos\left(a+b\,x\right)^6\left(3\cos\left(a+b\,x\right)^2-4\right)}{3\,b}$$

input
```
int(sin(2*a + 2*b*x)^5/sin(a + b*x)^2,x)
```

output
```
(4*cos(a + b*x)^6*(3*cos(a + b*x)^2 - 4))/(3*b)
```

3.49 $\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx$

3.49.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = x+\frac{\cos(a+bx)\sin(a+bx)}{b}+\frac{2\cos^3(a+bx)\sin(a+bx)}{3b}-\frac{8\cos^5(a+bx)\sin(a+bx)}{3b}$$

output
```
x+cos(b*x+a)*sin(b*x+a)/b+2/3*cos(b*x+a)^3*sin(b*x+a)/b-8/3*cos(b*x+a)^5*s
in(b*x+a)/b
```

3.49.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = -\frac{-12bx-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))}{12b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
-1/12*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)
])/b
```

3.49.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 4776, 3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^4}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$16\int \cos^4(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^4\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$16\left(\frac{1}{6}\int \cos^4(a+bx)dx-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$16\left(\frac{1}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3115}$$

$$16\left(\frac{1}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$16\left(\frac{1}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3115}$$

$$16\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{24}$$

$$16\left(\frac{1}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
16*(-1/6*(Cos[a + b*x]^5*Sin[a + b*x])/b + ((Cos[a + b*x]^3*Sin[a + b*x])/
(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4)/6)
```

3.49.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.49.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result	size
risch	$x-\frac{\sin(6xb+6a)}{12b}-\frac{\sin(4xb+4a)}{4b}+\frac{\sin(2xb+2a)}{4b}$	45
default	$\frac{-\frac{8\cos(xb+a)^{5}\sin(xb+a)}{3}+\frac{2\left(\cos(xb+a)^{3}+\frac{3\cos(xb+a)}{2}\right)\sin(xb+a)}{3}+xb+a}{b}$	55

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
x-1/12/b*sin(6*b*x+6*a)-1/4/b*sin(4*b*x+4*a)+1/4*sin(2*b*x+2*a)/b
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3\,bx-\left(8\,\cos\left(bx+a\right)^{5}-2\,\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/3*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*
x + a))/b
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.72

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\frac{12\,bx-\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)+3\,\sin\left(2\,bx+2\,a\right)}{12\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/12*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))
/b
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx=\frac{3\,bx+3\,a+\frac{3\,\tan\left(bx+a\right)^5+8\,\tan\left(bx+a\right)^3-3\,\tan\left(bx+a\right)}{\left(\tan\left(bx+a\right)^2+1\right)^3}}{3\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/3*(3*b*x + 3*a + (3*tan(b*x + a)^5 + 8*tan(b*x + a)^3 - 3*tan(b*x + a))/
(tan(b*x + a)^2 + 1)^3)/b
```

3.49.9 Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$=x+\frac{\tan(a+b\,x)^5+\frac{8\tan(a+b\,x)^3}{3}-\tan(a+b\,x)}{b\,\left(\tan\left(a+b\,x\right)^6+3\tan\left(a+b\,x\right)^4+3\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(sin(2*a + 2*b*x)^4/sin(a + b*x)^2,x)
```

output
```
x + ((8*tan(a + b*x)^3)/3 - tan(a + b*x) + tan(a + b*x)^5)/(b*(3*tan(a + b
*x)^2 + 3*tan(a + b*x)^4 + tan(a + b*x)^6 + 1))
```

3.50 $\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx$

3.50.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{2\cos^4(a+bx)}{b}$$

output
```
-2*cos(b*x+a)^4/b
```

3.50.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{2\cos^4(a+bx)}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-2*Cos[a + b*x]^4)/b
```

3.50.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4776, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^3}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^3\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{8\int \cos^3(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\cos^4(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-2*Cos[a + b*x]^4)/b
```

3.50.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.50.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2\cos(xb+a)^4}{b}$	14
risch	$-\frac{\cos(4xb+4a)}{4b}-\frac{\cos(2xb+2a)}{b}$	30

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*cos(b*x+a)^4/b
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{2\cos(bx+a)^4}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-2*cos(b*x + a)^4/b
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{\cos(4bx+4a)+4\cos(2bx+2a)}{4b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
-1/4*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{2\,\cos\left(bx+a\right)^4}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-2*cos(b*x + a)^4/b
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{2\cos\left(a+b\,x\right)^4}{b}$$

input
```
int(sin(2*a + 2*b*x)^3/sin(a + b*x)^2,x)
```

output
```
-(2*cos(a + b*x)^4)/b
```

3.51 $\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx$

3.51.1 Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = 2x+\frac{2\cos(a+bx)\sin(a+bx)}{b}$$

output
```
2*x+2*cos(b*x+a)*sin(b*x+a)/b
```

3.51.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{2(a+bx)+\sin(2(a+bx))}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
(2*(a + b*x) + Sin[2*(a + b*x)])/b
```

3.51.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4776, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^2}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$4\int \cos^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$4\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$4\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
4*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b))
```

3.51.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.51.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
risch	$2x + \frac{\sin(2xb+2a)}{b}$	18
default	$\frac{2\cos(xb+a)\sin(xb+a)+2xb+2a}{b}$	28

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x+sin(2*b*x+2*a)/b
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\,(bx+\cos{(bx+a)}\sin{(bx+a)})}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\,bx+\sin{(2\,bx+2\,a)}}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
(2*b*x + sin(2*b*x + 2*a))/b
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\left(bx+a+\frac{\tan(bx+a)}{\tan(bx+a)^2+1}\right)}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
2*(b*x + a + tan(b*x + a)/(tan(b*x + a)^2 + 1))/b
```

3.51.9 Mupad [B] (verification not implemented)

Time = 19.76 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = 2\,x+\frac{\sin\left(2\,a+2\,b\,x\right)}{b}$$

input
```
int(sin(2*a + 2*b*x)^2/sin(a + b*x)^2,x)
```

output
```
2*x + sin(2*a + 2*b*x)/b
```

3.52 $\int \csc^2(a+bx)\sin(2a+2bx)\,dx$

3.52.1 Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx=\frac{2\log(\sin(a+bx))}{b}$$

output
```
2*ln(sin(b*x+a))/b
```

3.52.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx=2\left(\frac{\log(\cos(a+bx))}{b}+\frac{\log(\tan(a+bx))}{b}\right)$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
2*(Log[Cos[a + b*x]]/b + Log[Tan[a + b*x]]/b)
```

3.52.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4776, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$2\int \cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-2\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{2\log(-\sin(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
(2*Log[-Sin[a + b*x]])/b
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.52.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\ln(\sin(xb+a))}{b}$	13
risch	$-2ix-\frac{4ia}{b}+\frac{2\ln(e^{2i(xb+a)}-1)}{b}$	30

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
2*ln(sin(b*x+a))/b
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\log\left(\frac{1}{2}\,\sin\left(bx+a\right)\right)}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*log(1/2*sin(b*x + a))/b
```

3.52.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6307 vs. $2(10) = 20$.

Time = 133.80 (sec) , antiderivative size = 18894, normalized size of antiderivative = 1574.50

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a),x)
```

output
```
4*Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (log(sin(b*x))/b, Eq(a, 0)), (0,
 Eq(b, 0)), (-4*b*x*tan(a/2)**5*tan(b*x/2)/(b*tan(a/2)**6*tan(b*x/2) + b*t
an(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*
tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b
*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) - 4*b*x*tan(a/2)**4*t
an(b*x/2)**2/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*t
an(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*
b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*ta
n(a/2) - b*tan(b*x/2)) + 4*b*x*tan(a/2)**4/(b*tan(a/2)**6*tan(b*x/2) + b*t
an(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*
tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b
*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) + 8*b*x*tan(a/2)**3*t
an(b*x/2)/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(
a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*t
an(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a
/2) - b*tan(b*x/2)) + 4*b*x*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2)**6*tan(b
*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*
x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan
(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) - 4*b*x*ta
n(a/2)**2/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*t...
```

3.52.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 6.75

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx$$
$$= \frac{\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + \log\left(\cos(bx)^2 - 2\right.}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*s
in(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*
x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
2*log(abs(sin(b*x + a)))/b
```

3.52.9 Mupad [B] (verification not implemented)

Time = 19.67 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx = \frac{\ln\left(\sin\left(a+b\,x\right)^2\right)}{b}$$

input
```
int(sin(2*a + 2*b*x)/sin(a + b*x)^2,x)
```

output
```
log(sin(a + b*x)^2)/b
```

3.53 $\int \csc^2(a+bx)\csc(2a+2bx)\,dx$

3.53.1 Optimal result

Integrand size = 18, antiderivative size = 30

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx=-\frac{\cot^2(a+bx)}{4b}+\frac{\log(\tan(a+bx))}{2b}$$

output
```
-1/4*cot(b*x+a)^2/b+1/2*ln(tan(b*x+a))/b
```

3.53.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx=-\frac{\csc^2(a+bx)}{4b}-\frac{\log(\cos(a+bx))}{2b}+\frac{\log(\sin(a+bx))}{2b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

output
```
-1/4*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/(2*b) + Log[Sin[a + b*x]]/(2*b)
```

3.53.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)}dx$$
$$\Big\downarrow \text{4776}$$
$$\frac{1}{2}\int \csc^3(a+bx)\sec(a+bx)dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2}\int \csc(a+bx)^3\sec(a+bx)dx$$
$$\Big\downarrow \text{3100}$$
$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{2b}$$
$$\Big\downarrow \text{244}$$
$$\frac{\int \left(\cot^3(a+bx)+\cot(a+bx)\right)d\tan(a+bx)}{2b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\log(\tan(a+bx))-\frac{1}{2}\cot^2(a+bx)}{2b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

output
```
(-1/2*Cot[a + b*x]^2 + Log[Tan[a + b*x]])/(2*b)
```

3.53.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.53.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{-\frac{1}{2\sin(xb+a)^2}+\ln(\tan(xb+a))}{2b}$	24
risch	$\frac{e^{2i(xb+a)}}{b\left(e^{2i(xb+a)}-1\right)^2}-\frac{\ln\left(e^{2i(xb+a)}+1\right)}{2b}+\frac{\ln\left(e^{2i(xb+a)}-1\right)}{2b}$	62

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx = -\frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\cos\left(bx+a\right)^2\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-1}{4\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-1/4*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(
-1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

3.53.6 Sympy [F]

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx = \int \csc^2\left(a+bx\right)\csc\left(2a+2bx\right)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a),x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x), x)
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(26) = 52$.

Time = 0.24 (sec) , antiderivative size = 656, normalized size of antiderivative = 21.87

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx = \frac{4\cos\left(4\,bx+4\,a\right)\cos\left(2\,bx+2\,a\right)-8\cos\left(2\,bx+2\,a\right)^2+\left(2\left(2\cos\left(2\,bx+2\,a\right)-1\right)\cos\left(4\,bx+4\,a\right)-\cos\left(\right.}{}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/4*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*co
s(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x +
2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(
2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*c
os(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)
- (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*co
s(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(
b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(
2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*
x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*
sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*
x + 4*a)*sin(2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*
cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b
*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx$$
$$= -\frac{\frac{1}{\sin(bx+a)^2} + \log\left(-\sin\left(bx+a\right)^2+1\right) - 2\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{4\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/4*(1/sin(b*x + a)^2 + log(-sin(b*x + a)^2 + 1) - 2*log(abs(sin(b*x + a)
)))/b
```

3.53.9 Mupad [B] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx = -\frac{\frac{\ln(\cos(a+b\,x))}{2}-\frac{\ln\left(\sin(a+b\,x)^2\right)}{4}+\frac{1}{4\sin(a+b\,x)^2}}{b}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)),x)
```

output
```
-(log(cos(a + b*x))/2 - log(sin(a + b*x)^2)/4 + 1/(4*sin(a + b*x)^2))/b
```

3.54 $\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$

3.54.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx=-\frac{\cot(a+bx)}{2b}-\frac{\cot^3(a+bx)}{12b}+\frac{\tan(a+bx)}{4b}$$

output
```
-1/2*cot(b*x+a)/b-1/12*cot(b*x+a)^3/b+1/4*tan(b*x+a)/b
```

3.54.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx=-\frac{5\cot(a+bx)}{12b}-\frac{\cot(a+bx)\csc^2(a+bx)}{12b}+\frac{\tan(a+bx)}{4b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

output
```
(-5*Cot[a + b*x])/(12*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(12*b) + Tan[a +
b*x]/(4*b)
```

3.54.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \csc^4(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)^4\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^4(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{4b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^4(a+bx)+2\cot^2(a+bx)+1\right)d\tan(a+bx)}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan(a+bx)-\frac{1}{3}\cot^3(a+bx)-2\cot(a+bx)}{4b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

output
```
(-2*Cot[a + b*x] - Cot[a + b*x]^3/3 + Tan[a + b*x])/(4*b)
```

3.54.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.54.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{4i\left(2\,\mathrm{e}^{2i(xb+a)}-1\right)}{3b\left(\mathrm{e}^{2i(xb+a)}-1\right)^{3}\left(\mathrm{e}^{2i(xb+a)}+1\right)}$	46
default	$\frac{-\frac{1}{3\cos(xb+a)\sin(xb+a)^{3}}+\frac{4}{3\sin(xb+a)\cos(xb+a)}-\frac{8\cot(xb+a)}{3}}{4b}$	51

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/3*I*(2*exp(2*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^3/(exp(2*I*(b*x+a))+1)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{8\cos(bx+a)^4-12\cos(bx+a)^2+3}{12\left(b\cos(bx+a)^3-b\cos(bx+a)\right)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-1/12*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*co
s(b*x + a))*sin(b*x + a))
```

3.54.6 Sympy [F]

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = \int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**2,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**2, x)
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(36) = 72.

Time = 0.31 (sec) , antiderivative size = 308, normalized size of antiderivative = 7.33

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$$
$$= \frac{4\left((2\right.}{3\left(b\cos(8\,bx+8\,a)^2+4\,b\cos(6\,bx+6\,a)^2+4\,b\cos(2\,bx+2\,a)^2+b\sin(8\,bx+8\,a)^2+4\,b\sin(6\,bx+6\,a)^2\right.}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
4/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1
)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6
*a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b
*cos(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*
sin(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*
b*x + 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x
 + 2*a) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*
a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\frac{6\tan(bx+a)^2+1}{\tan(bx+a)^3} - 3\tan(bx+a)}{12\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/12*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

3.54.9 Mupad [B] (verification not implemented)

Time = 19.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = \frac{\tan(a+b\,x)}{4\,b} - \frac{\frac{\tan(a+b\,x)^2}{2}+\frac{1}{12}}{b\tan(a+b\,x)^3}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^2),x)
```

output
```
tan(a + b*x)/(4*b) - (tan(a + b*x)^2/2 + 1/12)/(b*tan(a + b*x)^3)
```

3.55 $\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$

3.55.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx=-\frac{3\cot^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}+\frac{3\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}$$

output
```
-3/16*cot(b*x+a)^2/b-1/32*cot(b*x+a)^4/b+3/8*ln(tan(b*x+a))/b+1/16*tan(b*x
+a)^2/b
```

3.55.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx = -\frac{4\csc^2(a+bx)+\csc^4(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-2\sec^2(a+bx)}{32b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/32*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[S
in[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

3.55.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \ 4776$$

$$\frac{1}{8}\int \csc^5(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{8}\int \csc(a+bx)^5\sec(a+bx)^3dx$$

$$\Big\downarrow \ 3100$$

$$\frac{\int \cot^5(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{8b}$$

$$\Big\downarrow \ 243$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{16b}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(\cot^3(a+bx)+3\cot^2(a+bx)+3\cot(a+bx)+1\right)d\tan^2(a+bx)}{16b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\tan^2(a+bx)-\frac{1}{2}\cot^2(a+bx)-3\cot(a+bx)+3\log\left(\tan^2(a+bx)\right)}{16b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-3*Cot[a + b*x] - Cot[a + b*x]^2/2 + 3*Log[Tan[a + b*x]^2] + Tan[a + b*x]
^2)/(16*b)
```

3.55.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.55.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{-\frac{1}{4\sin(xb+a)^4\cos(xb+a)^2}+\frac{3}{4\sin(xb+a)^2\cos(xb+a)^2}-\frac{3}{2\sin(xb+a)^2}+3\ln(\tan(xb+a))}{8b}$	62
risch	$\frac{3\,\mathrm{e}^{10i(xb+a)}-6\,\mathrm{e}^{8i(xb+a)}-2\,\mathrm{e}^{6i(xb+a)}-6\,\mathrm{e}^{4i(xb+a)}+3\,\mathrm{e}^{2i(xb+a)}}{4b\left(\mathrm{e}^{2i(xb+a)}-1\right)^4\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}-\frac{3\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{8b}+\frac{3\ln\left(\mathrm{e}^{2i(xb+a)}-1\right)}{8b}$	123

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^2+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/si
n(b*x+a)^2+3*ln(tan(b*x+a)))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(52) = 104.

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$
$$= \frac{6\cos(bx+a)^4-9\cos(bx+a)^2-6\left(\cos(bx+a)^6-2\cos(bx+a)^4+\cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right)}{32\left(b\cos(bx+a)^6-2b\cos(bx+a)^4\right.}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
1/32*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b
*x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x
 + a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

3.55.6 Sympy [F]

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx = \int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**3,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**3, x)
```

3.55.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3188 vs. $2(52) = 104$.

Time = 0.29 (sec) , antiderivative size = 3188, normalized size of antiderivative = 53.13

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/16*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) -
6*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b
*x + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a)
 + 3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a
) - 12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*co
s(8*b*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*
b*x + 6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x
 + 4*a) + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b
*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*
cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8
*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) -
 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) -
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x +
8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) -
16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(
4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*
x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
 + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*si
n(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2...
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$
$$= -\frac{\frac{6\sin(bx+a)^4-3\sin(bx+a)^2-1}{\left(\sin(bx+a)^2-1\right)\sin(bx+a)^4}+6\log\left(-\sin(bx+a)^2+1\right)-12\log\left(\left|\sin(bx+a)\right|\right)}{32\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-1/32*((6*sin(b*x + a)^4 - 3*sin(b*x + a)^2 - 1)/((sin(b*x + a)^2 - 1)*sin
(b*x + a)^4) + 6*log(-sin(b*x + a)^2 + 1) - 12*log(abs(sin(b*x + a))))/b
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{3\ln\left(\sin(a+b\,x)^2\right)}{16\,b}-\frac{3\ln\left(\cos(a+b\,x)\right)}{8\,b}$$
$$+\frac{\frac{3\cos(a+b\,x)^4}{16}-\frac{9\cos(a+b\,x)^2}{32}+\frac{1}{16}}{b\,\left(\cos(a+b\,x)^6-2\cos(a+b\,x)^4+\cos(a+b\,x)^2\right)}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^3),x)
```

output
```
(3*log(sin(a + b*x)^2))/(16*b) - (3*log(cos(a + b*x)))/(8*b) + ((3*cos(a +
 b*x)^4)/16 - (9*cos(a + b*x)^2)/32 + 1/16)/(b*(cos(a + b*x)^2 - 2*cos(a +
 b*x)^4 + cos(a + b*x)^6))
```

3.56 $\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$

3.56.1 Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{3\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot^5(a+bx)}{80b}+\frac{\tan(a+bx)}{4b}+\frac{\tan^3(a+bx)}{48b}$$

output
```
-3/8*cot(b*x+a)/b-1/12*cot(b*x+a)^3/b-1/80*cot(b*x+a)^5/b+1/4*tan(b*x+a)/b
+1/48*tan(b*x+a)^3/b
```

3.56.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{73\cot(a+bx)}{240b}-\frac{7\cot(a+bx)\csc^2(a+bx)}{120b}-\frac{\cot(a+bx)\csc^4(a+bx)}{80b}+\frac{11\tan(a+bx)}{48b}+\frac{\sec^2(a+bx)\tan(a+bx)}{48b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-73*Cot[a + b*x])/(240*b) - (7*Cot[a + b*x]*Csc[a + b*x]^2)/(120*b) - (Co
t[a + b*x]*Csc[a + b*x]^4)/(80*b) + (11*Tan[a + b*x])/(48*b) + (Sec[a + b*
x]^2*Tan[a + b*x])/(48*b)
```

3.56.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^6(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^6\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^6(a+bx)\left(\tan^2(a+bx)+1\right)^4d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^6(a+bx)+4\cot^4(a+bx)+6\cot^2(a+bx)+\tan^2(a+bx)+4\right)d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\tan^3(a+bx)+4\tan(a+bx)-\frac{1}{5}\cot^5(a+bx)-\frac{4}{3}\cot^3(a+bx)-6\cot(a+bx)}{16b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-6*Cot[a + b*x] - (4*Cot[a + b*x]^3)/3 - Cot[a + b*x]^5/5 + 4*Tan[a + b*x
] + Tan[a + b*x]^3/3)/(16*b)
```

3.56.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.56.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{16 i \left(6 \,{\mathrm e}^{6 i \left(x b +a \right)}-2 \,{\mathrm e}^{4 i \left(x b +a \right)}-2 \,{\mathrm e}^{2 i \left(x b +a \right)}+1\right)}{15 b \left({\mathrm e}^{2 i \left(x b +a \right)}+1\right)^{3} \left({\mathrm e}^{2 i \left(x b +a \right)}-1\right)^{5}}$	68
default	$\frac{-\frac{1}{5 \sin \left(x b +a \right)^{5} \cos \left(x b +a \right)^{3}}+\frac{8}{15 \cos \left(x b +a \right)^{3} \sin \left(x b +a \right)^{3}}-\frac{16}{15 \cos \left(x b +a \right) \sin \left(x b +a \right)^{3}}+\frac{64}{15 \sin \left(x b +a \right) \cos \left(x b +a \right)}-\frac{128 \cot \left(x b +a \right)}{15}}{16 b}$	87

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-16/15*I*(6*exp(6*I*(b*x+a))-2*exp(4*I*(b*x+a))-2*exp(2*I*(b*x+a))+1)/b/(e
xp(2*I*(b*x+a))+1)^3/(exp(2*I*(b*x+a))-1)^5
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{128\cos(bx+a)^8-320\cos(bx+a)^6+240\cos(bx+a)^4-40\cos(bx+a)^2-5}{240\left(b\cos(bx+a)^7-2b\cos(bx+a)^5+b\cos(bx+a)^3\right)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-1/240*(128*cos(b*x + a)^8 - 320*cos(b*x + a)^6 + 240*cos(b*x + a)^4 - 40*
cos(b*x + a)^2 - 5)/((b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x +
a)^3)*sin(b*x + a))
```

3.56.6 Sympy [F]

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = \int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**4,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**4, x)
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(62) = 124.

Time = 0.25 (sec) , antiderivative size = 1227, normalized size of antiderivative = 17.04

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
16/15*(2*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(16
*b*x + 16*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a)
)*cos(14*b*x + 14*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*
x + 2*a))*cos(12*b*x + 12*a) + 12*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) -
 sin(2*b*x + 2*a))*cos(10*b*x + 10*a) - (6*cos(6*b*x + 6*a) - 2*cos(4*b*x
+ 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(16*b*x + 16*a) + 2*(6*cos(6*b*x + 6*a
) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(14*b*x + 14*a) + 2*(6
*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(12*b*
x + 12*a) - 6*(6*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a
) + 1)*sin(10*b*x + 10*a))/(b*cos(16*b*x + 16*a)^2 + 4*b*cos(14*b*x + 14*a
)^2 + 4*b*cos(12*b*x + 12*a)^2 + 36*b*cos(10*b*x + 10*a)^2 + 36*b*cos(6*b*
x + 6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(16*b*
x + 16*a)^2 + 4*b*sin(14*b*x + 14*a)^2 + 4*b*sin(12*b*x + 12*a)^2 + 36*b*s
in(10*b*x + 10*a)^2 + 36*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 + 8
*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos
(14*b*x + 14*a) + 2*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*co
s(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(16*b
*x + 16*a) + 4*(2*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*cos(
6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(14*b*x
 + 14*a) - 4*(6*b*cos(10*b*x + 10*a) - 6*b*cos(6*b*x + 6*a) + 2*b*cos(4...
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$$
$$= \frac{5\tan(bx+a)^3 - \frac{90\tan(bx+a)^4+20\tan(bx+a)^2+3}{\tan(bx+a)^5} + 60\tan(bx+a)}{240\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/240*(5*tan(b*x + a)^3 - (90*tan(b*x + a)^4 + 20*tan(b*x + a)^2 + 3)/tan(
b*x + a)^5 + 60*tan(b*x + a))/b
```

3.56.9 Mupad [B] (verification not implemented)

Time = 20.87 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$$
$$= -\frac{-5\tan(a+b\,x)^8-60\tan(a+b\,x)^6+90\tan(a+b\,x)^4+20\tan(a+b\,x)^2+3}{240\,b\tan\left(a+b\,x\right)^5}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^4),x)
```

output
```
-(20*tan(a + b*x)^2 + 90*tan(a + b*x)^4 - 60*tan(a + b*x)^6 - 5*tan(a + b*
x)^8 + 3)/(240*b*tan(a + b*x)^5)
```

3.57 $\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$

3.57.1 Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx=-\frac{5\cot^2(a+bx)}{32b}-\frac{5\cot^4(a+bx)}{128b}-\frac{\cot^6(a+bx)}{192b}+\frac{5\log(\tan(a+bx))}{16b}+\frac{5\tan^2(a+bx)}{64b}+\frac{\tan^4(a+bx)}{128b}$$

output
```
-5/32*cot(b*x+a)^2/b-5/128*cot(b*x+a)^4/b-1/192*cot(b*x+a)^6/b+5/16*ln(tan
(b*x+a))/b+5/64*tan(b*x+a)^2/b+1/128*tan(b*x+a)^4/b
```

3.57.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx=-\frac{36\csc^2(a+bx)+9\csc^4(a+bx)+2\csc^6(a+bx)+120\log(\cos(a+bx))-120\log(\sin(a+bx))-24\,\mathrm{se}}{384b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

output
```
-1/384*(36*Csc[a + b*x]^2 + 9*Csc[a + b*x]^4 + 2*Csc[a + b*x]^6 + 120*Log[
Cos[a + b*x]] - 120*Log[Sin[a + b*x]] - 24*Sec[a + b*x]^2 - 3*Sec[a + b*x]
^4)/b
```

3.57.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{32}\int \csc^7(a+bx)\sec^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^7\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^7(a+bx)\left(\tan^2(a+bx)+1\right)^5d\tan(a+bx)}{32b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^4(a+bx)\left(\tan^2(a+bx)+1\right)^5d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^4(a+bx)+5\cot^3(a+bx)+10\cot^2(a+bx)+10\cot(a+bx)+\tan^2(a+bx)+5\right)d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\tan^4(a+bx)+5\tan^2(a+bx)-\frac{1}{3}\cot^3(a+bx)-\frac{5}{2}\cot^2(a+bx)-10\cot(a+bx)+10\log\left(\tan^2(a+bx)\right)}{64b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-10*Cot[a + b*x] - (5*Cot[a + b*x]^2)/2 - Cot[a + b*x]^3/3 + 10*Log[Tan[a
 + b*x]^2] + 5*Tan[a + b*x]^2 + Tan[a + b*x]^4/2)/(64*b)
```

3.57.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.57.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

method	result
default	$\frac{-\frac{1}{6\sin(xb+a)^6\cos(xb+a)^4}+\frac{5}{12\sin(xb+a)^4\cos(xb+a)^4}-\frac{5}{6\sin(xb+a)^4\cos(xb+a)^2}+\frac{5}{2\sin(xb+a)^2\cos(xb+a)^2}-\frac{5}{\sin(xb+a)^2}+10\ln(\tan(xb+a))}{32b}$
risch	$\frac{15\,\mathrm{e}^{18i(xb+a)}-30\,\mathrm{e}^{16i(xb+a)}-40\,\mathrm{e}^{14i(xb+a)}+110\,\mathrm{e}^{12i(xb+a)}+18\,\mathrm{e}^{10i(xb+a)}+110\,\mathrm{e}^{8i(xb+a)}-40\,\mathrm{e}^{6i(xb+a)}-30\,\mathrm{e}^{4i(xb+a)}+15\,\mathrm{e}^{2i(xb+a)}}{24b\left(\mathrm{e}^{2i(xb+a)}-1\right)^6\left(\mathrm{e}^{2i(xb+a)}+1\right)^4}$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(-1/6/sin(b*x+a)^6/cos(b*x+a)^4+5/12/sin(b*x+a)^4/cos(b*x+a)^4-5/6/
sin(b*x+a)^4/cos(b*x+a)^2+5/2/sin(b*x+a)^2/cos(b*x+a)^2-5/sin(b*x+a)^2+10*
ln(tan(b*x+a)))
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.16

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$$
$$=\frac{60\cos(bx+a)^8-150\cos(bx+a)^6+110\cos(bx+a)^4-15\cos(bx+a)^2-60\left(\cos(bx+a)^{10}-3\cos(\right.}{384\,(b\,c}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
1/384*(60*cos(b*x + a)^8 - 150*cos(b*x + a)^6 + 110*cos(b*x + a)^4 - 15*co
s(b*x + a)^2 - 60*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)^6 -
 cos(b*x + a)^4)*log(cos(b*x + a)^2) + 60*(cos(b*x + a)^10 - 3*cos(b*x + a
)^8 + 3*cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2 + 1/4) -
3)/(b*cos(b*x + a)^10 - 3*b*cos(b*x + a)^8 + 3*b*cos(b*x + a)^6 - b*cos(b*
x + a)^4)
```

3.57.6 Sympy [F]

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = \int \csc^2{(a+bx)}\csc^5{(2a+2bx)}\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**5,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**5, x)
```

3.57.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7650 vs. $2(78) = 156$.

Time = 0.48 (sec) , antiderivative size = 7650, normalized size of antiderivative = 85.00

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/96*(4*(15*cos(18*b*x + 18*a) - 30*cos(16*b*x + 16*a) - 40*cos(14*b*x + 1
4*a) + 110*cos(12*b*x + 12*a) + 18*cos(10*b*x + 10*a) + 110*cos(8*b*x + 8*
a) - 40*cos(6*b*x + 6*a) - 30*cos(4*b*x + 4*a) + 15*cos(2*b*x + 2*a))*cos(
20*b*x + 20*a) + 4*(15*cos(16*b*x + 16*a) + 200*cos(14*b*x + 14*a) - 190*c
os(12*b*x + 12*a) - 216*cos(10*b*x + 10*a) - 190*cos(8*b*x + 8*a) + 200*co
s(6*b*x + 6*a) + 15*cos(4*b*x + 4*a) - 60*cos(2*b*x + 2*a) + 15)*cos(18*b*
x + 18*a) - 120*cos(18*b*x + 18*a)^2 - 12*(40*cos(14*b*x + 14*a) + 130*cos
(12*b*x + 12*a) - 102*cos(10*b*x + 10*a) + 130*cos(8*b*x + 8*a) + 40*cos(6
*b*x + 6*a) - 60*cos(4*b*x + 4*a) - 5*cos(2*b*x + 2*a) + 10)*cos(16*b*x +
16*a) + 360*cos(16*b*x + 16*a)^2 + 32*(100*cos(12*b*x + 12*a) + 78*cos(10*
b*x + 10*a) + 100*cos(8*b*x + 8*a) - 80*cos(6*b*x + 6*a) - 15*cos(4*b*x +
4*a) + 25*cos(2*b*x + 2*a) - 5)*cos(14*b*x + 14*a) - 1280*cos(14*b*x + 14*
a)^2 - 8*(642*cos(10*b*x + 10*a) - 220*cos(8*b*x + 8*a) - 400*cos(6*b*x +
6*a) + 195*cos(4*b*x + 4*a) + 95*cos(2*b*x + 2*a) - 55)*cos(12*b*x + 12*a)
 + 880*cos(12*b*x + 12*a)^2 - 24*(214*cos(8*b*x + 8*a) - 104*cos(6*b*x + 6
*a) - 51*cos(4*b*x + 4*a) + 36*cos(2*b*x + 2*a) - 3)*cos(10*b*x + 10*a) -
864*cos(10*b*x + 10*a)^2 + 40*(80*cos(6*b*x + 6*a) - 39*cos(4*b*x + 4*a) -
 19*cos(2*b*x + 2*a) + 11)*cos(8*b*x + 8*a) + 880*cos(8*b*x + 8*a)^2 - 160
*(3*cos(4*b*x + 4*a) - 5*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 1280*cos
(6*b*x + 6*a)^2 + 60*(cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a) + 360*cos(...
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = -\frac{\frac{60\sin(bx+a)^8-90\sin(bx+a)^6+20\sin(bx+a)^4+5\sin(bx+a)^2+2}{\left(\sin(bx+a)^2-1\right)^2\sin(bx+a)^6}+60\log\left(-\sin\left(bx+a\right)^2+1\right)-120\log\left(\left|\sin\left(bx+a\right)\right.\right.}{384\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
-1/384*((60*sin(b*x + a)^8 - 90*sin(b*x + a)^6 + 20*sin(b*x + a)^4 + 5*sin
(b*x + a)^2 + 2)/((sin(b*x + a)^2 - 1)^2*sin(b*x + a)^6) + 60*log(-sin(b*x
 + a)^2 + 1) - 120*log(abs(sin(b*x + a))))/b
```

3.57.9 Mupad [B] (verification not implemented)

Time = 19.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = \frac{5\ln\left(\sin\left(a+b\,x\right)^2\right)}{32\,b}-\frac{5\ln\left(\cos\left(a+b\,x\right)\right)}{16\,b}+\frac{-\frac{5\cos(a+b\,x)^8}{32}+\frac{25\cos(a+b\,x)^6}{64}-\frac{55\cos(a+b\,x)^4}{192}+\frac{5\cos(a+b\,x)^2}{128}+\frac{1}{128}}{b\,\left(-\cos\left(a+b\,x\right)^{10}+3\cos\left(a+b\,x\right)^8-3\cos\left(a+b\,x\right)^6+\cos\left(a+b\,x\right)^4\right)}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^5),x)
```

output
```
(5*log(sin(a + b*x)^2))/(32*b) - (5*log(cos(a + b*x)))/(16*b) + ((5*cos(a
+ b*x)^2)/128 - (55*cos(a + b*x)^4)/192 + (25*cos(a + b*x)^6)/64 - (5*cos(
a + b*x)^8)/32 + 1/128)/(b*(cos(a + b*x)^4 - 3*cos(a + b*x)^6 + 3*cos(a +
b*x)^8 - cos(a + b*x)^10))
```

3.58 $\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$

3.58.1 Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = -\frac{5\cot(a+bx)}{16b}-\frac{5\cot^3(a+bx)}{64b}-\frac{3\cot^5(a+bx)}{160b}-\frac{\cot^7(a+bx)}{448b}+\frac{15\tan(a+bx)}{64b}+\frac{\tan^3(a+bx)}{32b}+\frac{\tan^5(a+bx)}{320b}$$

output
```
-5/16*cot(b*x+a)/b-5/64*cot(b*x+a)^3/b-3/160*cot(b*x+a)^5/b-1/448*cot(b*x+
a)^7/b+15/64*tan(b*x+a)/b+1/32*tan(b*x+a)^3/b+1/320*tan(b*x+a)^5/b
```

3.58.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.29

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = -\frac{281\cot(a+bx)}{1120b}-\frac{53\cot(a+bx)\csc^2(a+bx)}{1120b}-\frac{27\cot(a+bx)\csc^4(a+bx)}{2240b}-\frac{\cot(a+bx)\csc^6(a+bx)}{448b}+\frac{33\tan(a+bx)}{160b}+\frac{\sec^2(a+bx)\tan(a+bx)}{40b}+\frac{\sec^4(a+bx)\tan(a+bx)}{320b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

output
```
(-281*Cot[a + b*x])/(1120*b) - (53*Cot[a + b*x]*Csc[a + b*x]^2)/(1120*b) -
 (27*Cot[a + b*x]*Csc[a + b*x]^4)/(2240*b) - (Cot[a + b*x]*Csc[a + b*x]^6)
/(448*b) + (33*Tan[a + b*x])/(160*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(40*b
) + (Sec[a + b*x]^4*Tan[a + b*x])/(320*b)
```

3.58.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^6}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{64}\int \csc^8(a+bx)\sec^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{64}\int \csc(a+bx)^8\sec(a+bx)^6dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^8(a+bx)\left(\tan^2(a+bx)+1\right)^6d\tan(a+bx)}{64b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^8(a+bx)+6\cot^6(a+bx)+15\cot^4(a+bx)+20\cot^2(a+bx)+\tan^4(a+bx)+6\tan^2(a+bx)+15\right)d\tan}{64b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\tan^5(a+bx)+2\tan^3(a+bx)+15\tan(a+bx)-\frac{1}{7}\cot^7(a+bx)-\frac{6}{5}\cot^5(a+bx)-5\cot^3(a+bx)-20\cot(a+}{64b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

output
```
(-20*Cot[a + b*x] - 5*Cot[a + b*x]^3 - (6*Cot[a + b*x]^5)/5 - Cot[a + b*x]
^7/7 + 15*Tan[a + b*x] + 2*Tan[a + b*x]^3 + Tan[a + b*x]^5/5)/(64*b)
```

3.58.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.58.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

method	result
risch	$\frac{32i\left(20\,\mathrm{e}^{10i(xb+a)}-5\,\mathrm{e}^{8i(xb+a)}-10\,\mathrm{e}^{6i(xb+a)}+4\,\mathrm{e}^{4i(xb+a)}+2\,\mathrm{e}^{2i(xb+a)}-1\right)}{35b\left(\mathrm{e}^{2i(xb+a)}-1\right)^{7}\left(\mathrm{e}^{2i(xb+a)}+1\right)^{5}}$
default	$\frac{-\frac{1}{7\sin(xb+a)^{7}\cos(xb+a)^{5}}+\frac{12}{35\sin(xb+a)^{5}\cos(xb+a)^{5}}-\frac{24}{35\sin(xb+a)^{5}\cos(xb+a)^{3}}+\frac{64}{35\cos(xb+a)^{3}\sin(xb+a)^{3}}-\frac{128}{35\cos(xb+a)\sin(xb+a)^{3}}+\frac{}{35\sin}}{64b}$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
32/35*I*(20*exp(10*I*(b*x+a))-5*exp(8*I*(b*x+a))-10*exp(6*I*(b*x+a))+4*exp
(4*I*(b*x+a))+2*exp(2*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^7/(exp(2*I*(b*x
+a))+1)^5
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx =$$
$$-\frac{1024\,\cos(bx+a)^{12}-3584\,\cos(bx+a)^{10}+4480\,\cos(bx+a)^{8}-2240\,\cos(bx+a)^{6}+280\,\cos(bx+a)}{2240\left(b\cos(bx+a)^{11}-3\,b\cos(bx+a)^{9}+3\,b\cos(bx+a)^{7}-b\cos(bx+a)^{5}\right)\sin(bx}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
-1/2240*(1024*cos(b*x + a)^12 - 3584*cos(b*x + a)^10 + 4480*cos(b*x + a)^8
 - 2240*cos(b*x + a)^6 + 280*cos(b*x + a)^4 + 28*cos(b*x + a)^2 + 7)/((b*c
os(b*x + a)^11 - 3*b*cos(b*x + a)^9 + 3*b*cos(b*x + a)^7 - b*cos(b*x + a)^
5)*sin(b*x + a))
```

3.58.6 Sympy [F]

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = \int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**6,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**6, x)
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2710 vs. $2(88) = 176$.

Time = 0.33 (sec) , antiderivative size = 2710, normalized size of antiderivative = 26.57

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
-32/35*((20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a)
+ 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(24*b*x + 24*a) - 2*(20*sin(
10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x +
4*a) + 2*sin(2*b*x + 2*a))*cos(22*b*x + 22*a) - 4*(20*sin(10*b*x + 10*a) -
 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b
*x + 2*a))*cos(20*b*x + 20*a) + 10*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x +
8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(
18*b*x + 18*a) + 5*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*
b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(16*b*x + 16*a) -
 20*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*
sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(14*b*x + 14*a) - (20*cos(10*b*x
 + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) +
 2*cos(2*b*x + 2*a) - 1)*sin(24*b*x + 24*a) + 2*(20*cos(10*b*x + 10*a) - 5
*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*sin(22*b*x + 22*a) + 4*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x +
 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)
*sin(20*b*x + 20*a) - 10*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*
cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(18*b*x
 + 18*a) - 5*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x +
6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(16*b*x + 16*a) ...
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$$
$$=\frac{7\tan(bx+a)^5+70\tan(bx+a)^3-\frac{700\tan(bx+a)^6+175\tan(bx+a)^4+42\tan(bx+a)^2+5}{\tan(bx+a)^7}+525\tan(bx+a)}{2240\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
1/2240*(7*tan(b*x + a)^5 + 70*tan(b*x + a)^3 - (700*tan(b*x + a)^6 + 175*t
an(b*x + a)^4 + 42*tan(b*x + a)^2 + 5)/tan(b*x + a)^7 + 525*tan(b*x + a))/
b
```

3.58.9 Mupad [B] (verification not implemented)

Time = 19.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$$
$$=\frac{15\tan(a+b\,x)}{64\,b}+\frac{\tan(a+b\,x)^3}{32\,b}+\frac{\tan(a+b\,x)^5}{320\,b}-\frac{\cot(a+b\,x)^7\left(\frac{5\tan(a+b\,x)^6}{16}+\frac{5\tan(a+b\,x)^4}{64}+\frac{3\tan(a+b\,x)^2}{160}+\frac{1}{448}\right)}{b}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^6),x)
```

output
```
(15*tan(a + b*x))/(64*b) + tan(a + b*x)^3/(32*b) + tan(a + b*x)^5/(320*b)
- (cot(a + b*x)^7*((3*tan(a + b*x)^2)/160 + (5*tan(a + b*x)^4)/64 + (5*tan
(a + b*x)^6)/16 + 1/448))/b
```

3.59 $\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx$

3.59.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = -\frac{1024\cos^{11}(a+bx)}{11b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{1024\cos^{17}(a+bx)}{17b}$$

output
```
-1024/11*cos(b*x+a)^11/b+3072/13*cos(b*x+a)^13/b-1024/5*cos(b*x+a)^15/b+10
24/17*cos(b*x+a)^17/b
```

3.59.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = -\frac{35\cos(a+bx)}{32b}-\frac{7\cos(3(a+bx))}{16b}+\frac{7\cos(5(a+bx))}{80b}+\frac{\cos(7(a+bx))}{8b}-\frac{5\cos(11(a+bx))}{176b}-\frac{\cos(13(a+bx))}{208b}+\frac{\cos(15(a+bx))}{320b}+\frac{\cos(17(a+bx))}{1088b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

output
```
(-35*Cos[a + b*x])/(32*b) - (7*Cos[3*(a + b*x)])/(16*b) + (7*Cos[5*(a + b*
x)])/(80*b) + Cos[7*(a + b*x)]/(8*b) - (5*Cos[11*(a + b*x)])/(176*b) - Cos
[13*(a + b*x)]/(208*b) + Cos[15*(a + b*x)]/(320*b) + Cos[17*(a + b*x)]/(10
88*b)
```

3.59.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{10}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$1024\int \cos^{10}(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$1024\int \cos(a+bx)^{10}\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{1024\int \cos^{10}(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{1024\int \left(-\cos^{16}(a+bx)+3\cos^{14}(a+bx)-3\cos^{12}(a+bx)+\cos^{10}(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1024\left(-\frac{1}{17}\cos^{17}(a+bx)+\frac{1}{5}\cos^{15}(a+bx)-\frac{3}{13}\cos^{13}(a+bx)+\frac{1}{11}\cos^{11}(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

output
```
(-1024*(Cos[a + b*x]^11/11 - (3*Cos[a + b*x]^13)/13 + Cos[a + b*x]^15/5 -
Cos[a + b*x]^17/17))/b
```

3.59.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.59.4 Maple [A] (verified)

Time = 168.67 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
default	$\frac{\frac{1024\cos(xb+a)^{17}}{17}-\frac{1024\cos(xb+a)^{15}}{5}+\frac{3072\cos(xb+a)^{13}}{13}-\frac{1024\cos(xb+a)^{11}}{11}}{b}$
risch	$-\frac{35\cos(xb+a)}{32b}+\frac{\cos(17xb+17a)}{1088b}+\frac{\cos(15xb+15a)}{320b}-\frac{\cos(13xb+13a)}{208b}-\frac{5\cos(11xb+11a)}{176b}+\frac{\cos(7xb+7a)}{8b}+\frac{7\cos(5xb+5a}{80b}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x,method=_RETURNVERBOSE)
```

output
```
1024/b*(1/17*cos(b*x+a)^17-1/5*cos(b*x+a)^15+3/13*cos(b*x+a)^13-1/11*cos(b
*x+a)^11)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = \frac{1024\left(715\cos\left(bx+a\right)^{17}-2431\cos\left(bx+a\right)^{15}+2805\cos\left(bx+a\right)^{13}-1105\cos\left(bx+a\right)^{11}\right)}{12155\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="fricas")
```

output
```
1024/12155*(715*cos(b*x + a)^17 - 2431*cos(b*x + a)^15 + 2805*cos(b*x + a)
^13 - 1105*cos(b*x + a)^11)/b
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**10,x)
```

output
```
Timed out
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = \frac{715\cos\left(17\,bx+17\,a\right)+2431\cos\left(15\,bx+15\,a\right)-3740\cos\left(13\,bx+13\,a\right)-22100\cos\left(11\,bx+11\,a\right)+9}{777920\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="maxima")
```

output
```
1/777920*(715*cos(17*b*x + 17*a) + 2431*cos(15*b*x + 15*a) - 3740*cos(13*b
*x + 13*a) - 22100*cos(11*b*x + 11*a) + 97240*cos(7*b*x + 7*a) + 68068*cos
(5*b*x + 5*a) - 340340*cos(3*b*x + 3*a) - 850850*cos(b*x + a))/b
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(53) = 106$.

Time = 0.37 (sec) , antiderivative size = 314, normalized size of antiderivative = 5.15

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx =$$

$$-\frac{32768\left(\frac{17(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{136(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{680(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{9775(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{71825(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{2210\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="giac")
```

output
```
-32768/12155*(17*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 136*(cos(b*x + a)
 - 1)^2/(cos(b*x + a) + 1)^2 + 680*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)
^3 + 9775*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 71825*(cos(b*x + a)
- 1)^5/(cos(b*x + a) + 1)^5 + 221000*(cos(b*x + a) - 1)^6/(cos(b*x + a) +
1)^6 + 486200*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 668525*(cos(b*x
+ a) - 1)^8/(cos(b*x + a) + 1)^8 + 692835*(cos(b*x + a) - 1)^9/(cos(b*x +
a) + 1)^9 + 466752*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 233376*(c
os(b*x + a) - 1)^11/(cos(b*x + a) + 1)^11 + 65637*(cos(b*x + a) - 1)^12/(c
os(b*x + a) + 1)^12 + 12155*(cos(b*x + a) - 1)^13/(cos(b*x + a) + 1)^13 -
1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^17)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 19.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx$$
$$= -\frac{-\frac{1024\cos(a+b\,x)^{17}}{17}+\frac{1024\cos(a+b\,x)^{15}}{5}-\frac{3072\cos(a+b\,x)^{13}}{13}+\frac{1024\cos(a+b\,x)^{11}}{11}}{b}$$

input
```
int(sin(2*a + 2*b*x)^10/sin(a + b*x)^3,x)
```

output
```
-((1024*cos(a + b*x)^11)/11 - (3072*cos(a + b*x)^13)/13 + (1024*cos(a + b*
x)^15)/5 - (1024*cos(a + b*x)^17)/17)/b
```

3.60 $\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx$

3.60.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \frac{512\sin^7(a+bx)}{7b} - \frac{2048\sin^9(a+bx)}{9b} + \frac{3072\sin^{11}(a+bx)}{11b} - \frac{2048\sin^{13}(a+bx)}{13b} + \frac{512\sin^{15}(a+bx)}{15b}$$

output
```
512/7*sin(b*x+a)^7/b-2048/9*sin(b*x+a)^9/b+3072/11*sin(b*x+a)^11/b-2048/13
*sin(b*x+a)^13/b+512/15*sin(b*x+a)^15/b
```

3.60.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \frac{512\sin^7(a+bx)}{7b} - \frac{2048\sin^9(a+bx)}{9b} + \frac{3072\sin^{11}(a+bx)}{11b} - \frac{2048\sin^{13}(a+bx)}{13b} + \frac{512\sin^{15}(a+bx)}{15b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

output
```
(512*Sin[a + b*x]^7)/(7*b) - (2048*Sin[a + b*x]^9)/(9*b) + (3072*Sin[a + b
*x]^11)/(11*b) - (2048*Sin[a + b*x]^13)/(13*b) + (512*Sin[a + b*x]^15)/(15
*b)
```

3.60.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^9(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^9}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$512\int \cos^9(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$512\int \cos(a+bx)^9\sin(a+bx)^6dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{512\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^4d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{512\int \left(\sin^{14}(a+bx)-4\sin^{12}(a+bx)+6\sin^{10}(a+bx)-4\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{512\left(\frac{1}{15}\sin^{15}(a+bx)-\frac{4}{13}\sin^{13}(a+bx)+\frac{6}{11}\sin^{11}(a+bx)-\frac{4}{9}\sin^9(a+bx)+\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

output
```
(512*(Sin[a + b*x]^7/7 - (4*Sin[a + b*x]^9)/9 + (6*Sin[a + b*x]^11)/11 - (
4*Sin[a + b*x]^13)/13 + Sin[a + b*x]^15/15))/b
```

3.60.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.60.4 Maple [A] (verified)

Time = 111.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
default	$\frac{\frac{512\sin(xb+a)^{15}}{15}-\frac{2048\sin(xb+a)^{13}}{13}+\frac{3072\sin(xb+a)^{11}}{11}-\frac{2048\sin(xb+a)^{9}}{9}+\frac{512\sin(xb+a)^{7}}{7}}{b}$
risch	$\frac{45\sin(xb+a)}{32b}-\frac{\sin(15xb+15a)}{480b}-\frac{3\sin(13xb+13a)}{416b}+\frac{3\sin(11xb+11a)}{352b}+\frac{17\sin(9xb+9a)}{288b}+\frac{3\sin(7xb+7a)}{224b}-\frac{39\sin(5xb+5a}{160b}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x,method=_RETURNVERBOSE)
```

output
```
512/b*(1/15*sin(b*x+a)^15-4/13*sin(b*x+a)^13+6/11*sin(b*x+a)^11-4/9*sin(b*
x+a)^9+1/7*sin(b*x+a)^7)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = -\frac{512\left(3003\cos(bx+a)^{14}-7161\cos(bx+a)^{12}+4473\cos(bx+a)^{10}-35\cos(bx+a)^{8}-40\cos(bx+\right.}{45045\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="fricas")
```

output
```
-512/45045*(3003*cos(b*x + a)^14 - 7161*cos(b*x + a)^12 + 4473*cos(b*x + a
)^10 - 35*cos(b*x + a)^8 - 40*cos(b*x + a)^6 - 48*cos(b*x + a)^4 - 64*cos(
b*x + a)^2 - 128)*sin(b*x + a)/b
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**9,x)
```

output
```
Timed out
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = -\frac{3003\sin(15\,bx+15\,a)+10395\sin(13\,bx+13\,a)-12285\sin(11\,bx+11\,a)-85085\sin(9\,bx+9\,a)-}{144144}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="maxima")
```

output
```
-1/1441440*(3003*sin(15*b*x + 15*a) + 10395*sin(13*b*x + 13*a) - 12285*sin
(11*b*x + 11*a) - 85085*sin(9*b*x + 9*a) - 19305*sin(7*b*x + 7*a) + 351351
*sin(5*b*x + 5*a) + 375375*sin(3*b*x + 3*a) - 2027025*sin(b*x + a))/b
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \frac{512\left(3003\sin\left(bx+a\right)^{15} - 13860\sin\left(bx+a\right)^{13} + 24570\sin\left(bx+a\right)^{11} - 20020\sin\left(bx+a\right)^{9} + 6435\sin(\right.}{45045\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="giac")
```

output
```
512/45045*(3003*sin(b*x + a)^15 - 13860*sin(b*x + a)^13 + 24570*sin(b*x +
a)^11 - 20020*sin(b*x + a)^9 + 6435*sin(b*x + a)^7)/b
```

3.60.9 Mupad [B] (verification not implemented)

Time = 19.55 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \frac{\frac{512\sin(a+b\,x)^{15}}{15} - \frac{2048\sin(a+b\,x)^{13}}{13} + \frac{3072\sin(a+b\,x)^{11}}{11} - \frac{2048\sin(a+b\,x)^{9}}{9} + \frac{512\sin(a+b\,x)^{7}}{7}}{b}$$

input
```
int(sin(2*a + 2*b*x)^9/sin(a + b*x)^3,x)
```

output
```
((512*sin(a + b*x)^7)/7 - (2048*sin(a + b*x)^9)/9 + (3072*sin(a + b*x)^11)
/11 - (2048*sin(a + b*x)^13)/13 + (512*sin(a + b*x)^15)/15)/b
```

3.61 $\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx$

3.61.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx=-\frac{256\cos^9(a+bx)}{9b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^{13}(a+bx)}{13b}$$

output
```
-256/9*cos(b*x+a)^9/b+512/11*cos(b*x+a)^11/b-256/13*cos(b*x+a)^13/b
```

3.61.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(46) = 92.

Time = 0.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.26

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx=-\frac{5\cos(a+bx)}{4b}-\frac{25\cos(3(a+bx))}{48b}+\frac{\cos(5(a+bx))}{16b}+\frac{\cos(7(a+bx))}{8b}+\frac{\cos(9(a+bx))}{72b}-\frac{3\cos(11(a+bx))}{176b}-\frac{\cos(13(a+bx))}{208b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

output
```
(-5*Cos[a + b*x])/(4*b) - (25*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/
(16*b) + Cos[7*(a + b*x)]/(8*b) + Cos[9*(a + b*x)]/(72*b) - (3*Cos[11*(a +
 b*x)])/(176*b) - Cos[13*(a + b*x)]/(208*b)
```

3.61.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^8}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$256\int \cos^8(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$256\int \cos(a+bx)^8\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{256\int \cos^8(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{256\int \left(\cos^{12}(a+bx)-2\cos^{10}(a+bx)+\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{256\left(\frac{1}{13}\cos^{13}(a+bx)-\frac{2}{11}\cos^{11}(a+bx)+\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

output
```
(-256*(Cos[a + b*x]^9/9 - (2*Cos[a + b*x]^11)/11 + Cos[a + b*x]^13/13))/b
```

3.61.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.61.4 Maple [A] (verified)

Time = 68.81 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
default	$-\frac{256\left(\frac{\cos(xb+a)^{13}}{13}-\frac{2\cos(xb+a)^{11}}{11}+\frac{\cos(xb+a)^{9}}{9}\right)}{b}$
risch	$-\frac{5\cos(xb+a)}{4b}-\frac{\cos(13xb+13a)}{208b}-\frac{3\cos(11xb+11a)}{176b}+\frac{\cos(9xb+9a)}{72b}+\frac{\cos(7xb+7a)}{8b}+\frac{\cos(5xb+5a)}{16b}-\frac{25\cos(3xb+3a)}{48b}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

output
```
-256/b*(1/13*cos(b*x+a)^13-2/11*cos(b*x+a)^11+1/9*cos(b*x+a)^9)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = -\frac{256\left(99\cos(bx+a)^{13}-234\cos(bx+a)^{11}+143\cos(bx+a)^{9}\right)}{1287\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

output
```
-256/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/
b
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**8,x)
```

output
```
Timed out
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = -\frac{99\cos(13\,bx+13\,a)+351\cos(11\,bx+11\,a)-286\cos(9\,bx+9\,a)-2574\cos(7\,bx+7\,a)-1287\cos}{20592\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

output
```
-1/20592*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x +
 9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x +
3*a) + 25740*cos(b*x + a))/b
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(40) = 80$.

Time = 0.34 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.39

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx =$$

$$-\frac{4096\left(\frac{13(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{78(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{572(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{3718(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-\frac{7722(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-\frac{13728(c}{(\cos(}\right.}{1287\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right.}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

output
```
-4096/1287*(13*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 78*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 - 572*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3
- 3718*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 7722*(cos(b*x + a) - 1)
^5/(cos(b*x + a) + 1)^5 - 13728*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6
- 12012*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 9009*(cos(b*x + a) - 1
)^8/(cos(b*x + a) + 1)^8 - 3003*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1)^9
- 858*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 - 1)/(b*((cos(b*x + a) -
 1)/(cos(b*x + a) + 1) - 1)^13)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = -\frac{256\left(99\cos(a+b\,x)^{13}-234\cos(a+b\,x)^{11}+143\cos(a+b\,x)^9\right)}{1287\,b}$$

input
```
int(sin(2*a + 2*b*x)^8/sin(a + b*x)^3,x)
```

output
```
-(256*(143*cos(a + b*x)^9 - 234*cos(a + b*x)^11 + 99*cos(a + b*x)^13))/(12
87*b)
```

3.62 $\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx$

3.62.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx=\frac{128\sin^5(a+bx)}{5b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^9(a+bx)}{3b}-\frac{128\sin^{11}(a+bx)}{11b}$$

output
```
128/5*sin(b*x+a)^5/b-384/7*sin(b*x+a)^7/b+128/3*sin(b*x+a)^9/b-128/11*sin(
b*x+a)^11/b
```

3.62.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx=\frac{128\sin^5(a+bx)}{5b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^9(a+bx)}{3b}-\frac{128\sin^{11}(a+bx)}{11b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*Sin[a + b*x]^5)/(5*b) - (384*Sin[a + b*x]^7)/(7*b) + (128*Sin[a + b*x
]^9)/(3*b) - (128*Sin[a + b*x]^11)/(11*b)
```

3.62.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^7}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$128\int \cos^7(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$128\int \cos(a+bx)^7\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{128\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{128\int \left(-\sin^{10}(a+bx)+3\sin^8(a+bx)-3\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{128\left(-\frac{1}{11}\sin^{11}(a+bx)+\frac{1}{3}\sin^9(a+bx)-\frac{3}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*(Sin[a + b*x]^5/5 - (3*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/3 - Sin[a +
 b*x]^11/11))/b
```

3.62.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.62.4 Maple [A] (verified)

Time = 41.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{128\left(\frac{\sin(xb+a)^{11}}{11}-\frac{\sin(xb+a)^{9}}{3}+\frac{3\sin(xb+a)^{7}}{7}-\frac{\sin(xb+a)^{5}}{5}\right)}{b}$	47
risch	$\frac{7\sin(xb+a)}{4b}+\frac{\sin(11xb+11a)}{88b}+\frac{\sin(9xb+9a)}{24b}-\frac{\sin(7xb+7a)}{56b}-\frac{11\sin(5xb+5a)}{40b}-\frac{\sin(3xb+3a)}{4b}$	83

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
-128/b*(1/11*sin(b*x+a)^11-1/3*sin(b*x+a)^9+3/7*sin(b*x+a)^7-1/5*sin(b*x+a
)^5)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{128\left(105\cos(bx+a)^{10}-140\cos(bx+a)^{8}+5\cos(bx+a)^{6}+6\cos(bx+a)^{4}+8\cos(bx+a)^{2}+16\right)\sin}{1155\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
128/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*
cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**7,x)
```

output
```
Timed out
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{105\sin(11\,bx+11\,a)+385\sin(9\,bx+9\,a)-165\sin(7\,bx+7\,a)-2541\sin(5\,bx+5\,a)-2310\sin(3\,bx}{9240\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
1/9240*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*
a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = -\frac{128\left(105\sin\left(bx+a\right)^{11}-385\sin\left(bx+a\right)^{9}+495\sin\left(bx+a\right)^{7}-231\sin\left(bx+a\right)^{5}\right)}{1155\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
-128/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 -
 231*sin(b*x + a)^5)/b
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = \frac{-\frac{128\sin(a+b\,x)^{11}}{11}+\frac{128\sin(a+b\,x)^9}{3}-\frac{384\sin(a+b\,x)^7}{7}+\frac{128\sin(a+b\,x)^5}{5}}{b}$$

input
```
int(sin(2*a + 2*b*x)^7/sin(a + b*x)^3,x)
```

output
```
((128*sin(a + b*x)^5)/5 - (384*sin(a + b*x)^7)/7 + (128*sin(a + b*x)^9)/3
- (128*sin(a + b*x)^11)/11)/b
```

3.63 $\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx$

3.63.1 Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx=-\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}$$

output
```
-64/7*cos(b*x+a)^7/b+64/9*cos(b*x+a)^9/b
```

3.63.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx=\frac{32\cos^7(a+bx)(-11+7\cos(2(a+bx)))}{63b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

output
```
(32*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

3.63.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^6}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$64\int \cos^6(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^6\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{64\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{64\int \left(\cos^6(a+bx)-\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{64\left(\frac{1}{7}\cos^7(a+bx)-\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

output
```
(-64*(Cos[a + b*x]^7/7 - Cos[a + b*x]^9/9))/b
```

3.63.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.63.4 Maple [A] (verified)

Time = 23.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{64 \cos(xb+a)^9}{9}-\frac{64 \cos(xb+a)^7}{7}}{b}$	27
risch	$-\frac{3 \cos(xb+a)}{2b}+\frac{\cos(9xb+9a)}{36b}+\frac{3 \cos(7xb+7a)}{28b}-\frac{2 \cos(3xb+3a)}{3b}$	55

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
64/b*(1/9*cos(b*x+a)^9-1/7*cos(b*x+a)^7)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = \frac{64\left(7\cos(bx+a)^9-9\cos(bx+a)^7\right)}{63\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
64/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**6,x)
```

output
```
Timed out
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = \frac{7\cos(9\,bx+9\,a)+27\cos(7\,bx+7\,a)-168\cos(3\,bx+3\,a)-378\cos(bx+a)}{252\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
1/252*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) - 3
78*cos(b*x + a))/b
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(27) = 54$.

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 5.87

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = -\frac{256\left(\frac{9\,(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{27\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{189\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{189\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + \frac{315\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} + \frac{105\,(\cos(bx+}{(\cos(bx+a)}\right.}{63\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^9}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
-256/63*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 27*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 + 189*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 18
9*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 315*(cos(b*x + a) - 1)^5/(co
s(b*x + a) + 1)^5 + 105*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 63*(co
s(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*
x + a) + 1) - 1)^9)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 20.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\left(9\cos\left(a+b\,x\right)^7-7\cos\left(a+b\,x\right)^9\right)}{63\,b}$$

input
```
int(sin(2*a + 2*b*x)^6/sin(a + b*x)^3,x)
```

output
```
-(64*(9*cos(a + b*x)^7 - 7*cos(a + b*x)^9))/(63*b)
```

3.64 $\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx$

3.64.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx=\frac{32\sin^3(a+bx)}{3b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^7(a+bx)}{7b}$$

output
```
32/3*sin(b*x+a)^3/b-64/5*sin(b*x+a)^5/b+32/7*sin(b*x+a)^7/b
```

3.64.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$=\frac{4(157+108\cos(2(a+bx))+15\cos(4(a+bx)))\sin^3(a+bx)}{105b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*(157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(105
*b)
```

3.64.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^5}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{32\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{32\int \left(\sin^6(a+bx)-2\sin^4(a+bx)+\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{32\left(\frac{1}{7}\sin^7(a+bx)-\frac{2}{5}\sin^5(a+bx)+\frac{1}{3}\sin^3(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*(Sin[a + b*x]^3/3 - (2*Sin[a + b*x]^5)/5 + Sin[a + b*x]^7/7))/b
```

3.64.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.64.4 Maple [A] (verified)

Time = 12.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\frac{32 \sin(xb+a)^7}{7}-\frac{64 \sin(xb+a)^5}{5}+\frac{32 \sin(xb+a)^3}{3}}{b}$	37
risch	$\frac{5 \sin(xb+a)}{2b}-\frac{\sin(7xb+7a)}{14b}-\frac{3 \sin(5xb+5a)}{10b}-\frac{\sin(3xb+3a)}{6b}$	55

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
32/b*(1/7*sin(b*x+a)^7-2/5*sin(b*x+a)^5+1/3*sin(b*x+a)^3)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(15\cos(bx+a)^6-3\cos(bx+a)^4-4\cos(bx+a)^2-8\right)\sin(bx+a)}{105\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-32/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(
b*x + a)/b
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{15\sin(7\,bx+7\,a)+63\sin(5\,bx+5\,a)+35\sin(3\,bx+3\,a)-525\sin(bx+a)}{210\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/210*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
525*sin(b*x + a))/b
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(15\,\sin\left(bx+a\right)^7 - 42\,\sin\left(bx+a\right)^5 + 35\,\sin\left(bx+a\right)^3\right)}{105\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
32/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(15\sin\left(a+b\,x\right)^7 - 42\sin\left(a+b\,x\right)^5 + 35\sin\left(a+b\,x\right)^3\right)}{105\,b}$$

input
```
int(sin(2*a + 2*b*x)^5/sin(a + b*x)^3,x)
```

output
```
(32*(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7))/(105*b)
```

3.65 $\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx$

3.65.1 Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx=-\frac{16\cos^5(a+bx)}{5b}$$

output
```
-16/5*cos(b*x+a)^5/b
```

3.65.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx=-\frac{16\cos^5(a+bx)}{5b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*Cos[a + b*x]^5)/(5*b)
```

3.65.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4776, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\csc^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2a+2bx)^4}{\sin(a+bx)^3}dx$$
$$\Big\downarrow \text{4776}$$
$$16\int \cos^4(a+bx)\sin(a+bx)dx$$
$$\Big\downarrow \text{3042}$$
$$16\int \cos(a+bx)^4\sin(a+bx)dx$$
$$\Big\downarrow \text{3045}$$
$$-\frac{16\int \cos^4(a+bx)d\cos(a+bx)}{b}$$
$$\Big\downarrow \text{15}$$
$$-\frac{16\cos^5(a+bx)}{5b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*Cos[a + b*x]^5)/(5*b)
```

3.65.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.65.4 Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{16 \cos(xb+a)^5}{5b}$	14
risch	$-\frac{2\cos(xb+a)}{b} - \frac{\cos(5xb+5a)}{5b} - \frac{\cos(3xb+3a)}{b}$	41

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-16/5*cos(b*x+a)^5/b
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\cos(bx+a)^5}{5b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-16/5*cos(b*x + a)^5/b
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{\cos(5\,bx+5\,a)+5\,\cos(3\,bx+3\,a)+10\,\cos(bx+a)}{5\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
-1/5*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 4.93

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{32\left(\frac{10\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{5\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+1\right)}{5\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^5}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
32/5*(10*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 5*(cos(b*x + a) - 1)^
4/(cos(b*x + a) + 1)^4 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)
^5)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\cos\left(a+b\,x\right)^5}{5\,b}$$

input
```
int(sin(2*a + 2*b*x)^4/sin(a + b*x)^3,x)
```

output
```
-(16*cos(a + b*x)^5)/(5*b)
```

3.66 $\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx$

3.66.1 Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}$$

output
```
8*sin(b*x+a)/b-8/3*sin(b*x+a)^3/b
```

3.66.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx=8\left(\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\right)$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
8*(Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b))
```

3.66.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^3}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \sin\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$-\frac{8\int \left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-8*(-Sin[a + b*x] + Sin[a + b*x]^3/3))/b
```

3.66.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.66.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{8\left(2+\cos(xb+a)^{2}\right)\sin(xb+a)}{3b}$	22
risch	$\frac{6\sin(xb+a)}{b}+\frac{2\sin(3xb+3a)}{3b}$	27

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
8/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
8/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{2\left(\sin\left(3\,bx+3\,a\right)+9\,\sin\left(bx+a\right)\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
2/3*(sin(3*b*x + 3*a) + 9*sin(b*x + a))/b
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(\sin\left(bx+a\right)^3-3\,\sin\left(bx+a\right)\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-8/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.66.9 Mupad [B] (verification not implemented)

Time = 19.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(3\,\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^3\right)}{3\,b}$$

input
```
int(sin(2*a + 2*b*x)^3/sin(a + b*x)^3,x)
```

output
```
(8*(3*sin(a + b*x) - sin(a + b*x)^3))/(3*b)
```

3.67 $\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx$

3.67.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx=-\frac{4\text{arctanh}(\cos(a+bx))}{b}+\frac{4\cos(a+bx)}{b}$$

output
```
-4*arctanh(cos(b*x+a))/b+4*cos(b*x+a)/b
```

3.67.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx=4\left(\frac{\cos(a+bx)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
4*(Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)
```

3.67.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 25, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^2}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$4\int \cos(a+bx)\cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int -\sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-4\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{4\int \frac{\cos^2(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{4\left(\int \frac{1}{1-\cos^2(a+bx)}d\cos(a+bx)-\cos(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{4(\text{arctanh}(\cos(a+bx))-\cos(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*(ArcTanh[Cos[a + b*x]] - Cos[a + b*x]))/b
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.67.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{4\cos(xb+a)+4\ln(\csc(xb+a)-\cot(xb+a))}{b}$	29
risch	$\frac{2\,\mathrm{e}^{i(xb+a)}}{b}+\frac{2\,\mathrm{e}^{-i(xb+a)}}{b}+\frac{4\ln(\mathrm{e}^{i(xb+a)}-1)}{b}-\frac{4\ln(\mathrm{e}^{i(xb+a)}+1)}{b}$	64

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/b*(cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\left(2\,\cos\left(bx+a\right)-\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)\right)}{b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) +
1/2))/b
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.83

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \frac{2\left(2\cos\left(bx+a\right)-\log\left(\cos\left(bx\right)^2+2\cos\left(bx\right)\cos\left(a\right)+\cos\left(a\right)^2+\sin\left(bx\right)^2-2\sin\left(bx\right)\sin\left(a\right)+\sin\left(a\right)^2\right)\right.}{b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
2*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*
x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
+ cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{2\left(\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)\right)}{b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(-(cos(b*x + a) - 1
)/(cos(b*x + a) + 1)))/b
```

3.67.9 Mupad [B] (verification not implemented)

Time = 21.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{4\,\cos(a+b\,x)-4\,\mathrm{atanh}(\cos(a+b\,x))}{b}$$

input
```
int(sin(2*a + 2*b*x)^2/sin(a + b*x)^3,x)
```

output
```
(4*cos(a + b*x) - 4*atanh(cos(a + b*x)))/b
```

3.68 $\int \csc^3(a+bx)\sin(2a+2bx)\,dx$

3.68.1 Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\csc(a+bx)}{b}$$

output
```
-2*csc(b*x+a)/b
```

3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\csc(a+bx)}{b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Csc[a + b*x])/b
```

3.68.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(2a+2bx)}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \; 4776$$

$$2\int \cot(a+bx)\csc(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$2\int -\sec\left(a+bx-\frac{\pi}{2}\right)\tan\left(a+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 25$$

$$-2\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)\tan\left(\frac{1}{2}(2a-\pi)+bx\right)dx$$

$$\Big\downarrow \; 3086$$

$$-\frac{2\int 1d\csc(a+bx)}{b}$$

$$\Big\downarrow \; 24$$

$$-\frac{2\csc(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Csc[a + b*x])/b
```

3.68.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.68.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{2}{\sin(xb+a)b}$	14
risch	$-\frac{4i\mathrm{e}^{i(xb+a)}}{b\left(\mathrm{e}^{2i(xb+a)}-1\right)}$	29

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-2/sin(b*x+a)/b
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2}{b\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2/(b*sin(b*x + a))
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a),x)
```

output
```
Timed out
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(11) = 22.

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 7.64

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = -\frac{4\left(\cos(bx+a)\sin(2bx+2a)-\cos(2bx+2a)\sin(bx+a)+\sin(bx+a)\right)}{b\cos(2bx+2a)^2+b\sin(2bx+2a)^2-2b\cos(2bx+2a)+b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-4*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*
x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a
) + b)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx=-\frac{2}{b\sin{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-2/(b*sin(b*x + a))
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx=-\frac{2}{b\,\sin{(a+b\,x)}}$$

input
```
int(sin(2*a + 2*b*x)/sin(a + b*x)^3,x)
```

output
```
-2/(b*sin(a + b*x))
```

3.69 $\int \csc^3(a+bx)\csc(2a+2bx)\,dx$

3.69.1 Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}-\frac{\csc^3(a+bx)}{6b}$$

output
```
1/2*arctanh(sin(b*x+a))/b-1/2*csc(b*x+a)/b-1/6*csc(b*x+a)^3/b
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(a+bx)\right)}{6b}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

output
```
-1/6*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

3.69.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4776, 3042, 3101, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{2}\int \csc^4(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc(a+bx)^4\sec(a+bx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\text{arctanh}(\csc(a+bx))+\frac{1}{3}\csc^3(a+bx)+\csc(a+bx)}{2b}$$

input
```
Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*(-ArcTanh[Csc[a + b*x]] + Csc[a + b*x] + Csc[a + b*x]^3/3)/b
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.69.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\frac{1}{3\sin(xb+a)^3}-\frac{1}{\sin(xb+a)}+\ln(\sec(xb+a)+\tan(xb+a))}{2b}$	41
risch	$-\frac{i(3\,\mathrm{e}^{5i(xb+a)}-10\,\mathrm{e}^{3i(xb+a)}+3\,\mathrm{e}^{i(xb+a)})}{3b(\mathrm{e}^{2i(xb+a)}-1)^3}-\frac{\ln(\mathrm{e}^{i(xb+a)}-i)}{2b}+\frac{\ln(i+\mathrm{e}^{i(xb+a)})}{2b}$	91

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-1/3/sin(b*x+a)^3-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \frac{3\left(\cos\left(bx+a\right)^2-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin}{12\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/12*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b
*x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)
/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

3.69.6 Sympy [F]

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \int \csc^3\left(a+bx\right)\csc\left(2a+2bx\right)dx$$

input
```
integrate(csc(b*x+a)**3*csc(2*b*x+2*a),x)
```

output
```
Integral(csc(a + b*x)**3*csc(2*a + 2*b*x), x)
```

3.69.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(37) = 74$.

Time = 0.33 (sec) , antiderivative size = 834, normalized size of antiderivative = 19.40

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/12*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*
b*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 1
2*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*
b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2
 + 6*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*
cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6
*a) - sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x
 + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(
b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(
b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(
3*cos(5*b*x + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a
) - 12*(3*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12
*(10*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x
 + 2*a) - 1)*sin(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36
*cos(b*x + a)*sin(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin
(b*x + a))/(b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x
+ 2*a)^2 + b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x
+ 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a)
 - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a)...
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx$$
$$= -\frac{\frac{2\left(3\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(-\sin\left(bx+a\right)+1\right)}{12\,b}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/12*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) +
 3*log(-sin(b*x + a) + 1))/b
```

3.69.9 Mupad [B] (verification not implemented)

Time = 19.91 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx=\frac{\operatorname{atanh}(\sin{(a+b\,x)})}{2\,b}-\frac{\frac{{\sin(a+b\,x)}^2}{2}+\frac{1}{6}}{b\,{\sin{(a+b\,x)}}^3}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)),x)
```

output
```
atanh(sin(a + b*x))/(2*b) - (sin(a + b*x)^2/2 + 1/6)/(b*sin(a + b*x)^3)
```

3.70 $\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$

3.70.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{15\operatorname{arctanh}(\cos(a+bx))}{32b}+\frac{15\sec(a+bx)}{32b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{32b}-\frac{\csc^4(a+bx)\sec(a+bx)}{16b}$$

output
```
-15/32*arctanh(cos(b*x+a))/b+15/32*sec(b*x+a)/b-5/32*csc(b*x+a)^2*sec(b*x+
a)/b-1/16*csc(b*x+a)^4*sec(b*x+a)/b
```

3.70.2 Mathematica [A] (verified)

Time = 4.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.84

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{14\csc^2\left(\frac{1}{2}(a+bx)\right)+\csc^4\left(\frac{1}{2}(a+bx)\right)+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(78+\cos(a+bx)\left(-8\left(8+15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right.\right.\right.}{-1+\tan^2\left(\frac{1}{2}(a+bx)\right)}}{256b}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/256*(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(
78 + Cos[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)
/2]]) + Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/
2]^2))/b
```

3.70.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4776, 3042, 3102, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \csc^5(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)^5\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{4b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\sec(a+bx))-\sec(a+bx))\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{4b}$$

input
```
Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

output
```
(-1/4*Sec[a + b*x]^5/(1 - Sec[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Sec[a + b*x
]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Sec[a + b*x]^2))))/4)/(4*b)
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.70.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{-\frac{1}{4\cos(xb+a)\sin(xb+a)^4}-\frac{5}{8\sin(xb+a)^2\cos(xb+a)}+\frac{15}{8\cos(xb+a)}+\frac{15\ln(\csc(xb+a)-\cot(xb+a))}{8}}{4b}$	71
risch	$\frac{15\,\mathrm{e}^{9i(xb+a)}-40\,\mathrm{e}^{7i(xb+a)}+18\,\mathrm{e}^{5i(xb+a)}-40\,\mathrm{e}^{3i(xb+a)}+15\,\mathrm{e}^{i(xb+a)}}{16b\left(\mathrm{e}^{2i(xb+a)}-1\right)^4\left(\mathrm{e}^{2i(xb+a)}+1\right)}-\frac{15\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{32b}+\frac{15\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{32b}$	123

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(-1/4/cos(b*x+a)/sin(b*x+a)^4-5/8/sin(b*x+a)^2/cos(b*x+a)+15/8/cos(b
*x+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(62) = 124.

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.89

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$
$$=\frac{30\cos(bx+a)^4-50\cos(bx+a)^2-15\left(\cos(bx+a)^5-2\cos(bx+a)^3+\cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+\right.}{64\left(b\cos(bx+a)^5-2\,b\cos(bx+\right.}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/64*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b
*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5
 - 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*
cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

3.70.6 Sympy [F]

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**2,x)
```

output
```
Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**2, x)
```

3.70.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. $2(62) = 124$.

Time = 0.25 (sec) , antiderivative size = 2237, normalized size of antiderivative = 31.96

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/64*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) -
 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*b
*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) -
 1)*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40*
cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x +
6*a) + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*(
18*cos(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x + 6
*a) + 72*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) -
40*(8*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*b
*x + 2*a) - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*(
2*(3*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*
b*x + 2*a) - 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x
 + 6*a) + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) -
9*cos(8*b*x + 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos
(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*
x + 4*a) - 4*cos(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x +
8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(1
0*b*x + 10*a) - sin(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x
 + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*
(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*...
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(62) = 124$.

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.29

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$
$$= \frac{\frac{\left(\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{90(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+60\log\left(-\frac{\cos(\cdots}{\cos(\cdots}\right.}{256\,b}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/256*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 90*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*
(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a)
 + 1)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + 60*log(-(cos(b
*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \frac{\frac{15\cos(a+b\,x)^4}{32}-\frac{25\cos(a+b\,x)^2}{32}+\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)} - \frac{15\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{32\,b}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^2),x)
```

output
```
((15*cos(a + b*x)^4)/32 - (25*cos(a + b*x)^2)/32 + 1/4)/(b*(cos(a + b*x) -
 2*cos(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(32*b)
```

3.71 $\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$

3.71.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx=\frac{7\text{arctanh}(\sin(a+bx))}{16b}-\frac{7\csc(a+bx)}{16b}-\frac{7\csc^3(a+bx)}{48b}-\frac{7\csc^5(a+bx)}{80b}+\frac{\csc^5(a+bx)\sec^2(a+bx)}{16b}$$

output
```
7/16*arctanh(sin(b*x+a))/b-7/16*csc(b*x+a)/b-7/48*csc(b*x+a)^3/b-7/80*csc(
b*x+a)^5/b+1/16*csc(b*x+a)^5*sec(b*x+a)^2/b
```

3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.38

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx=-\frac{\csc^5(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},2,-\frac{3}{2},\sin^2(a+bx)\right)}{40b}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/40*(Csc[a + b*x]^5*Hypergeometric2F1[-5/2, 2, -3/2, Sin[a + b*x]^2])/b
```

3.71.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc^6(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^6\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^8(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^7(a+bx)}{2(1-\csc^2(a+bx))}-\frac{7}{2}\int \frac{\csc^6(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\csc^7(a+bx)}{2(1-\csc^2(a+bx))}-\frac{7}{2}\int \left(-\csc^4(a+bx)-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\csc^7(a+bx)}{2(1-\csc^2(a+bx))}-\frac{7}{2}\left(\text{arctanh}(\csc(a+bx))-\frac{1}{5}\csc^5(a+bx)-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)}{8b}$$

input
```
Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/8*(Csc[a + b*x]^7/(2*(1 - Csc[a + b*x]^2)) - (7*(ArcTanh[Csc[a + b*x]]
- Csc[a + b*x] - Csc[a + b*x]^3/3 - Csc[a + b*x]^5/5))/2)/b
```

3.71.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.71.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

method	result
default	$\frac{-\frac{1}{5\sin(xb+a)^5\cos(xb+a)^2}-\frac{7}{15\sin(xb+a)^3\cos(xb+a)^2}+\frac{7}{6\sin(xb+a)\cos(xb+a)^2}-\frac{7}{2\sin(xb+a)}+\frac{7\ln(\sec(xb+a)+\tan(xb+a))}{2}}{8b}$
risch	$-\frac{i\left(105\,\mathrm{e}^{13i(xb+a)}-350\,\mathrm{e}^{11i(xb+a)}+231\,\mathrm{e}^{9i(xb+a)}+412\,\mathrm{e}^{7i(xb+a)}+231\,\mathrm{e}^{5i(xb+a)}-350\,\mathrm{e}^{3i(xb+a)}+105\,\mathrm{e}^{i(xb+a)}\right)}{120b\left(\mathrm{e}^{2i(xb+a)}-1\right)^5\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}-\frac{7\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{16b}$

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/5/sin(b*x+a)^5/cos(b*x+a)^2-7/15/sin(b*x+a)^3/cos(b*x+a)^2+7/6/s
in(b*x+a)/cos(b*x+a)^2-7/2/sin(b*x+a)+7/2*ln(sec(b*x+a)+tan(b*x+a)))
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.05

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = -\frac{210\,\cos(bx+a)^6-490\,\cos(bx+a)^4-105\left(\cos(bx+a)^6-2\,\cos(bx+a)^4+\cos(bx+a)^2\right)\log(\sin(b}{480\left(b\cos(bx+\right.}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/480*(210*cos(b*x + a)^6 - 490*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - 2*
cos(b*x + a)^4 + cos(b*x + a)^2)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*
(cos(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*log(-sin(b*x + a) + 1
)*sin(b*x + a) + 322*cos(b*x + a)^2 - 30)/((b*cos(b*x + a)^6 - 2*b*cos(b*x
 + a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

3.71.6 Sympy [F]

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**3,x)
```

output
```
Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**3, x)
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3095 vs. $2(71) = 142$.

Time = 0.43 (sec) , antiderivative size = 3095, normalized size of antiderivative = 38.21

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/480*(4*(105*sin(13*b*x + 13*a) - 350*sin(11*b*x + 11*a) + 231*sin(9*b*x
+ 9*a) + 412*sin(7*b*x + 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a
) + 105*sin(b*x + a))*cos(14*b*x + 14*a) + 420*(3*sin(12*b*x + 12*a) - sin
(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) + 5*sin(6*b*x + 6*a) + sin(4*b*x + 4*
a) - 3*sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 12*(350*sin(11*b*x + 11*a) -
 231*sin(9*b*x + 9*a) - 412*sin(7*b*x + 7*a) - 231*sin(5*b*x + 5*a) + 350*
sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(12*b*x + 12*a) + 1400*(sin(10*b*x
 + 10*a) + 5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 3*
sin(2*b*x + 2*a))*cos(11*b*x + 11*a) + 4*(231*sin(9*b*x + 9*a) + 412*sin(7
*b*x + 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x +
a))*cos(10*b*x + 10*a) - 924*(5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - si
n(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 20*(412*sin(7*b*x
+ 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x + a))*c
os(8*b*x + 8*a) + 1648*(5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*
x + 2*a))*cos(7*b*x + 7*a) - 140*(33*sin(5*b*x + 5*a) - 50*sin(3*b*x + 3*a
) + 15*sin(b*x + a))*cos(6*b*x + 6*a) + 924*(sin(4*b*x + 4*a) - 3*sin(2*b*
x + 2*a))*cos(5*b*x + 5*a) + 140*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*co
s(4*b*x + 4*a) + 105*(2*(3*cos(12*b*x + 12*a) - cos(10*b*x + 10*a) - 5*cos
(8*b*x + 8*a) + 5*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a)
 + 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 + 6*(cos(10*b*x + 10*a)...
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\frac{30\sin(bx+a)}{\sin(bx+a)^2-1}+\frac{4\left(45\sin(bx+a)^4+10\sin(bx+a)^2+3\right)}{\sin(bx+a)^5}-105\log\left(\sin\left(bx+a\right)+1\right)+105\log\left(-\sin\left(bx+a\right)+1\right)}{480\,b}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-1/480*(30*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(45*sin(b*x + a)^4 + 10*s
in(b*x + a)^2 + 3)/sin(b*x + a)^5 - 105*log(sin(b*x + a) + 1) + 105*log(-s
in(b*x + a) + 1))/b
```

3.71.9 Mupad [B] (verification not implemented)

Time = 19.50 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{7\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{16\,b} - \frac{-\frac{7\sin(a+b\,x)^6}{16}+\frac{7\sin(a+b\,x)^4}{24}+\frac{7\sin(a+b\,x)^2}{120}+\frac{1}{40}}{b\,\left(\sin\left(a+b\,x\right)^5-\sin\left(a+b\,x\right)^7\right)}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^3),x)
```

output
```
(7*atanh(sin(a + b*x)))/(16*b) - ((7*sin(a + b*x)^2)/120 + (7*sin(a + b*x)
^4)/24 - (7*sin(a + b*x)^6)/16 + 1/40)/(b*(sin(a + b*x)^5 - sin(a + b*x)^7
))
```

3.72 $\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$

3.72.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx=-\frac{105\text{arctanh}(\cos(a+bx))}{256b}+\frac{105\sec(a+bx)}{256b}+\frac{35\sec^3(a+bx)}{256b}-\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b}-\frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b}-\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b}$$

output
```
-105/256*arctanh(cos(b*x+a))/b+105/256*sec(b*x+a)/b+35/256*sec(b*x+a)^3/b-
21/256*csc(b*x+a)^2*sec(b*x+a)^3/b-3/128*csc(b*x+a)^4*sec(b*x+a)^3/b-1/96*
csc(b*x+a)^6*sec(b*x+a)^3/b
```

3.72.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 278 vs. 2(112) = 224.

Time = 1.15 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.48

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$
$$=\frac{\csc^{12}(a+bx)\left(1150-4752\cos(2(a+bx))+1600\cos(3(a+bx))+504\cos(4(a+bx))+1680\cos(6(a+b\right.}{}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

output
```
(Csc[a + b*x]^12*(1150 - 4752*Cos[2*(a + b*x)] + 1600*Cos[3*(a + b*x)] + 5
04*Cos[4*(a + b*x)] + 1680*Cos[6*(a + b*x)] - 600*Cos[7*(a + b*x)] - 630*C
os[8*(a + b*x)] + 200*Cos[9*(a + b*x)] + 2520*Cos[3*(a + b*x)]*Log[Cos[(a
+ b*x)/2]] - 945*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 315*Cos[9*(a + b
*x)]*Log[Cos[(a + b*x)/2]] - 30*Cos[a + b*x]*(40 + 63*Log[Cos[(a + b*x)/2]
] - 63*Log[Sin[(a + b*x)/2]]) - 2520*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]
] + 945*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]] - 315*Cos[9*(a + b*x)]*Log[
Sin[(a + b*x)/2]]))/(3072*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

3.72.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4776, 3042, 3102, 252, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^7(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^7\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^{10}(a+bx)}{(1-\sec^2(a+bx))^4}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\int \frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \textbf{252}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\left(\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}-\frac{7}{4}\int\frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)\right)}{16b}$$

$$\Big\downarrow \textbf{252}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\left(\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}-\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int\frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)\right)}{16b}$$

$$\Big\downarrow \textbf{254}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\left(\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}-\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int\left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)\right)\right)}{16b}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\left(\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}-\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)\right)\right)}{16b}$$

input
```
Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

output
```
(Sec[a + b*x]^9/(6*(1 - Sec[a + b*x]^2)^3) - (3*(Sec[a + b*x]^7/(4*(1 - Se
c[a + b*x]^2)^2) - (7*(Sec[a + b*x]^5/(2*(1 - Sec[a + b*x]^2)) - (5*(ArcTa
nh[Sec[a + b*x]] - Sec[a + b*x] - Sec[a + b*x]^3/3))/2))/4))/2)/(16*b)
```

3.72.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.72.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

method	result
default	$\frac{-\frac{1}{6\sin(xb+a)^6\cos(xb+a)^3}-\frac{3}{8\sin(xb+a)^4\cos(xb+a)^3}+\frac{7}{8\sin(xb+a)^2\cos(xb+a)^3}-\frac{35}{16\sin(xb+a)^2\cos(xb+a)}+\frac{105}{16\cos(xb+a)}+\frac{105\ln(\csc(xb+a)-\mathrm{co}}{16}}{16b}$
risch	$\frac{315\,\mathrm{e}^{17i(xb+a)}-840\,\mathrm{e}^{15i(xb+a)}-252\,\mathrm{e}^{13i(xb+a)}+2376\,\mathrm{e}^{11i(xb+a)}-1150\,\mathrm{e}^{9i(xb+a)}+2376\,\mathrm{e}^{7i(xb+a)}-252\,\mathrm{e}^{5i(xb+a)}-840\,\mathrm{e}^{3i(xb+a)}+315}{384b\left(\mathrm{e}^{2i(xb+a)}-1\right)^6\left(\mathrm{e}^{2i(xb+a)}+1\right)^3}$

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(-1/6/sin(b*x+a)^6/cos(b*x+a)^3-3/8/sin(b*x+a)^4/cos(b*x+a)^3+7/8/s
in(b*x+a)^2/cos(b*x+a)^3-35/16/sin(b*x+a)^2/cos(b*x+a)+105/16/cos(b*x+a)+1
05/16*ln(csc(b*x+a)-cot(b*x+a)))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.73

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$
$$= \frac{630\cos(bx+a)^8 - 1680\cos(bx+a)^6 + 1386\cos(bx+a)^4 - 288\cos(bx+a)^2 - 315\left(\cos(bx+a)^9 - 3\right.}{15}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/1536*(630*cos(b*x + a)^8 - 1680*cos(b*x + a)^6 + 1386*cos(b*x + a)^4 - 2
88*cos(b*x + a)^2 - 315*(cos(b*x + a)^9 - 3*cos(b*x + a)^7 + 3*cos(b*x + a
)^5 - cos(b*x + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 315*(cos(b*x + a)^9 -
3*cos(b*x + a)^7 + 3*cos(b*x + a)^5 - cos(b*x + a)^3)*log(-1/2*cos(b*x + a
) + 1/2) - 32)/(b*cos(b*x + a)^9 - 3*b*cos(b*x + a)^7 + 3*b*cos(b*x + a)^5
 - b*cos(b*x + a)^3)
```

3.72.6 Sympy [F]

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**4,x)
```

output
```
Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**4, x)
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4268 vs. 2(100) = 200.

Time = 0.38 (sec) , antiderivative size = 4268, normalized size of antiderivative = 38.11

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/1536*(4*(315*cos(17*b*x + 17*a) - 840*cos(15*b*x + 15*a) - 252*cos(13*b*
x + 13*a) + 2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7*b
*x + 7*a) - 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x + a)
)*cos(18*b*x + 18*a) - 1260*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) +
 6*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*
b*x + 2*a) - 1)*cos(17*b*x + 17*a) + 12*(840*cos(15*b*x + 15*a) + 252*cos(
13*b*x + 13*a) - 2376*cos(11*b*x + 11*a) + 1150*cos(9*b*x + 9*a) - 2376*co
s(7*b*x + 7*a) + 252*cos(5*b*x + 5*a) + 840*cos(3*b*x + 3*a) - 315*cos(b*x
 + a))*cos(16*b*x + 16*a) - 3360*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10
*a) - 6*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*co
s(15*b*x + 15*a) - 1008*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10*a) - 6*c
os(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(13*b*x
+ 13*a) + 32*(2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7
*b*x + 7*a) - 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x +
a))*cos(12*b*x + 12*a) - 9504*(6*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) -
 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 24*(115
0*cos(9*b*x + 9*a) - 2376*cos(7*b*x + 7*a) + 252*cos(5*b*x + 5*a) + 840*co
s(3*b*x + 3*a) - 315*cos(b*x + a))*cos(10*b*x + 10*a) + 4600*(6*cos(8*b*x
+ 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(9*b*x + 9*a) - 7
2*(792*cos(7*b*x + 7*a) - 84*cos(5*b*x + 5*a) - 280*cos(3*b*x + 3*a) + ...
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(100) = 200$.

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.39

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx =$$

$$-\frac{\frac{285\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{21\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{\frac{18\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{225\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{2966\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{3513\,(\cos(bx+\cdots}{(\cos(bx+a)\cdots}}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-\cdots}{(\cos(bx+a)+\cdots}\right.}}{6144\,b}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/6144*(285*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 21*(cos(b*x + a) - 1)
^2/(cos(b*x + a) + 1)^2 + (cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + (18*
(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 225*(cos(b*x + a) - 1)^2/(cos(b*x
+ a) + 1)^2 - 2966*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 3513*(cos(b
*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 660*(cos(b*x + a) - 1)^5/(cos(b*x +
a) + 1)^5 + 1155*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1)/((cos(b*x
+ a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^
3 - 1260*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

3.72.9 Mupad [B] (verification not implemented)

Time = 19.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{-\frac{105\cos(a+b\,x)^8}{256}+\frac{35\cos(a+b\,x)^6}{32}-\frac{231\cos(a+b\,x)^4}{256}+\frac{3\cos(a+b\,x)^2}{16}+\frac{1}{48}}{b\,\left(-\cos\left(a+b\,x\right)^9+3\cos\left(a+b\,x\right)^7-3\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)} - \frac{105\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^4),x)
```

output
```
((3*cos(a + b*x)^2)/16 - (231*cos(a + b*x)^4)/256 + (35*cos(a + b*x)^6)/32
 - (105*cos(a + b*x)^8)/256 + 1/48)/(b*(cos(a + b*x)^3 - 3*cos(a + b*x)^5
+ 3*cos(a + b*x)^7 - cos(a + b*x)^9)) - (105*atanh(cos(a + b*x)))/(256*b)
```

3.73 $\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.73.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\begin{aligned}\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx =& -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{32b}\\&+\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b}\\&-\frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b}\\&+\frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}\\&-\frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\end{aligned}$$

output
```
-5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b+5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+5/24*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-1/6*cos(b*x+a)*si
n(2*b*x+2*a)^(5/2)/b-5/16*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.73.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{15\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(14\,\mathrm{co}}{96b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(15*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]]) - 2*(14*Cos[a + b*x] + 3*Cos[3*(a + b*x)] - 2
*Cos[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/(96*b)
```

3.73.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4790, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^{5/2}dx$$

$$\Big\downarrow \text{4790}$$

$$\frac{5}{6}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$$\Big\downarrow \text{4789}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 3042

$$\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 4790

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 3042

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 4793

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a\cdots}}{}\right.\right.$$
$$\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/6*(Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/b + (5*((3*((-1/2*ArcSin[Cos[a
+ b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a
+ 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 + (S
in[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/6
```

3.73. $\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.73.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.73.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 76.22 (sec) , antiderivative size = 183661410, normalized size of antiderivative = 1350451.54

method	result	size
default	Expression too large to display	183661410

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(118) = 236$.

Time = 0.26 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.14

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{8\sqrt{2}\left(32\cos(bx+a)^5 - 52\cos(bx+a)^3 + 5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)} + 30\arctan\left(-\frac{\sqrt{2}\sqrt{}}{}\right.}{}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
1/384*(8*sqrt(2)*(32*cos(b*x + a)^5 - 52*cos(b*x + a)^3 + 5*cos(b*x + a))*
sqrt(cos(b*x + a)*sin(b*x + a)) + 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(co
s(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 30*arctan(-(2*sqrt(2)*s
qrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a
) - sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)
^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/
b
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.73.7 Maxima [F]

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

3.73.8 Giac [F]

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)\,\sin\left(2\,a+2\,b\,x\right)^{5/2}\,dx$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2), x)
```

3.74 $\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.74.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{16b} - \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b} + \frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b} - \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

output
```
-3/16*arcsin(cos(b*x+a)-sin(b*x+a))/b-3/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-1/4*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+3/8*sin(b*x+a)*sin
(2*b*x+2*a)^(1/2)/b
```

3.74.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{-3\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin(2}}{16b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-3*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(2*Sin[a + b*x] - S
in[3*(a + b*x)]))/(16*b)
```

3.74.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx$$

$$\Big\downarrow \ 4790$$

$$\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \ 4789$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \ 4794$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx)}{}\right.$$

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)))/4 - (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

3.74.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.74.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 24.15 (sec) , antiderivative size = 73677072, normalized size of antiderivative = 669791.56

method	result	size
default	Expression too large to display	73677072

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(96) = 192.

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.55

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx =$$

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(}{\cos(bx+a)^2+2}\right.}{}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(8*sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a))*si
n(b*x + a) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
 a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b
*x + a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*
x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*
log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 +
 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*co
s(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.74.7 Maxima [F]

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

3.74.8 Giac [F]

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2), x)
```

3.75 $\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.75.1 Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} - \frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{2b}$$

output
```
-1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b-1/2*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.75.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\cos(a+bx)}{4b}$$

input
```
Integrate[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] +
Sqrt[Sin[2*(a + b*x)]]] - 2*Cos[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

3.75.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{ 4790}$$

$$\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx - \frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx - \frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{ 4793}$$

$$\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b} - \frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right) - \frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b
*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*
x]])/(2*b)
```

3.75.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.75.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 3.39 (sec) , antiderivative size = 6800166, normalized size of antiderivative = 80954.36

method	result	size
default	Expression too large to display	6800166

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.17

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx =$$
$$-\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a) - 2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) - 2*arctan(-
(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + c
os(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) -
1)) + 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a)
- sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 +
4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(
b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x
 + a)*sin(b*x + a) + 1))/b
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)\;dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

3.75.8 Giac [F]

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)\;dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sin(a+b\,x)\;\sqrt{\sin(2\,a+2\,b\,x)}\,dx$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2), x)
```

3.76 $\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

3.76.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

output
```
-1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b
```

3.76.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{2b}$$

input
```
Integrate[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]])/b
```

3.76.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \ 4794$$

$$-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

input
```
Int[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*
x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

3.76.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.76.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 3.74 (sec) , antiderivative size = 18281230, normalized size of antiderivative = 315193.62

method	result	size
default	Expression too large to display	18281230

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(52) = 104.

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.14

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-c}{\cos(bx+a)-\sin(}\right.}{}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))
- cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*co
s(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*
x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)
^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.76.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

3.76.8 Giac [F]

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(a+b\,x)}{\sqrt{\sin(2\,a+2\,b\,x)}}\,dx$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(1/2), x)
```

3.77 $\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.77.1 Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\frac{\sin(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
Sin[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]])
```

3.77.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx$$
$$\Big\downarrow \text{ 4780}$$
$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

3.77.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

3.77.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 12.36 (sec) , antiderivative size = 65687946, normalized size of antiderivative = 2855997.65

method	result	size
default	Expression too large to display	65687946

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\sqrt{2}\sqrt{\cos{(bx+a)}\sin{(bx+a)}}+\cos{(bx+a)}}{2\,b\cos{(bx+a)}}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + cos(b*x + a))/(b*cos(b*x +
a))
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.77.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

3.77.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2029 vs. 2(21) = 42.

Time = 14.02 (sec) , antiderivative size = 2029, normalized size of antiderivative = 88.22

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*((2*(sqrt(2)*tan(1/2*a)^25 + 10*sqrt(2)*tan(1/2*a)^2
3 + 44*sqrt(2)*tan(1/2*a)^21 + 110*sqrt(2)*tan(1/2*a)^19 + 165*sqrt(2)*tan
(1/2*a)^17 + 132*sqrt(2)*tan(1/2*a)^15 - 132*sqrt(2)*tan(1/2*a)^11 - 165*s
qrt(2)*tan(1/2*a)^9 - 110*sqrt(2)*tan(1/2*a)^7 - 44*sqrt(2)*tan(1/2*a)^5 -
 10*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^24
 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a
)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan
(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1) + (s
qrt(2)*tan(1/2*a)^26 + 5*sqrt(2)*tan(1/2*a)^24 - 10*sqrt(2)*tan(1/2*a)^22
- 154*sqrt(2)*tan(1/2*a)^20 - 605*sqrt(2)*tan(1/2*a)^18 - 1353*sqrt(2)*tan
(1/2*a)^16 - 1980*sqrt(2)*tan(1/2*a)^14 - 1980*sqrt(2)*tan(1/2*a)^12 - 135
3*sqrt(2)*tan(1/2*a)^10 - 605*sqrt(2)*tan(1/2*a)^8 - 154*sqrt(2)*tan(1/2*a
)^6 - 10*sqrt(2)*tan(1/2*a)^4 + 5*sqrt(2)*tan(1/2*a)^2 + sqrt(2))/(tan(1/2
*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan
(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 4
95*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 ...
```

3.77.9 Mupad [B] (verification not implemented)

Time = 19.95 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\cos(a+b\,x)\,\sqrt{\sin(2\,a+2\,b\,x)}}{b\,(\cos(2\,a+2\,b\,x)+1)}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2))/(b*(cos(2*a + 2*b*x) + 1))
```

3.78 $\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.78.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

output
```
1/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-2/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2
)
```

3.78.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sqrt{\sin(2(a+bx))}\left(-\frac{1}{4}\csc(a+bx)+\frac{1}{12}\sec(a+bx)\tan(a+bx)\right)}{b}$$

input
```
Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Sqrt[Sin[2*(a + b*x)]]*(-1/4*Csc[a + b*x] + (Sec[a + b*x]*Tan[a + b*x])/1
2))/b
```

3.78.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{ 4792}$$

$$\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 4779}$$

$$\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt[Sin
[2*a + 2*b*x]])
```

3.78.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.78.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 27.10 (sec) , antiderivative size = 597, normalized size of antiderivative = 11.26

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(6\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\operatorname{EllipticE}\left(\sqrt{\tan\right.\right.}{}$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(6*(tan(1/2*a+1
/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*t
an(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*
a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-3*(tan(1/2*a+1/2*x*b
)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2
*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*
x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2+6*(tan(1/2*a+1/2*x*b)*(tan
(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2
*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1
)^(1/2),1/2*2^(1/2))-3*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)
*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+
1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+2*(tan
(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^4+2*(ta
n(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4-2*(tan(1
/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2-2*(tan(
1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2))/tan(1/2*a+1/2*x*b)/(tan(1/2*a+
1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)/(1+tan(1/2*a+1/2*x*b)^2)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{4\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\left(4\cos(bx+a)^2-1\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{12\,b\cos(bx+a)^2\sin(bx+a)}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(4*cos(b*x + a)^2*sin(b*x + a) + sqrt(2)*(4*cos(b*x + a)^2 - 1)*sqrt
(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^2*sin(b*x + a))
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.78.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

3.78.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7875 vs. $2(45) = 90$.

Time = 52.85 (sec) , antiderivative size = 7875, normalized size of antiderivative = 148.58

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
-1/24*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a
)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/
2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan
(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(
1/2*b*x) + tan(1/2*a))*((((((2*(sqrt(2)*tan(1/2*a)^56 + 23*sqrt(2)*tan(1/2
*a)^54 + 251*sqrt(2)*tan(1/2*a)^52 + 1725*sqrt(2)*tan(1/2*a)^50 + 8350*sqr
t(2)*tan(1/2*a)^48 + 30130*sqrt(2)*tan(1/2*a)^46 + 83490*sqrt(2)*tan(1/2*a
)^44 + 179630*sqrt(2)*tan(1/2*a)^42 + 297275*sqrt(2)*tan(1/2*a)^40 + 36052
5*sqrt(2)*tan(1/2*a)^38 + 264385*sqrt(2)*tan(1/2*a)^36 - 37145*sqrt(2)*tan
(1/2*a)^34 - 445740*sqrt(2)*tan(1/2*a)^32 - 742900*sqrt(2)*tan(1/2*a)^30 -
 742900*sqrt(2)*tan(1/2*a)^28 - 445740*sqrt(2)*tan(1/2*a)^26 - 37145*sqrt(
2)*tan(1/2*a)^24 + 264385*sqrt(2)*tan(1/2*a)^22 + 360525*sqrt(2)*tan(1/2*a
)^20 + 297275*sqrt(2)*tan(1/2*a)^18 + 179630*sqrt(2)*tan(1/2*a)^16 + 83490
*sqrt(2)*tan(1/2*a)^14 + 30130*sqrt(2)*tan(1/2*a)^12 + 8350*sqrt(2)*tan(1/
2*a)^10 + 1725*sqrt(2)*tan(1/2*a)^8 + 251*sqrt(2)*tan(1/2*a)^6 + 23*sqrt(2
)*tan(1/2*a)^4 + sqrt(2)*tan(1/2*a)^2)*tan(1/2*b*x)/(tan(1/2*a)^51 + 23*ta
n(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43
+ 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 38936
7*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan
(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1...
```

3.78.9 Mupad [B] (verification not implemented)

Time = 24.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.04

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{3\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
-(2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2)*(exp(a*2i + b*x*2i)*1i + exp(a*4i + b*x*4i)*1i + 1i))/(3*b*(e
xp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i) + 1)^2)
```

3.79 $\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

3.79.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

output
```
1/5*sin(b*x+a)/b/sin(2*b*x+2*a)^(5/2)-4/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/
2)+8/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx =\frac{(-5\cot(a+bx)\csc(a+bx)+3\sec(a+bx)\left(9+\sec^2(a+bx)\right))\sqrt{\sin(2(a+bx))}}{120b}$$

input
```
Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
((-5*Cot[a + b*x]*Csc[a + b*x] + 3*Sec[a + b*x]*(9 + Sec[a + b*x]^2))*Sqrt
[Sin[2*(a + b*x)]])/(120*b)
```

3.79.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4792, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4792}$$

$$\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)$$

input
```
Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(4*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*S
qrt[Sin[2*a + 2*b*x]])))/5 + Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))
```

3.79.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.79.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 247.66 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.90

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(5\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+\cdots\right.\right.}{48b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sqrt{\tan\left(\cdots\right.}}$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/
2*x*b)^2-1)/tan(1/2*a+1/2*x*b)*(5*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2
*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*
x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b)^6+5*(ta
n(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*
x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+
1/2*x*b)-7*tan(1/2*a+1/2*x*b)^4+7*tan(1/2*a+1/2*x*b)^2+1)/(tan(1/2*a+1/2*x
*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*
b))^(1/2)/(1+tan(1/2*a+1/2*x*b)^2)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{32\,\cos(bx+a)^5 - 32\,\cos(bx+a)^3 + \sqrt{2}\left(32\,\cos(bx+a)^4 - 24\,\cos(bx+a)^2 - 3\right)\sqrt{\cos(bx+a)\sin(bx}}{120\,\left(b\cos(bx+a)^5 - b\cos(bx+a)^3\right)}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(32*cos(b*x + a)^5 - 32*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4
- 24*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^
5 - b*cos(b*x + a)^3)
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.79.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18022 vs. $2(67) = 134$.

Time = 181.63 (sec) , antiderivative size = 18022, normalized size of antiderivative = 228.13

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
1/480*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a
)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/
2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan
(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(
1/2*b*x) + tan(1/2*a))*((((((((((2*(sqrt(2)*tan(1/2*a)^87 - 92*sqrt(2)*tan
(1/2*a)^85 - 3213*sqrt(2)*tan(1/2*a)^83 - 48008*sqrt(2)*tan(1/2*a)^81 - 44
3462*sqrt(2)*tan(1/2*a)^79 - 2875040*sqrt(2)*tan(1/2*a)^77 - 13907802*sqrt
(2)*tan(1/2*a)^75 - 51781432*sqrt(2)*tan(1/2*a)^73 - 149943911*sqrt(2)*tan
(1/2*a)^71 - 333456564*sqrt(2)*tan(1/2*a)^69 - 536442973*sqrt(2)*tan(1/2*a
)^67 - 482080288*sqrt(2)*tan(1/2*a)^65 + 316221080*sqrt(2)*tan(1/2*a)^63 +
 2190937152*sqrt(2)*tan(1/2*a)^61 + 4607763368*sqrt(2)*tan(1/2*a)^59 + 574
2984608*sqrt(2)*tan(1/2*a)^57 + 3316624962*sqrt(2)*tan(1/2*a)^55 - 3241815
576*sqrt(2)*tan(1/2*a)^53 - 11030972730*sqrt(2)*tan(1/2*a)^51 - 1471202712
0*sqrt(2)*tan(1/2*a)^49 - 10524179460*sqrt(2)*tan(1/2*a)^47 + 10524179460*
sqrt(2)*tan(1/2*a)^43 + 14712027120*sqrt(2)*tan(1/2*a)^41 + 11030972730*sq
rt(2)*tan(1/2*a)^39 + 3241815576*sqrt(2)*tan(1/2*a)^37 - 3316624962*sqrt(2
)*tan(1/2*a)^35 - 5742984608*sqrt(2)*tan(1/2*a)^33 - 4607763368*sqrt(2)*ta
n(1/2*a)^31 - 2190937152*sqrt(2)*tan(1/2*a)^29 - 316221080*sqrt(2)*tan(1/2
*a)^27 + 482080288*sqrt(2)*tan(1/2*a)^25 + 536442973*sqrt(2)*tan(1/2*a)^23
 + 333456564*sqrt(2)*tan(1/2*a)^21 + 149943911*sqrt(2)*tan(1/2*a)^19 + ...
```

3.79.9 Mupad [B] (verification not implemented)

Time = 24.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-3\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+2\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}+2\,\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}+2\right)}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^3}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
(4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(2*exp(a*2i + b*x*2i) - 3*exp(a*4i + b*x*4i) + 2*exp(a*6i + b*
x*6i) + 2*exp(a*8i + b*x*8i) + 2))/(15*b*(exp(a*2i + b*x*2i) - 1)^2*(exp(a
*2i + b*x*2i) + 1)^3)
```

3.80 $\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

3.80.1 Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

output
```
1/7*sin(b*x+a)/b/sin(2*b*x+2*a)^(7/2)-6/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/
2)+8/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-16/35*cos(b*x+a)/b/sin(2*b*x+2*a
)^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{(-5-10\cos(2(a+bx))+4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^3(a+bx)\sec^4(a+bx)\sqrt{\sin(2(a+bx))}}{560b}$$

input
```
Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
((-5 - 10*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[
a + b*x]^3*Sec[a + b*x]^4*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

3.80.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4792, 3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{ 4792}$$

$$\frac{6}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{7}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 4791}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 4792}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 4779}$$

$$\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{6}{7}\left(\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

input
```
Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(6*((4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*
Sqrt[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))))/
7 + Sin[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2))
```

3.80.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.80.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sin(xb+a)}{\sin(2xb+2a)^{\frac{9}{2}}}dx$$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(128\cos(bx+a)^6-160\cos(bx+a)^4+20\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+128\,(\cos}{560\left(b\cos(bx+a)^6-b\cos(bx+a)^4\right)\sin(bx+a)}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
-1/560*(sqrt(2)*(128*cos(b*x + a)^6 - 160*cos(b*x + a)^4 + 20*cos(b*x + a)
^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - cos(b*x +
a)^4)*sin(b*x + a))/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

3.80.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.80.9 Mupad [B] (verification not implemented)

Time = 24.96 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.34

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = &-\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^4}\\&+\frac{16\,e^{a\,3i+b\,x\,3i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{35\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}\\&-\frac{e^{a\,1i+b\,x\,1i}\left(\frac{1i}{7\,b}+\frac{e^{a\,2i+b\,x\,2i}\,8i}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)^2\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^2}\\&-\frac{e^{a\,1i+b\,x\,1i}\left(\frac{16}{35\,b}-\frac{44\,e^{a\,2i+b\,x\,2i}}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)^3\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^3}\end{aligned}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(9/2),x)
```

output
```
(16*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2))/(35*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i)*1i + 1i))
 - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2)*1i)/(7*b*(exp(a*2i + b*x*2i)*1i + 1i)^4) - (exp(a*1i + b*x*1i
)*(1i/(7*b) + (exp(a*2i + b*x*2i)*8i)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2
 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) - 1)^2*(exp(a*2i
 + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*(16/(35*b) - (44*exp(a*2i + b
*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^
(1/2))/((exp(a*2i + b*x*2i) - 1)^3*(exp(a*2i + b*x*2i)*1i + 1i)^3)
```

3.80. $\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

3.81 $\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

3.81.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\frac{5\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{42b}-\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b}-\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b}-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

output
```
-5/42*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi
+b*x),2^(1/2))/b-1/14*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b-1/18*sin(2*b*x
+2*a)^(9/2)/b-5/42*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

3.81.2 Mathematica [A] (verified)

Time = 3.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$=\frac{240\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}-70\sin(2(a+bx))-156\sin(4(a+bx))+35\sin(6(a+bx$$
$$2016b\sqrt{\sin(2(a+bx))}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 70*Sin[2*(a + b
*x)] - 156*Sin[4*(a + b*x)] + 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] -
7*Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

3.81.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4786, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^{7/2}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)dx-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{7/2}dx-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{5}{7}\int \sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{7}\int \sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/18*Sin[2*a + 2*b*x]^(9/2)/b + ((5*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) -
 (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b)))/7 - (Cos[2*a + 2*b*x]*S
in[2*a + 2*b*x]^(5/2))/(7*b))/2
```

3.81.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.81.4 Maple [F(-1)]

Timed out.

$$\int \sin(xb+a)^2 \sin(2xb+2a)^{\frac{7}{2}} dx$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

3.81.5 Fricas [F]

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*si
n(2*b*x + 2*a)^(3/2), x)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(7/2)*sin(b*x + a)^2, x)
```

3.81.8 Giac [F]

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(7/2)*sin(b*x + a)^2, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^{7/2}\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2),x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2), x)
```

3.82 $\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.82.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} - \frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b} - \frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

output
```
-3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))/b-1/10*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b-1/14*sin(2*b*x
+2*a)^(7/2)/b
```

3.82.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{84E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}(-15\sin(2(a+bx))-14\sin(4(a+bx))+5\sin(6(a+bx)))}{280b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(-15*Sin[2*(a +
b*x)] - 14*Sin[4*(a + b*x)] + 5*Sin[6*(a + b*x)]))/(280*b)
```

3.82.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4786, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^{5/2}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)dx-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{5/2}dx-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{2}\left(\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/14*Sin[2*a + 2*b*x]^(7/2)/b + ((3*EllipticE[a - Pi/4 + b*x, 2])/(5*b) -
 (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(3/2))/(5*b))/2
```

3.82.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.82.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 176.98 (sec) , antiderivative size = 312731247, normalized size of antiderivative = 4532336.91

method	result	size
default	Expression too large to display	312731247

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.82.5 Fricas [F]

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*sq
rt(sin(2*b*x + 2*a)), x)
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.82.7 Maxima [F]

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

3.82.8 Giac [F]

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^{5/2}\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2), x)
```

3.83 $\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.83.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{6b} - \frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b} - \frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

output
```
-1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/10*sin(2*b*x+2*a)^(5/2)/b-1/6*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^(1/2)/b
```

3.83.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{20\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}-9\sin(2(a+bx))-10\sin(4(a+bx))+3\sin(6(a+bx))}{120b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 9*Sin[2*(a + b*x
)] - 10*Sin[4*(a + b*x)] + 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)
]])
```

3.83.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4786, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^{3/2}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{2}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b
*x]])/(3*b))/2 - Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

3.83.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.83.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 65.32 (sec) , antiderivative size = 185748620, normalized size of antiderivative = 2692008.99

method	result	size
default	Expression too large to display	185748620

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.83.5 Fricas [F]

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^(3/2), x)
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.83.7 Maxima [F]

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

3.83.8 Giac [F]

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^{\frac{3}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin(a+b\,x)^2\sin(2\,a+2\,b\,x)^{3/2}\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2), x)
```

3.84 $\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.84.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

output
```
-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/6*sin(2*b*x+2*a)^(3/2)/b
```

3.84.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{-3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))}{6b}$$

input
```
Integrate[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/6*(-3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/b
```

3.84.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4786, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^2\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{ 4786}$$

$$\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{E\left(a+bx-\frac{\pi}{4}|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

input
```
Int[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

3.84.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.84.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 7.40 (sec) , antiderivative size = 17183759, normalized size of antiderivative = 429593.98

method	result	size
default	Expression too large to display	17183759

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.84.5 Fricas [F]

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a)), x)
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.84.7 Maxima [F]

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

3.84.8 Giac [F]

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sin\left(a+b\,x\right)^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2), x)
```

3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}} \, dx$

3.85.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}} \, dx = \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{2b} - \frac{\sqrt{\sin(2a+2bx)}}{2b}$$

output
```
-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/2*sin(2*b*x+2*a)^(1/2)/b
```

3.85.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}} \, dx = -\frac{2\sqrt{\sin(2(a+bx))} + \frac{\sqrt{2}\,\text{EllipticF}\left(\arcsin(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{4b}$$

input
```
Integrate[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/4*(2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] -
Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x
)]])/b
```

3.85.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4786, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

input
```
Int[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
EllipticF[a - Pi/4 + b*x, 2]/(2*b) - Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

3.85.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.85.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 14.82 (sec) , antiderivative size = 61245868, normalized size of antiderivative = 1531146.70

method	result	size
default	Expression too large to display	61245868

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.85.5 Fricas [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin{(bx+a)}^2}{\sqrt{\sin{(2\,bx+2\,a)}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)/sqrt(sin(2*b*x + 2*a)), x)
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.85.7 Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

3.85.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(1/2), x)
```

3.86 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx$

3.86.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx = -\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b+sin(b*x+a)^2/b/sin(2*b*x+2*a)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx = \frac{-E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}\tan(a+bx)}{2b}$$

input
```
Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*Tan[a + b*x])/(2*b
)
```

3.86.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4784, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

input
```
Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/2*EllipticE[a - Pi/4 + b*x, 2]/b + Sin[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b
*x]])
```

3.86.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

3.86.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 26.14 (sec) , antiderivative size = 108031867, normalized size of antiderivative = 2400708.16

method	result	size
default	Expression too large to display	108031867

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.86.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.47

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{-i\sqrt{2i}\cos(bx+a)\,E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+i\sqrt{-2i}\cos(bx+a)\,E(\arcsin(\cos(bx}{}$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(-I*sqrt(2*I)*cos(b*x + a)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x
+ a)), -1) + I*sqrt(-2*I)*cos(b*x + a)*elliptic_e(arcsin(cos(b*x + a) - I*
sin(b*x + a)), -1) + I*sqrt(2*I)*cos(b*x + a)*elliptic_f(arcsin(cos(b*x +
a) + I*sin(b*x + a)), -1) - I*sqrt(-2*I)*cos(b*x + a)*elliptic_f(arcsin(co
s(b*x + a) - I*sin(b*x + a)), -1) + 2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
a))*sin(b*x + a))/(b*cos(b*x + a))
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.86.7 Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

3.86.8 Giac [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(3/2), x)
```

3.87 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.87.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{6b}+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
-1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b+1/3*sin(b*x+a)^2/b/sin(2*b*x+2*a)^(3/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx =\frac{\sec^2(a+bx)\sqrt{\sin(2(a+bx))}-\frac{\sqrt{2}\,\text{EllipticF}\left(\arcsin(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{12b}$$

input
```
Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Sec[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a +
 b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*
(a + b*x)]])/(12*b)
```

3.87.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4784, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{6b}$$

input
```
Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
EllipticF[a - Pi/4 + b*x, 2]/(6*b) + Sin[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^
(3/2))
```

3.87.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

3.87.4 Maple [A] (verified)

Time = 28.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.56

method	result
default	$\frac{\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\ \mathrm{EllipticF}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)\sin(2xb+2a)-2\cos(2xb+2a)^2+2\cos(2x}{12\sin(2xb+2a)^{\frac{3}{2}}\cos(2xb+2a)b}$

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin
(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^
(1/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2+2*cos(2*b*x+2*a))/b
```

3.87.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2i}\cos(bx+a)^2 F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+\sqrt{-2i}\cos(bx+a)^2 F(\arcsin(\cos(bx}{12\,b\cos(bx+a)^2}$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(2*I)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
 + a)), -1) + sqrt(-2*I)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) - I
*sin(b*x + a)), -1) - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x
+ a)^2)
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.87.7 Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

3.87.8 Giac [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin(a+b\,x)^2}{\sin(2\,a+2\,b\,x)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(5/2), x)
```

3.88 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

3.88.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=-\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

output
```
3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b+1/5*sin(b*x+a)^2/b/sin(2*b*x+2*a)^(5/2)-3/10*cos(2*b*x+2*a
)/b/sin(2*b*x+2*a)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=-\frac{12E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{4(1+6\cos(2(a+bx))+3\cos(4(a+bx)))\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2(a+bx))}}{40b}$$

input
```
Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/40*(12*EllipticE[a - Pi/4 + b*x, 2] + (4*(1 + 6*Cos[2*(a + b*x)] + 3*Co
s[4*(a + b*x)])*Sin[a + b*x]^2)/Sin[2*(a + b*x)]^(5/2))/b
```

3.88.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4784, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\int \frac{1}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{10}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{3}{10}\left(-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)$$

input
```
Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(3*(-(EllipticE[a - Pi/4 + b*x, 2]/b) - Cos[2*a + 2*b*x]/(b*Sqrt[Sin[2*a +
 2*b*x]])))/10 + Sin[a + b*x]^2/(5*b*Sin[2*a + 2*b*x]^(5/2))
```

3.88.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(92) = 184$.

Time = 58.82 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.95

method	result
default	$\frac{\sqrt{2}\left(\frac{8\sqrt{2}}{5\sin(2xb+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\sin(2xb+2a)^{2}\,\mathrm{EllipticE}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(2xb}{5\sin(2xb-}}{32b}$

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*2^(1/2)*(8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^
(5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+
2*a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2
))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a)
)^(1/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6
*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

3.88.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.75

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{-6i\sqrt{2i}\cos(bx+a)^3 E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+6i\sqrt{-2i}\cos(bx+a)^3 E}{}$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/40*(-6*I*sqrt(2*I)*cos(b*x + a)^3*elliptic_e(arcsin(cos(b*x + a) + I*sin
(b*x + a)), -1)*sin(b*x + a) + 6*I*sqrt(-2*I)*cos(b*x + a)^3*elliptic_e(ar
csin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + 6*I*sqrt(2*I)*cos(
b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x +
 a) - 6*I*sqrt(-2*I)*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) - I*sin
(b*x + a)), -1)*sin(b*x + a) - sqrt(2)*(12*cos(b*x + a)^4 - 6*cos(b*x + a)
^2 - 1)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^3*sin(b*x + a))
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.88.7 Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

3.88.8 Giac [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sin(a+b\,x)^2}{\sin(2\,a+2\,b\,x)^{7/2}}\,dx$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(7/2), x)
```

3.89 $\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.89.1 Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{7\arcsin(\cos(a+bx)-\sin(a+bx))}{64b} - \frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{64b} + \frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b} - \frac{7\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} - \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

output
```
-7/64*arcsin(cos(b*x+a)-sin(b*x+a))/b-7/64*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-7/48*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-1/12*sin(b*x+a)*s
in(2*b*x+2*a)^(5/2)/b+7/32*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.89.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{-7\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin(2}}{64b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-7*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(10*Sin[a + b*x] -
 9*Sin[3*(a + b*x)] + 2*Sin[5*(a + b*x)]))/3)/(64*b)
```

3.89.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4786, 3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(a+bx)^3\sin(2a+2bx)^{3/2}dx$$
$$\Big\downarrow \text{4786}$$
$$\frac{7}{12}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{7}{12}\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$
$$\Big\downarrow \text{4790}$$

$$\frac{7}{12}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$\downarrow$ 3042

$$\frac{7}{12}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$\downarrow$ 4789

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$\downarrow$ 3042

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$\downarrow$ 4794

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a-\right.\right.\right.$$
$$-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/12*(Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/b + (7*((3*((-1/2*ArcSin[Cos[a
 + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a
 + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 - (
Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/12
```

3.89. $\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.89.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.89.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 95.12 (sec) , antiderivative size = 404845695, normalized size of antiderivative = 2976806.58

method	result	size
default	Expression too large to display	404845695

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(118) = 236.

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.13

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{8\sqrt{2}(32\cos(bx+a)^4-60\cos(bx+a)^2+21)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+42\arctan\left(-\frac{\sqrt{2}}{}\right.}{}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/768*(8*sqrt(2)*(32*cos(b*x + a)^4 - 60*cos(b*x + a)^2 + 21)*sqrt(cos(b*x
 + a)*sin(b*x + a))*sin(b*x + a) + 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*s
in(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(c
os(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 42*arctan(-(2*sqrt(2)*
sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x +
a) - sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a
)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
 a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))
/b
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.89.7 Maxima [F]

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

3.89.8 Giac [F]

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)^3\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2), x)
```

3.90 $\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.90.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{32b} + \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b} - \frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b} - \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

output
```
-5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b+5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-1/8*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-5/16*cos(b*x+a)*si
n(2*b*x+2*a)^(1/2)/b
```

3.90.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{5\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2(-6\,\mathrm{co}}{32b}$$

input
```
Integrate[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(5*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + 2*(-6*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[
Sin[2*(a + b*x)]])/(32*b)
```

3.90.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4786, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(a+bx)^3\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \; 4786$$

$$\frac{5}{8}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{8}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

$$\Big\downarrow \; 4790$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

$$\Big\downarrow \; 4793$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a+2b}}{2}\right.$$

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

input
```
Int[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(5*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)))/8 - (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

3.90.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.90.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 16.22 (sec) , antiderivative size = 57690707, normalized size of antiderivative = 524460.97

method	result	size
default	Expression too large to display	57690707

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(96) = 192.

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.55

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{8\sqrt{2}\left(4\cos(bx+a)^3-9\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(b}{\cos(bx+a)^2+2}\right.}{}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(8*sqrt(2)*(4*cos(b*x + a)^3 - 9*cos(b*x + a))*sqrt(cos(b*x + a)*sin
(b*x + a)) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x
+ a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(
b*x + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) -
5*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2
 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*
cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.90.7 Maxima [F]

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^3, x)
```

3.90.8 Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]e
xt_reduce Error: Bad Argument TypeDone
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sin\left(a+b\,x\right)^3\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2), x)
```

3.91 $\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

3.91.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{8b} - \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b} - \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

output
```
-3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b-3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b-1/4*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.91.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+2\sin(a+}{8b}$$

input
```
Integrate[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/8*(3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a +
 b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

3.91.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4786, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^3}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 4786}$$

$$\frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

$$\Big\downarrow \text{ 4794}$$

$$\frac{3}{4}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

input
```
Int[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(3*(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
+ b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)))/4 - (Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b)
```

3.91.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.91.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 39.74 (sec) , antiderivative size = 202764172, normalized size of antiderivative = 2413859.19

method	result	size
default	Expression too large to display	202764172

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.19

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) - 6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) - 6*arctan(-
(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + c
os(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) -
1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a)
- sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4
+ 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*co
s(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b
*x + a)*sin(b*x + a) + 1))/b
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.91.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

3.91.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(1/2), x)
```

3.92 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.92.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} + \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\arcsin(\cos(a+bx)-\sin(a+bx)) - \log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right) + 2\sec(a+bx)\sqrt{}}{4b}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(ArcSin[Cos[a + b*x] - Sin[a + b*x]] - Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] + 2*Sec[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

3.92.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4782, 3042, 4796, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4782}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{4}\int \csc(a+bx)\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{4}\int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4796}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{2}\left(\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) - Log[Cos[a + b*x] + Sin[a + b*
x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b
*x]])
```

3.92.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4782
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(p + 1))), x] + Simp[e^4*((m + p - 1)/(4*g^2*(p + 1)))   Int[(e
*Sin[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c,
d, e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m,
 2] && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.92.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 28.41 (sec) , antiderivative size = 178923370, normalized size of antiderivative = 2208930.49

method	result	size
default	Expression too large to display	178923370

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.65

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\cos(bx+a) - 2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos}}{}\right.}{}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) -
sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a
)*sin(b*x + a) - 1))*cos(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)
*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)
))*cos(b*x + a) - cos(b*x + a)*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b
*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos
(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a)
 + 1) - 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - 8*cos(b*x + a))/(b*cos
(b*x + a))
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.92.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

3.92.8 Giac [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(3/2), x)
```

3.93 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.93.1 Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
1/3*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(3/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2(a+bx))}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
Sin[a + b*x]^3/(3*b*Sin[2*(a + b*x)]^(3/2))
```

3.93.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{5/2}}dx$$
$$\Big\downarrow \text{4780}$$
$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
Sin[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))
```

3.93.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

3.93.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 83.16 (sec) , antiderivative size = 727, normalized size of antiderivative = 25.96

method	result	size
default	Expression too large to display	727

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*
x*b)^2-1)*(6*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*
(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(
1/2))*tan(1/2*a+1/2*x*b)^6-3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/
2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+
1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^6+18*(tan(1/2*a+1/2*x*b)+1)^(1/2)
*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((ta
n(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^4-9*(tan(1/2*a+1
/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/
2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^
4+6*tan(1/2*a+1/2*x*b)^8+18*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2
*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1
)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-9*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(
-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(
1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-2*tan(1/2*a+1/2*
x*b)^6+6*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-ta
n(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2)
)-3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2
*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+10*
tan(1/2*a+1/2*x*b)^4-14*tan(1/2*a+1/2*x*b)^2)/(tan(1/2*a+1/2*x*b)*(tan(...
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos{(bx+a)}^2-\sqrt{2}\sqrt{\cos{(bx+a)}\sin{(bx+a)}}\sin{(bx+a)}}{12\,b\cos{(bx+a)}^2}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(cos(b*x + a)^2 - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x +
a))/(b*cos(b*x + a)^2)
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.93.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15648 vs. 2(24) = 48.

Time = 102.08 (sec) , antiderivative size = 15648, normalized size of antiderivative = 558.86

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*((((2*(4*(2*(sqrt(2)*tan(1/2*a)^54 + 17*sqrt(2)*tan(
1/2*a)^52 + 132*sqrt(2)*tan(1/2*a)^50 + 612*sqrt(2)*tan(1/2*a)^48 + 1842*s
qrt(2)*tan(1/2*a)^46 + 3570*sqrt(2)*tan(1/2*a)^44 + 3668*sqrt(2)*tan(1/2*a
)^42 - 1292*sqrt(2)*tan(1/2*a)^40 - 11457*sqrt(2)*tan(1/2*a)^38 - 19057*sq
rt(2)*tan(1/2*a)^36 - 12920*sqrt(2)*tan(1/2*a)^34 + 7752*sqrt(2)*tan(1/2*a
)^32 + 27132*sqrt(2)*tan(1/2*a)^30 + 27132*sqrt(2)*tan(1/2*a)^28 + 7752*sq
rt(2)*tan(1/2*a)^26 - 12920*sqrt(2)*tan(1/2*a)^24 - 19057*sqrt(2)*tan(1/2*
a)^22 - 11457*sqrt(2)*tan(1/2*a)^20 - 1292*sqrt(2)*tan(1/2*a)^18 + 3668*sq
rt(2)*tan(1/2*a)^16 + 3570*sqrt(2)*tan(1/2*a)^14 + 1842*sqrt(2)*tan(1/2*a)
^12 + 612*sqrt(2)*tan(1/2*a)^10 + 132*sqrt(2)*tan(1/2*a)^8 + 17*sqrt(2)*ta
n(1/2*a)^6 + sqrt(2)*tan(1/2*a)^4)*tan(1/2*b*x)/(tan(1/2*a)^51 + 23*tan(1/
2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 31
878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*ta
n(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2
*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^2
3 - 653752*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 ...
```

3.93.9 Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(2\,\sin\left(a+b\,x\right)+3\,\sin\left(3\,a+3\,b\,x\right)+\sin\left(5\,a+5\,b\,x\right)\right)}{6\,b\,\left(30\,{\sin\left(a+b\,x\right)}^2+12\,{\sin\left(2\,a+2\,b\,x\right)}^2+2\,{\sin\left(3\,a+3\,b\,x\right)}^2-32\right)}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
-(sin(2*a + 2*b*x)^(1/2)*(2*sin(a + b*x) + 3*sin(3*a + 3*b*x) + sin(5*a +
5*b*x)))/(6*b*(12*sin(2*a + 2*b*x)^2 + 2*sin(3*a + 3*b*x)^2 + 30*sin(a + b
*x)^2 - 32))
```

3.94 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

3.94.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

output
```
1/5*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(5/2)+1/5*sin(b*x+a)/b/sin(2*b*x+2*a)^(1
/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{\sec(a+bx)\left(4+\sec^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{40b}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(Sec[a + b*x]*(4 + Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(40*b)
```

3.94.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4784, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{1}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
Sin[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) + Sin[a + b*x]/(5*b*Sqrt[Sin[2
*a + 2*b*x]])
```

3.94.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

3.94.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 183.40 (sec) , antiderivative size = 4684, normalized size of antiderivative = 85.16

method	result	size
default	Expression too large to display	4684

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(56*tan(1/2*a+1/2*x*b)+8*tan(1/2*a+1/2*x*b)^5-8*tan(1/2*a+1/2*x*b)^
3-24*arctan(((tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)+tan(1/2*a+1/2
*x*b))/tan(1/2*a+1/2*x*b))*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)
*tan(1/2*a+1/2*x*b)^5-18*arctan(((tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))
^(1/2)-tan(1/2*a+1/2*x*b))/tan(1/2*a+1/2*x*b))*(tan(1/2*a+1/2*x*b)^3-tan(1
/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4-12*ln(1/tan(1/2*a+1/2*x*b)*(tan(
1/2*a+1/2*x*b)^2+2*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)+2*tan(1
/2*a+1/2*x*b)-1))*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(3/2)*tan(1/2*
a+1/2*x*b)^2+9*ln(1/tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2+2*(tan(1/2*a+
1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)+2*tan(1/2*a+1/2*x*b)-1))*(tan(1/2*a+1
/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4+24*arctan(((tan(1
/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)+tan(1/2*a+1/2*x*b))/tan(1/2*a+1/
2*x*b))*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(3/2)*tan(1/2*a+1/2*x*b)
^2-18*arctan(((tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)+tan(1/2*a+1/
2*x*b))/tan(1/2*a+1/2*x*b))*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2
)*tan(1/2*a+1/2*x*b)^4-9*ln(-1/tan(1/2*a+1/2*x*b)*(-tan(1/2*a+1/2*x*b)^2+2
*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)-2*tan(1/2*a+1/2*x*b)+1))*
(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2-18*ar
ctan(((tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)-tan(1/2*a+1/2*x*b))/
tan(1/2*a+1/2*x*b))*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan...
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{4\cos(bx+a)^3+\sqrt{2}\left(4\cos(bx+a)^2+1\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{40\,b\cos(bx+a)^3}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/40*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 + 1)*sqrt(cos(b*x + a)*
sin(b*x + a)))/(b*cos(b*x + a)^3)
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.94.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

3.94.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\text{Timed out}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.94.9 Mupad [B] (verification not implemented)

Time = 25.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,\left(3\,e^{a\,2i+b\,x\,2i}+e^{a\,4i+b\,x\,4i}+1\right)}{5\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^3}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*(3*exp(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1))/(5*b*(exp(a*2i
+ b*x*2i) + 1)^3)
```

3.95 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

3.95.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

output
```
1/7*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(7/2)+2/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(
3/2)-4/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{(5+12\cos(2(a+bx))+4\cos(4(a+bx)))\csc(a+bx)\sec^4(a+bx)\sqrt{\sin(2(a+bx))}}{336b}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
-1/336*((5 + 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]*Sec[a
+ b*x]^4*Sqrt[Sin[2*(a + b*x)]])/b
```

3.95.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4784, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{2}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{2}{7}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2}{7}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(2*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt
[Sin[2*a + 2*b*x]])))/7 + Sin[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2))
```

3.95.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.95.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sin{(xb+a)}^3}{\sin{(2xb+2a)}^{\frac{9}{2}}}dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{32\cos(bx+a)^4\sin(bx+a)+\sqrt{2}\left(32\cos(bx+a)^4-8\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{336\,b\cos(bx+a)^4\sin(bx+a)}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
-1/336*(32*cos(b*x + a)^4*sin(b*x + a) + sqrt(2)*(32*cos(b*x + a)^4 - 8*co
s(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^4*sin(b
*x + a))
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

3.95.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

3.95.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.95.9 Mupad [B] (verification not implemented)

Time = 24.64 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.70

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = &-\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,5i}{84\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^2}\\ &+\frac{3\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{14\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^3}\\ &-\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^4}\\ &+\frac{e^{a\,1i+b\,x\,1i}\left(\frac{5}{84\,b}+\frac{4\,e^{a\,2i+b\,x\,2i}}{21\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}\end{aligned}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(9/2),x)
```

output
```
(3*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2))/(14*b*(exp(a*2i + b*x*2i)*1i + 1i)^3) - (exp(a*1i + b*x*1i)*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*5i)/(84*b*(
exp(a*2i + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i
)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*1i)/(7*b*(exp(a*2i + b*x*2i)*1i
 + 1i)^4) + (exp(a*1i + b*x*1i)*(5/(84*b) + (4*exp(a*2i + b*x*2i))/(21*b))
*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*
2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i)*1i + 1i))
```

3.96 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

3.96.1 Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} + \frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

output
```
1/9*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(9/2)+1/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(
5/2)-4/45*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+8/45*sin(b*x+a)/b/sin(2*b*x+2*
a)^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \frac{(-15\cot(a+bx)\csc(a+bx)+113\sec(a+bx)+17\sec^3(a+bx)+5\sec^5(a+bx))\sqrt{\sin(2(a+bx))}}{1440b}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

output
```
((-15*Cot[a + b*x]*Csc[a + b*x] + 113*Sec[a + b*x] + 17*Sec[a + b*x]^3 + 5
*Sec[a + b*x]^5)*Sqrt[Sin[2*(a + b*x)]])/(1440*b)
```

3.96.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4784, 3042, 4792, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{11/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{1}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{7/2}}dx+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{1}{3}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{1}{3}\left(\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)\right)$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

output
```
((4*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*
Sqrt[Sin[2*a + 2*b*x]])))/5 + Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)))/3
 + Sin[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2))
```

3.96.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.96.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sin (xb+a)^3}{\sin (2xb+2a)^{\frac{11}{2}}}dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

output
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \frac{128\,\cos{(bx+a)}^7 - 128\,\cos{(bx+a)}^5 + \sqrt{2}\big(128\,\cos{(bx+a)}^6 - 96\,\cos{(bx+a)}^4 - 12\,\cos{(bx+a)}^2 - 5\big)}{1440\,\left(b\cos{(bx+a)}^7 - b\cos{(bx+a)}^5\right)}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

output
```
1/1440*(128*cos(b*x + a)^7 - 128*cos(b*x + a)^5 + sqrt(2)*(128*cos(b*x + a
)^6 - 96*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x
 + a)))/(b*cos(b*x + a)^7 - b*cos(b*x + a)^5)
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

output
```
Timed out
```

3.96.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

3.96.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

output
```
Timed out
```

3.96.9 Mupad [B] (verification not implemented)

Time = 25.91 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.58

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{60\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^3} - \frac{2\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^4} + \frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^5} + \frac{e^{a\,3i+b\,x\,3i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,8i}{45\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)} - \frac{e^{a\,1i+b\,x\,1i}\,\left(\frac{49}{180\,b}-\frac{19\,e^{a\,2i+b\,x\,2i}}{180\,b}\right)\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)^2\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^2}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(11/2),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*1i)/(9*b*(exp(a*2i + b*x*2i)*1i + 1i)^5) - (2*exp(a*1i + b*x*1i)
*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(9*b*(ex
p(a*2i + b*x*2i)*1i + 1i)^4) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*
1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*1i)/(60*b*(exp(a*2i + b*x*2i)*1i
+ 1i)^3) + (exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i +
b*x*2i)*1i)/2)^(1/2)*8i)/(45*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i
)*1i + 1i)) - (exp(a*1i + b*x*1i)*(49/(180*b) - (19*exp(a*2i + b*x*2i))/(1
80*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((
exp(a*2i + b*x*2i) - 1)^2*(exp(a*2i + b*x*2i)*1i + 1i)^2)
```

3.97 $\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

3.97.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{16b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b} + \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{12b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}$$

output
```
-5/16*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-5/12*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/3*sin(b*x+a)*si
n(2*b*x+2*a)^(5/2)/b+5/8*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.97.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{-5\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin(2}}{16b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] -
 3*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(16*b)
```

3.97.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4796, 3042, 4789, 3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2a+2bx)^{7/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{ 4796}$$

$$2\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \cos(a+bx)\sin(2a+2bx)^{5/2}dx$$

$$\Big\downarrow \text{ 4789}$$

$$2\left(\frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{5}{6}\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)$$

$$\Big\downarrow \text{4790}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)$$

$$\Big\downarrow \text{4789}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\,\text{s}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\,\text{s}}{}\right.$$

$$\Big\downarrow \text{4794}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(}{}\right.\right.\right.$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
2*((Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b) + (5*((3*((-1/2*ArcSin[Cos[
a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*
a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 -
(Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/6)
```

3.97. $\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

3.97.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.97.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 7.12 (sec) , antiderivative size = 973, normalized size of antiderivative = 7.15

method	result	size
default	Expression too large to display	973

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-16/5*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(6*(tan(1/2*a+1
/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/
2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)
*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)+1))^(1/2)*tan(1/2*a+1/2*x*b)^4
-3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*
a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan
(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)+1))^(1/2)*tan(1
/2*a+1/2*x*b)^4-12*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^
(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1
/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)
+1))^(1/2)*tan(1/2*a+1/2*x*b)^2+6*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2
*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*
x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan
(1/2*a+1/2*x*b)+1))^(1/2)*tan(1/2*a+1/2*x*b)^2+6*(tan(1/2*a+1/2*x*b)^3-tan
(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^6+6*(tan(1/2*a+1/2*x*b)+1)^(1/2)
*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((ta
n(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*
x*b)-1)*(tan(1/2*a+1/2*x*b)+1))^(1/2)-3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*t
an(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*
a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b...
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(118) = 236$.

Time = 0.28 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.13

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx =$$

$$-\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-12\cos\left(bx+a\right)^{2}-15\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\sin\left(bx+a\right)-30\arctan\left(-\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/192*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*
x + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*
sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(
cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)
*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x +
 a) - sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x +
a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x
+ a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)
)/b
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.97.7 Maxima [F]

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(7/2), x)
```

3.97.8 Giac [F]

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(7/2), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{7/2}}{\sin(a+b\,x)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x), x)
```

3.98 $\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.98.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b} - \frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b}$$

output
```
-3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+1/2*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-3/4*cos(b*x+a)*sin(2
*b*x+2*a)^(1/2)/b
```

3.98.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{3\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(2\cos(}{8b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[S
in[2*(a + b*x)]])/(8*b)
```

3.98.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4796, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{5/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx$$

$$\Big\downarrow \text{4789}$$

$$2\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{4790}$$

$$2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{4793}$$

$$2\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a-}}{}\right.\right.$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
2*((3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Si
n[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a
 + 2*b*x]])/(2*b)))/4 + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b))
```

3.98.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.98.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 12.12 (sec) , antiderivative size = 26835015, normalized size of antiderivative = 243954.68

method	result	size
default	Expression too large to display	26835015

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(96) = 192.

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.55

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx =$$

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^3-\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(b}{\cos(bx+a)^2+2\,c}\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b
*x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*lo
g(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1
)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(
b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.98.7 Maxima [F]

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

3.98.8 Giac [F]

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{5/2}}{\sin(a+b\,x)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x), x)
```

3.99 $\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.99.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b} + \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}$$

output
```
-1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.99.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\sin(a+bx}{2b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

3.99.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4796, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{3/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4789}$$

$$2\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4794}$$

$$2\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx)\cdot}{}\right.$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
2*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
+ b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2
*b*x]])/(2*b))
```

3.99.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.99.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 4.74 (sec) , antiderivative size = 5540414, normalized size of antiderivative = 68400.17

method	result	size
default	Expression too large to display	5540414

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(73) = 146$.

Time = 0.27 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.28

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) + 2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\,\mathrm{s}}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)
) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) -
sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*
sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*
x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x +
 a)*sin(b*x + a) + 1))/b
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.99.7 Maxima [F]

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \csc\left(bx+a\right)\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

3.99.8 Giac [F]

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \csc\left(bx+a\right)\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \frac{\sin\left(2\,a+2\,b\,x\right)^{3/2}}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x), x)
```

3.100 $\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.100.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b}$$

output
```
-arcsin(cos(b*x+a)-sin(b*x+a))/b+ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)^(
1/2))/b
```

3.100.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{b}$$

input
```
Integrate[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]/b
```

3.100.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4796, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4793}$$

$$2\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)$$

input
```
Int[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
2*(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a +
 b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))
```

3.100.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.100.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.96

method	result
default	$\frac{2\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1},\frac{\sqrt{2}}{2}\right)}{b\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}}$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*
b)^2-1)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2
*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)
/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*
x*b)+1)^(1/2),1/2*2^(1/2))
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(51) = 102.

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.57

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-c}{\cos(bx+a)-\sin(}\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))
- cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*co
s(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*
x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)
^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.100.7 Maxima [F]

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)\,\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

3.100.8 Giac [F]

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)\,\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2\,a+2\,b\,x)}}{\sin(a+b\,x)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x), x)
```

3.101 $\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

3.101.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc(a+bx)\sqrt{\sin(2a+2bx)}}{b}$$

output
```
-csc(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.101.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc(a+bx)\sqrt{\sin(2(a+bx))}}{b}$$

input
```
Integrate[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-((Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b)
```

3.101.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 4780}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-((Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b)
```

3.101.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

3.101.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.38 (sec) , antiderivative size = 308, normalized size of antiderivative = 12.83

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\operatorname{EllipticE}\left(\sqrt{\tan\ldots}\right.\right.}{}$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(2*(tan(1/2*a+1/2
*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan
(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+
1/2*x*b)+1)^(1/2),1/2*2^(1/2))-(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1
))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-ta
n(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2)
)+(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2-(ta
n(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2))/tan(1/2*a+1/2*x*b)/(tan(1/2*
a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+\sin\left(bx+a\right)}{b\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x + a))
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.101.7 Maxima [F]

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc(bx+a)}{\sqrt{\sin(2bx+2a)}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

3.101.8 Giac [F]

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc(bx+a)}{\sqrt{\sin(2bx+2a)}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 21.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{b\,\sin\left(a+b\,x\right)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2)),x)
```

output
```
-sin(2*a + 2*b*x)^(1/2)/(b*sin(a + b*x))
```

3.102 $\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.102.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

output
```
-2/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+4/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/
2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\left(-\frac{1}{6}\cot(a+bx)\csc(a+bx) + \frac{1}{2}\sec(a+bx)\right)\sqrt{\sin(2(a+bx))}}{b}$$

input
```
Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
((-1/6*(Cot[a + b*x]*Csc[a + b*x]) + Sec[a + b*x]/2)*Sqrt[Sin[2*(a + b*x)]
])/b
```

3.102.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4796, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$2\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4780}$$

$$2\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)$$

input
```
Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
2*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*Sq
rt[Sin[2*a + 2*b*x]]))
```

3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.102.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 5.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.66

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+\right.\right.}{12b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}}$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/
2*x*b)^2-1)/tan(1/2*a+1/2*x*b)*(2*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2
*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*
x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)-tan(1/2*a+1/2*x*b)^4+1)/(tan
(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(
1/2*a+1/2*x*b))^(1/2)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{4\cos(bx+a)^3+\sqrt{2}\left(4\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}-4\cos(bx+a)}{6\left(b\cos(bx+a)^3-b\cos(bx+a)\right)}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*s
in(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.102.7 Maxima [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

3.102.8 Giac [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 23.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.94

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(1+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}{3\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2)),x)
```

output
```
(4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*4i + b*x*4i) - exp(a*2i + b*x*2i) + 1))/(3*b*(exp(a*2i
+ b*x*2i) - 1)^2*(exp(a*2i + b*x*2i) + 1))
```

3.103 $\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.103.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

output
```
-2/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)+8/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(3
/2)-16/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin(2(a+bx))}(27\csc(a+bx)+3\csc^3(a+bx)-5\sec(a+bx)\tan(a+bx))}{60b}$$

input
```
Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/60*(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec[
a + b*x]*Tan[a + b*x]))/b
```

3.103.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4796, 3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$2\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4792}$$

$$2\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4779}$$

$$2\left(\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

input
```
Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
2*((4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*S
qrt[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)))
```

3.103.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p]
&& IntegerQ[2*p]
```

3.103.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 16.42 (sec) , antiderivative size = 481, normalized size of antiderivative = 6.09

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-1}}\left(24\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticE}\left(\right.\right.}{}$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/80/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/tan(1/2*a+1/2
*x*b)^3*(24*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a
+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(
1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b
)^2-12*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*
x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*
EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2+(
tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^6+12
*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4-(tan
(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^4-12*(t
an(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2-(tan(1/
2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2+(tan(1/2
*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2))/(tan(1/2*a+1/2*x*b)^3-tan(1/2
*a+1/2*x*b))^(1/2)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.30

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(32\cos(bx+a)^4-40\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\left(\cos(bx+a)^4-\cos(bx+\right.}{60\left(b\cos(bx+a)^4-b\cos(bx+a)^2\right)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/60*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b*
cos(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.103.7 Maxima [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

3.103.8 Giac [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 23.76 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= \frac{8\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}\right)}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^2}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2)),x)
```

output
```
(8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*
x*6i)*2i - exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(
exp(a*2i + b*x*2i) + 1)^2)
```

3.104 $\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

3.104.1 Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} -\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

output
```
-2/7*cos(b*x+a)/b/sin(2*b*x+2*a)^(7/2)+12/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(
5/2)-16/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+32/35*sin(b*x+a)/b/sin(2*b*x+
2*a)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{(5-10\cos(2(a+bx))-4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^4(a+bx)\sec^3(a+bx)\sqrt{\sin(2(a+bx))}}{280b}$$

input
```
Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
 + b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(280*b)
```

3.104.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4796, 3042, 4791, 3042, 4792, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$2\left(\frac{6}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{6}{7}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{7/2}}dx-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4792}$$

$$2\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4791}$$

$$2\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4780}$$

$$2\left(\frac{6}{7}\left(\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

input
```
Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
2*((6*((4*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])
/(3*b*Sqrt[Sin[2*a + 2*b*x]])))/5 + Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/
2))))/7 - Cos[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)))
```

3.104.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.104.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 89.89 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.11

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{8}+40\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\right.\right.}}{1344b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}}$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/1344/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1
/2*x*b)^2-1)/tan(1/2*a+1/2*x*b)^3*(3*tan(1/2*a+1/2*x*b)^8+40*(tan(1/2*a+1/
2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2
)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^3
-26*tan(1/2*a+1/2*x*b)^6+26*tan(1/2*a+1/2*x*b)^2-3)/(tan(1/2*a+1/2*x*b)*(t
an(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1
/2)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= \frac{128\,\cos\left(bx+a\right)^{7}-256\,\cos\left(bx+a\right)^{5}+128\,\cos\left(bx+a\right)^{3}+\sqrt{2}\big(128\,\cos\left(bx+a\right)^{6}-224\,\cos\left(bx+a\right)^{4}+}{280\,\left(b\cos\left(bx+a\right)^{7}-2\,b\cos\left(bx+a\right)^{5}+b\cos\left(bx+a\right)^{3}\right)}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/280*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt
(2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt
(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*co
s(b*x + a)^3)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.104.7 Maxima [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

3.104.8 Giac [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 24.44 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.33

$$\begin{aligned}\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = & -\frac{2\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}\\&+\frac{e^{a\,3i+b\,x\,3i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,32i}{35\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}\\&-\frac{e^{a\,1i+b\,x\,1i}\left(\frac{2}{7\,b}-\frac{16\,e^{a\,2i+b\,x\,2i}}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)^2\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}\\&+\frac{e^{a\,1i+b\,x\,1i}\left(\frac{32i}{35\,b}+\frac{e^{a\,2i+b\,x\,2i}\,88i}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}\end{aligned}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(7/2)),x)
```

output
```
(exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*32i)/(35*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i)
) - (2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2
i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i
)*(2/(7*b) - (16*exp(a*2i + b*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2
- (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i
+ b*x*2i)*1i - 1i)^2) + (exp(a*1i + b*x*1i)*(32i/(35*b) + (exp(a*2i + b*x*
2i)*88i)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)
^(1/2))/((exp(a*2i + b*x*2i) + 1)^3*(exp(a*2i + b*x*2i)*1i - 1i)^3)
```

3.105 $\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

3.105.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b}-\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b}-\frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b}+\frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{7b}$$

output
```
-6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-2/5*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b-2/7*cos(2*b*x+2*
a)*sin(2*b*x+2*a)^(7/2)/b+1/7*csc(b*x+a)^2*sin(2*b*x+2*a)^(11/2)/b
```

3.105.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \frac{84E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}(15\sin(2(a+bx))-14\sin(4(a+bx))-5\sin(6(a+bx)))}{70b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b
*x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(70*b)
```

3.105.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4788, 3042, 3115, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{9/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{18}{7}\int \sin^{\frac{9}{2}}(2a+2bx)dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{7}\int \sin(2a+2bx)^{9/2}dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{18}{7}\left(\frac{7}{9}\int \sin^{\frac{5}{2}}(2a+2bx)dx-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{7}\left(\frac{7}{9}\int \sin(2a+2bx)^{5/2}dx-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{18}{7}\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{7}\left(\frac{7}{9}\left(\frac{3}{5}\int\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}+\frac{18}{7}\left(\frac{7}{9}\left(\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(11/2))/(7*b) + (18*(-1/9*(Cos[2*a + 2*b*
x]*Sin[2*a + 2*b*x]^(7/2))/b + (7*((3*EllipticE[a - Pi/4 + b*x, 2])/(5*b)
- (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(3/2))/(5*b)))/9))/7
```

3.105.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.105.4 Maple [A] (verified)

Time = 22.56 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.92

method	result
default	$\frac{8\sqrt{2}\left(\frac{\sqrt{2}\,\sin(2xb+2a)^{\frac{7}{2}}}{56}-\frac{\sqrt{2}\left(6\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\operatorname{EllipticE}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(2xb+2a)+1}\,\sqrt{-}\right.}{80\cos(2xb+2a)\sqrt{\sin(2}}\right.}{b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
8*2^(1/2)*(1/56*2^(1/2)*sin(2*b*x+2*a)^(7/2)-1/80*2^(1/2)*(6*(sin(2*b*x+2*
a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE(
(sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2
*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1
/2),1/2*2^(1/2))-2*sin(2*b*x+2*a)^4+2*sin(2*b*x+2*a)^2)/cos(2*b*x+2*a)/sin
(2*b*x+2*a)^(1/2))/b
```

3.105.5 Fricas [F]

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx=\int \csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
integral((cos(2*b*x + 2*a)^4 - 2*cos(2*b*x + 2*a)^2 + 1)*csc(b*x + a)^2*sq
rt(sin(2*b*x + 2*a)), x)
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

3.105.7 Maxima [F]

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{9}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(9/2), x)
```

3.105.8 Giac [F]

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{9}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(9/2), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{9/2}}{\sin(a+b\,x)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^2, x)
```

3.106 $\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

3.106.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{3b} - \frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b} - \frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b} + \frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{5b}$$

output
```
-2/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-2/5*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b+1/5*csc(b*x+a)^2
*sin(2*b*x+2*a)^(9/2)/b-2/3*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

3.106.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{20\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}+9\sin(2(a+bx))-10\sin(4(a+bx))-3\sin(6(a+bx))}{30b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x
)] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(30*b*Sqrt[Sin[2*(a + b*x)]
])
```

3.106.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4788, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{7/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{14}{5}\int \sin^{\frac{7}{2}}(2a+2bx)dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{5}\int \sin(2a+2bx)^{7/2}dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{14}{5}\left(\frac{5}{7}\int \sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{5}\left(\frac{5}{7}\int \sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{14}{5}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{5}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}+\frac{14}{5}\left(\frac{5}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2))/(5*b) + (14*((5*(EllipticF[a - Pi/
4 + b*x, 2]/(3*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b)))/7 -
(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(7*b)))/5
```

3.106.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.106. $\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.106.4 Maple [A] (verified)

Time = 20.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

method	result
default	$\frac{4\sqrt{2}\left(\frac{\sqrt{2}\,\sin(2xb+2a)^{\frac{5}{2}}}{20}+\frac{\sqrt{2}\left(\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)+2\sin(2xb+2a)^{3}-2\sin(2\right.}{24\cos(2xb+2a)\sqrt{\sin(2xb+2a)}}\right.}{b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
4*2^(1/2)*(1/20*2^(1/2)*sin(2*b*x+2*a)^(5/2)+1/24*2^(1/2)*((sin(2*b*x+2*a)
+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((s
in(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+2*sin(2*b*x+2*a)^3-2*sin(2*b*x+2*a))/c
os(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

3.106.5 Fricas [F]

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \csc\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2),
x)
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.106.7 Maxima [F]

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

3.106.8 Giac [F]

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \frac{\sin{(2\,a+2\,b\,x)}^{7/2}}{\sin{(a+b\,x)}^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^2, x)
```

3.107 $\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.107.1 Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx=\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}+\frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b}$$

output
```
-2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*
x),2^(1/2))/b-2/3*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b+1/3*csc(b*x+a)^2*s
in(2*b*x+2*a)^(7/2)/b
```

3.107.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.45

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx=\frac{2\left(3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))\right)}{3b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(2*(3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2)))/(3*b)
```

3.107.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4788, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{5/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{10}{3}\int \sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{3}\int \sin(2a+2bx)^{5/2}dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{10}{3}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{3}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}+\frac{10}{3}\left(\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2))/(3*b) + (10*((3*EllipticE[a - Pi/4
 + b*x, 2])/(5*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(3/2))/(5*b)))/3
```

3.107.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.107.4 Maple [A] (verified)

Time = 19.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.83

method	result
default	$\frac{2\sqrt{2}\left(\frac{\sqrt{2}\,\sin(2xb+2a)^{\frac{3}{2}}}{6}-\frac{\sqrt{2}\,\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{\sin(2xb+\cdots}\right.\right.}{4\cos(2xb+2a)\sqrt{\sin(2xb+2a)}}\right.}{b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*(1/6*2^(1/2)*sin(2*b*x+2*a)^(3/2)-1/4*2^(1/2)*(sin(2*b*x+2*a)+1)
^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*(2*EllipticE((s
in(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2
*2^(1/2)))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

3.107.5 Fricas [F]

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)),
x)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.107.7 Maxima [F]

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

3.107.8 Giac [F]

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{5/2}}{\sin(a+b\,x)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^2, x)
```

3.108 $\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.108.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\frac{2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{b}-\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}$$

output
```
-2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*
x),2^(1/2))/b+csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2)/b-2*cos(2*b*x+2*a)*sin(2*b
*x+2*a)^(1/2)/b
```

3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\frac{2\left(1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\sin(2(a+bx))}}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(2*(1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x
]^2])*Sqrt[Sin[2*(a + b*x)]])/b
```

3.108.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4788, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{3/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4788}$$

$$6\int \sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$6\int \sin(2a+2bx)^{3/2}dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3115}$$

$$6\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$6\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}+6\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
6*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2
*b*x]])/(3*b)) + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2))/b
```

3.108.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.108.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.59

method	result	size
default	$\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{\sin(2xb+2a)}+\frac{\sqrt{2}\,\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\ \operatorname{EllipticF}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)}{2\cos(2xb+2a)\sqrt{\sin(2xb+2a)}}\right)}{b}$	111

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(2^(1/2)*sin(2*b*x+2*a)^(1/2)+1/2*2^(1/2)*(sin(2*b*x+2*a)+1)^(1/2)
*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+
2*a)+1)^(1/2),1/2*2^(1/2))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

3.108.5 Fricas [F]

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.108.7 Maxima [F]

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

3.108.8 Giac [F]

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{3/2}}{\sin(a+b\,x)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^2, x)
```

3.109 $\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.109.1 Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}$$

output
```
2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x
),2^(1/2))/b-csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2)/b
```

3.109.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{2\left(E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\cot(a+bx)\sqrt{\sin(2(a+bx))}\right)}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/
b
```

3.109.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4788, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4788}$$

$$-2\int \sqrt{\sin(2a+2bx)}dx - \frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-2\int \sqrt{\sin(2a+2bx)}dx - \frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b} - \frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-2*EllipticE[a - Pi/4 + b*x, 2])/b - (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/
2))/b
```

3.109.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(67) = 134$.

Time = 4.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 4.00

method	result
default	$\frac{2\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\operatorname{EllipticE}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+}}{\cos(2xb+2a)\sqrt{\sin(2xb+2a)}\,b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2)*(2*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE((sin(2*b*x+2*a)+1)^(
1/2),1/2*2^(1/2))-(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-s
in(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-2*cos
(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

3.109.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.52

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{-i\sqrt{2i}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+i\sqrt{-2i}E(\arcsin(\cos(bx+a)-i\sin(}{}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2*I)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*
x + a) + I*sqrt(-2*I)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1
)*sin(b*x + a) + I*sqrt(2*I)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x +
a)), -1)*sin(b*x + a) - I*sqrt(-2*I)*elliptic_f(arcsin(cos(b*x + a) - I*si
n(b*x + a)), -1)*sin(b*x + a) - 2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*
cos(b*x + a))/(b*sin(b*x + a))
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.109.7 Maxima [F]

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)^2\sqrt{\sin(2bx+2a)}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

3.109.8 Giac [F]

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc\left(bx+a\right)^2\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{\sin\left(a+b\,x\right)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^2, x)
```

3.110 $\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

3.110.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=\frac{2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{3b}-\frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}$$

output
```
-2/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/3*csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2)/b
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\left(\csc^2(a+bx)-2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\sin(2(a+bx))}}{3b}$$

input
```
Integrate[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/3*((Csc[a + b*x]^2 - 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2
]*Sqrt[Sec[a + b*x]^2])*Sqrt[Sin[2*(a + b*x)]])/b
```

3.110.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4788, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 4788}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

input
```
Int[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*
b*x]])/(3*b)
```

3.110.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.110.4 Maple [A] (verified)

Time = 6.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.56

method	result
default	$\frac{\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)\sin(2xb+2a)-2\cos(2xb+2a)^2-2\cos(2x}{3\sin(2xb+2a)^{\frac{3}{2}}\cos(2xb+2a)b}$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(
1/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

3.110.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.15

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sqrt{2i}\left(\cos\left(bx+a\right)^2-1\right)F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+\sqrt{-2i}\left(\cos\left(bx+a\right)^2-1\right)F(\text{arcs}}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(2*I)*(cos(b*x + a)^2 - 1)*elliptic_f(arcsin(cos(b*x + a) + I*si
n(b*x + a)), -1) + sqrt(-2*I)*(cos(b*x + a)^2 - 1)*elliptic_f(arcsin(cos(b
*x + a) - I*sin(b*x + a)), -1) - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))/
(b*cos(b*x + a)^2 - b)
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.110.7 Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

3.110.8 Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=\int \frac{\csc\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2)), x)
```

3.111 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.111.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=-\frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

output
```
6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b-6/5*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(1/2)-1/5*csc(b*x+a)^2/
b/sin(2*b*x+2*a)^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\frac{-12E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{2(1-6\cos(2(a+bx))+3\cos(4(a+bx)))\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))}}{10b}$$

input
```
Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(
a + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(10*b)
```

3.111.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4788, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{6}{5}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\int \frac{1}{\sin(2a+2bx)^{3/2}}dx-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{6}{5}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6}{5}\left(-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(6*(-(EllipticE[a - Pi/4 + b*x, 2]/b) - Cos[2*a + 2*b*x]/(b*Sqrt[Sin[2*a +
 2*b*x]])))/5 - Csc[a + b*x]^2/(5*b*Sqrt[Sin[2*a + 2*b*x]])
```

3.111.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(92) = 184$.

Time = 21.94 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.95

method	result
default	$\frac{\sqrt{2}\left(-\frac{8\sqrt{2}}{5\sin(2xb+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\sin(2xb+2a)^{2}\,\text{EllipticE}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(2}{5\sin(2x}\right.}{8b}$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^
(5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+
2*a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2
))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a)
)^(1/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6
*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

3.111.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.45

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{6\sqrt{2i}\left(i\cos(bx+a)^3 - i\cos(bx+a)\right)E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+6\sqrt{}}{}$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(6*sqrt(2*I)*(I*cos(b*x + a)^3 - I*cos(b*x + a))*elliptic_e(arcsin(c
os(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 6*sqrt(-2*I)*(-I*cos(b*x
 + a)^3 + I*cos(b*x + a))*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a))
, -1)*sin(b*x + a) + 6*sqrt(2*I)*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*elli
ptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 6*sqrt(-2
*I)*(I*cos(b*x + a)^3 - I*cos(b*x + a))*elliptic_f(arcsin(cos(b*x + a) - I
*sin(b*x + a)), -1)*sin(b*x + a) + sqrt(2)*(12*cos(b*x + a)^4 - 18*cos(b*x
 + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)))/((b*cos(b*x + a)^3 - b*cos(b
*x + a))*sin(b*x + a))
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.111.7 Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

3.111.8 Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\int \frac{1}{\sin(a+b\,x)^2\,\sin(2\,a+2\,b\,x)^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2)), x)
```

3.112 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.112.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{10\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
-10/21*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*P
i+b*x),2^(1/2))/b-10/21*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(3/2)-1/7*csc(b*x+
a)^2/b/sin(2*b*x+2*a)^(3/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{40\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)+\left(-13\csc^2(a+bx)-3\csc^4(a+bx)+7\sec^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{84b}$$

input
```
Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(40*EllipticF[a - Pi/4 + b*x, 2] + (-13*Csc[a + b*x]^2 - 3*Csc[a + b*x]^4
+ 7*Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(84*b)
```

3.112.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4788, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{10}{7}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}\int \frac{1}{\sin(2a+2bx)^{5/2}}dx-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{10}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{10}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(10*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - Cos[2*a + 2*b*x]/(3*b*Sin[2*a +
2*b*x]^(3/2))))/7 - Csc[a + b*x]^2/(7*b*Sin[2*a + 2*b*x]^(3/2))
```

3.112.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.112.4 Maple [A] (verified)

Time = 39.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.00

method	result
default	$\frac{\sqrt{2}\left(-\frac{16\sqrt{2}}{7\sin(2xb+2a)^{\frac{7}{2}}}+\frac{8\sqrt{2}\left(5\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\mathrm{EllipticF}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)\sin(2xb+2a)^{3}+10\sin(2a \cdots\right.}{21\sin(2xb+2a)^{\frac{7}{2}}\cos(2xb+2a)}\right.}{16b}$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*2^(1/2)*(-16/7*2^(1/2)/sin(2*b*x+2*a)^(7/2)+8/21*2^(1/2)/sin(2*b*x+2*
a)^(7/2)*(5*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b
*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))*sin(2*b*x+2
*a)^3+10*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-6)/cos(2*b*x+2*a))/b
```

3.112.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.32

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{20\sqrt{2i}\left(\cos(bx+a)^6 - 2\cos(bx+a)^4 + \cos(bx+a)^2\right)F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+}{}$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/84*(20*sqrt(2*I)*(cos(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*e
lliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + 20*sqrt(-2*I)*(cos(
b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*elliptic_f(arcsin(cos(b*x
+ a) - I*sin(b*x + a)), -1) - sqrt(2)*(20*cos(b*x + a)^4 - 30*cos(b*x + a)
^2 + 7)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^6 - 2*b*cos(b*x +
 a)^4 + b*cos(b*x + a)^2)
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.112.7 Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

3.112.8 Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{5/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2)), x)
```

3.113 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

3.113.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{14E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)} -\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}$$

output
```
14/15*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))/b-14/45*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(5/2)-1/9*csc(b*x+a
)^2/b/sin(2*b*x+2*a)^(5/2)-14/15*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{336E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{(-9+98\cos(2(a+bx))-28\cos(4(a+bx))-42\cos(6(a+bx))+21\cos(8(a+bx)))\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2(a+bx))}}{360b}$$

input
```
Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/360*(336*EllipticE[a - Pi/4 + b*x, 2] + ((-9 + 98*Cos[2*(a + b*x)] - 28
*Cos[4*(a + b*x)] - 42*Cos[6*(a + b*x)] + 21*Cos[8*(a + b*x)])*Csc[a + b*x
]^2)/Sin[2*(a + b*x)]^(5/2))/b
```

3.113.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4788, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{14}{9}\int \frac{1}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{9}\int \frac{1}{\sin(2a+2bx)^{7/2}}dx-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{14}{9}\left(\frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{9}\left(\frac{3}{5}\int \frac{1}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{14}{9}\left(\frac{3}{5}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{9}\left(\frac{3}{5}\left(-\int\sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{14}{9}\left(\frac{3}{5}\left(-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

input
```
Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(14*((3*(-(EllipticE[a - Pi/4 + b*x, 2]/b) - Cos[2*a + 2*b*x]/(b*Sqrt[Sin[
2*a + 2*b*x]])))/5 - Cos[2*a + 2*b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))))/9 - C
sc[a + b*x]^2/(9*b*Sin[2*a + 2*b*x]^(5/2))
```

3.113.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
 + 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.113.4 Maple [F(-1)]

Timed out.

$$\int \frac{\csc(xb+a)^2}{\sin(2xb+2a)^{\frac{7}{2}}}dx$$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

3.113.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.26

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx =$$
$$-\frac{168\sqrt{2i}\left(i\cos(bx+a)^7-2i\cos(bx+a)^5+i\cos(bx+a)^3\right)E(\arcsin(\cos(bx+a)+i\sin(bx+a))\mid -$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/360*(168*sqrt(2*I)*(I*cos(b*x + a)^7 - 2*I*cos(b*x + a)^5 + I*cos(b*x +
 a)^3)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a)
+ 168*sqrt(-2*I)*(-I*cos(b*x + a)^7 + 2*I*cos(b*x + a)^5 - I*cos(b*x + a)^
3)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + 16
8*sqrt(2*I)*(-I*cos(b*x + a)^7 + 2*I*cos(b*x + a)^5 - I*cos(b*x + a)^3)*el
liptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 168*sqr
t(-2*I)*(I*cos(b*x + a)^7 - 2*I*cos(b*x + a)^5 + I*cos(b*x + a)^3)*ellipti
c_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + sqrt(2)*(336
*cos(b*x + a)^8 - 840*cos(b*x + a)^6 + 644*cos(b*x + a)^4 - 126*cos(b*x +
a)^2 - 9)*sqrt(cos(b*x + a)*sin(b*x + a)))/((b*cos(b*x + a)^7 - 2*b*cos(b*
x + a)^5 + b*cos(b*x + a)^3)*sin(b*x + a))
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.113.7 Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

3.113.8 Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{7/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2)), x)
```

3.114 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

3.114.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{30\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{77b} - \frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
-30/77*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*P
i+b*x),2^(1/2))/b-18/77*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(7/2)-1/11*csc(b*x
+a)^2/b/sin(2*b*x+2*a)^(7/2)-30/77*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(3/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{480\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)+\left(-141\csc^2(a+bx)-32\csc^4(a+bx)-7\csc^6(a+bx)+11\sec^2(a+bx)\right.(}{1232b}$$

input
```
Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(480*EllipticF[a - Pi/4 + b*x, 2] + (-141*Csc[a + b*x]^2 - 32*Csc[a + b*x]
^4 - 7*Csc[a + b*x]^6 + 11*Sec[a + b*x]^2*(9 + Sec[a + b*x]^2))*Sqrt[Sin[2
*(a + b*x)]])/(1232*b)
```

3.114.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4788, 3042, 3116, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{18}{11}\int \frac{1}{\sin^{\frac{9}{2}}(2a+2bx)}dx-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{11}\int \frac{1}{\sin(2a+2bx)^{9/2}}dx-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{18}{11}\left(\frac{5}{7}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{11}\left(\frac{5}{7}\int \frac{1}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{18}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\downarrow 3120$$

$$\frac{18}{11}\left(\frac{5}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

input
```
Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(18*((5*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - Cos[2*a + 2*b*x]/(3*b*Sin[2*
a + 2*b*x]^(3/2))))/7 - Cos[2*a + 2*b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2))))/11
 - Csc[a + b*x]^2/(11*b*Sin[2*a + 2*b*x]^(7/2))
```

3.114.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.114.4 Maple [F(-1)]

Timed out.

$$\int \frac{\csc(xb+a)^2}{\sin(2xb+2a)^{\frac{9}{2}}}dx$$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

3.114.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.22

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{240\sqrt{2i}\left(\cos(bx+a)^{10} - 3\cos(bx+a)^8 + 3\cos(bx+a)^6 - \cos(bx+a)^4\right)F(\arcsin(\cos(bx+a)+i\,\mathrm{s}}{}$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
-1/1232*(240*sqrt(2*I)*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a
)^6 - cos(b*x + a)^4)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1
) + 240*sqrt(-2*I)*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)^6
- cos(b*x + a)^4)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) -
sqrt(2)*(240*cos(b*x + a)^8 - 600*cos(b*x + a)^6 + 444*cos(b*x + a)^4 - 66
*cos(b*x + a)^2 - 11)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^10
- 3*b*cos(b*x + a)^8 + 3*b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

3.114.7 Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

3.114.8 Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{9/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(9/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(9/2)), x)
```

3.115 $\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

3.115.1 Optimal result

Integrand size = 22, antiderivative size = 190

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = -\frac{7\arcsin(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b} - \frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b} - \frac{14\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{15b} + \frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b} + \frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b}$$

output
```
-7/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+7/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+7/6*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-14/15*cos(b*x+a)*sin
(2*b*x+2*a)^(5/2)/b+4/5*sin(b*x+a)*sin(2*b*x+2*a)^(7/2)/b+1/5*csc(b*x+a)^3
*sin(2*b*x+2*a)^(11/2)/b-7/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.115.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$$
$$=\frac{7\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-\frac{2}{3}(10\cos}{8b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(7*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2
*Cos[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(8*b)
```

3.115.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 4788, 3042, 4796, 3042, 4789, 3042, 4790, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2a+2bx)^{9/2}}{\sin(a+bx)^3}dx$$
$$\Big\downarrow \text{4788}$$
$$\frac{16}{5}\int \csc(a+bx)\sin^{\frac{9}{2}}(2a+2bx)dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{16}{5}\int \frac{\sin(2a+2bx)^{9/2}}{\sin(a+bx)}dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$
$$\Big\downarrow \text{4796}$$

$$\frac{32}{5}\int \cos(a+bx)\sin^{\frac{7}{2}}(2a+2bx)dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{32}{5}\int \cos(a+bx)\sin(2a+2bx)^{7/2}dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 4789}$$

$$\frac{32}{5}\left(\frac{7}{8}\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{8b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{32}{5}\left(\frac{7}{8}\int \sin(a+bx)\sin(2a+2bx)^{5/2}dx+\frac{\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{8b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 4790}$$

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}\right)+\frac{\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{8b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}\right)+\frac{\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{8b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 4789}$$

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}\right)+ \ldots\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

3.115. $\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}\right)+\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\big\downarrow$ 4790

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2\right.\right.\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\big\downarrow$ 3042

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2\right.\right.\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\big\downarrow$ 4793

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\right.\right.\right.\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(11/2))/(5*b) + (32*((Sin[a + b*x]*Sin[2*
a + 2*b*x]^(7/2))/(8*b) + (7*(-1/6*(Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/b
 + (5*((3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x]
+ Sin[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin
[2*a + 2*b*x]])/(2*b)))/4 + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/
6))/8))/5
```

3.115. $\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

3.115.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.115.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 105.30 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.32

method	result
default	$-\frac{64\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\right.}{}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-64/21*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*((tan(1/2*a+1/
2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2
)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^6
-3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*
a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(
1/2*a+1/2*x*b)^4+2*tan(1/2*a+1/2*x*b)^7+3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2
*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/
2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2+10*tan(1/2*a+1/2*x
*b)^5-(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1
/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+1
0*tan(1/2*a+1/2*x*b)^3+2*tan(1/2*a+1/2*x*b))/(tan(1/2*a+1/2*x*b)*(tan(1/2*
a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)-1)^2/(tan(1/2*a+1/2*x*b)^3-tan(
1/2*a+1/2*x*b))^(1/2)/(tan(1/2*a+1/2*x*b)+1)^2/b
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.53

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx =$$

$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^5-4\cos(bx+a)^3-7\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-42\arctan\left(-\frac{\sqrt{2}}{}\right.}{}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
-1/96*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*s
qrt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin
(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos
(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*sq
rt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a)
 - sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^
3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a
)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

3.115.7 Maxima [F]

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{9}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(9/2), x)
```

3.115.8 Giac [F]

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^3 \sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(9/2), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \frac{\sin\left(2\,a+2\,b\,x\right)^{9/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^3, x)
```

3.116 $\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

3.116.1 Optimal result

Integrand size = 22, antiderivative size = 164

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} + \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b} + \frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b}$$

output
```
-5/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b-5/3*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+4/3*sin(b*x+a)*sin(2
*b*x+2*a)^(5/2)/b+1/3*csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2)/b+5/2*sin(b*x+a)*s
in(2*b*x+2*a)^(1/2)/b
```

3.116.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.51

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{-5\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin(2}{4b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + S
in[3*(a + b*x)]))/(4*b)
```

3.116.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4788, 3042, 4796, 3042, 4789, 3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{7/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4788}$$

$$4\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$4\int \frac{\sin(2a+2bx)^{7/2}}{\sin(a+bx)}dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4796}$$

$$8\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)\sin(2a+2bx)^{5/2}dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4789}$$

$$8\left(\frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{5}{6}\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4790}$$

$$8\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4789}$$

$$8\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\ldots}{\ldots}\right.$$
$$\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

3.116. $\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

$$8\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)}{}\right.$$
$$\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4794}$$

$$8\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(}{}\right.\right.\right.$$
$$\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2))/(3*b) + 8*((Sin[a + b*x]*Sin[2*a +
 2*b*x]^(5/2))/(6*b) + (5*((3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b
 - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (S
in[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 - (Cos[a + b*x]*Sin[2*a + 2*
b*x]^(3/2))/(4*b)))/6)
```

3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.116.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 102.09 (sec) , antiderivative size = 973, normalized size of antiderivative = 5.93

method	result	size
default	Expression too large to display	973

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
32/5*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(2*(tan(1/2*a+1/
2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2
)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*
(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)+1))^(1/2)*tan(1/2*a+1/2*x*b)^4-
(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1
/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/
2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)+1))^(1/2)*tan(1/2*
a+1/2*x*b)^4+2*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1
/2*x*b)^6-4*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(
-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1
/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)+1))^(1
/2)*tan(1/2*a+1/2*x*b)^2+2*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*
x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)
^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+
1/2*x*b)+1))^(1/2)*tan(1/2*a+1/2*x*b)^2-4*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+
1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4+2*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*ta
n(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a
+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)
*(tan(1/2*a+1/2*x*b)+1))^(1/2)-(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+
1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*...
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.71

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{8\sqrt{2}\left(4\cos{(bx+a)}^2+5\right)\sqrt{\cos{(bx+a)}\sin{(bx+a)}}\sin{(bx+a)}+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx}{\cos(bx+a)^2+2\,c}\right.}{}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/16*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*sin
(b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
 a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b
*x + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b
*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 5
*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2
+ 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*c
os(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.116.7 Maxima [F]

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(7/2), x)
```

3.116.8 Giac [F]

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(7/2), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \frac{\sin\left(2\,a+2\,b\,x\right)^{7/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^3, x)
```

3.117 $\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.117.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b} - \frac{6\cos(a+bx)\sqrt{\sin(2a+2bx)}}{b} + \frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b} + \frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b}$$

output
```
-3*arcsin(cos(b*x+a)-sin(b*x+a))/b+3*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*
a)^(1/2))/b+4*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+csc(b*x+a)^3*sin(2*b*x+2*a
)^(7/2)/b-6*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.117.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$=\frac{-3\arcsin(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+\csc(a+b}{b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/b
```

3.117.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4788, 3042, 4796, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{5/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4788}$$

$$8\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$8\int \frac{\sin(2a+2bx)^{5/2}}{\sin(a+bx)}dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4796}$$

$$16\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$16\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \; 4789$$

$$16\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$16\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \; 4790$$

$$16\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$16\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \; 4793$$

$$16\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a}{}\right.\right.$$
$$\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

3.117. $\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

output
```
(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2))/b + 16*((3*((-1/2*ArcSin[Cos[a + b
*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a + 2
*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 + (Sin[
a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b))
```

3.117.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.117.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 107.88 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.91

method	result
default	$\frac{16\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-1}}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-\right.}{3\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\sqrt{\tan}}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
16/3*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*((tan(1/2*a+1/2*
x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*
EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-(
tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/
2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))-tan(1/2*
a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))
^(1/2)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/b
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(119) = 238.

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.11

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\,s}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)
) - 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) -
sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 +
4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(
b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x
 + a)*sin(b*x + a) + 1))/b
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.117.7 Maxima [F]

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)
```

3.117.8 Giac [F]

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{5/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^3, x)
```

3.118 $\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.118.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{2\arcsin(\cos(a+bx)-\sin(a+bx))}{b} + \frac{2\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b} - \frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b} - \frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}$$

output
```
2*arcsin(cos(b*x+a)-sin(b*x+a))/b+2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a
)^(1/2))/b-csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2)/b-4*sin(b*x+a)*sin(2*b*x+2*a)
^(1/2)/b
```

3.118.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{2\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\csc(a+bx\right.}{b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/b
```

3.118.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4788, 3042, 4796, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{3/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4788}$$

$$-4\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-4\int \frac{\sin(2a+2bx)^{3/2}}{\sin(a+bx)}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4796}$$

$$-8\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-8\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4789}$$

$$-8\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-8\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4794}$$

$$-8\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+ba}{}\right.$$
$$\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-8*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)) - (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2))/b
```

3.118.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.118.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 14.65 (sec) , antiderivative size = 542, normalized size of antiderivative = 5.21

method	result
default	$\frac{4\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(4\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right.}}{}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(4*(tan(1/2*a+1/2*x
*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*(
tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b)*(ta
n(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)+1))^(1/2)*EllipticE((tan(1/2*a+1/2
*x*b)+1)^(1/2),1/2*2^(1/2))-2*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1
/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)*(tan(1/2*
a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2
*a+1/2*x*b)+1))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+
(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)*(tan(1
/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)+1))^(1/2)*tan(1/2*a+1/2*x*b)^2+2*tan(
1/2*a+1/2*x*b)^2*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(
1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)-(tan(1/2*a+1/2*x*b)^3-tan(1/2*a
+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2
*x*b)+1))^(1/2))/tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*
b))^(1/2)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)-1)*(tan(1/2*a+1/2*x*b)+1
))^(1/2)/b
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(98) = 196$.

Time = 0.27 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.84

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx =$$

$$-\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\cdots}}{}\right.}{}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - s
in(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)
*sin(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*
sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))
)*sin(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4
*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x
+ a) + 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin(
b*x + a))
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.118.7 Maxima [F]

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

3.118.8 Giac [F]

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{3/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^3, x)
```

3.119 $\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.119.1 Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}$$

output
```
-1/3*csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2)/b
```

3.119.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2(a+bx))}{3b}$$

input
```
Integrate[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/3*(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/b
```

3.119.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\csc^3(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)^3}dx$$
$$\Big\downarrow \text{ 4780}$$
$$-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

input
```
Int[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/3*(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2))/b
```

3.119.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

3.119.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 13.02 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.86

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(4\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1},\right.\right.}{3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,b}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*
b)^2-1)/tan(1/2*a+1/2*x*b)*(4*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1
/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)
+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)+tan(1/2*a+1/2*x*b)^4-1)/(tan(1/2
*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*
a+1/2*x*b))^(1/2)/b
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{2\left(\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+\cos(bx+a)^2-1\right)}{3\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^2
 - 1)/(b*cos(b*x + a)^2 - b)
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)^3\,\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

3.119.8 Giac [F]

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)^3\,\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 21.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.39

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$=\frac{4\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(4\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-6\sin\left(\frac{3\,a}{2}+\frac{3\,b\,x}{2}\right)^2+2\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2\right)}{3\,b\,\left(30\sin\left(a+b\,x\right)^2-12\sin\left(2\,a+2\,b\,x\right)^2+2\sin\left(3\,a+3\,b\,x\right)^2\right)}$$

input
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^3,x)
```

output
```
(4*sin(2*a + 2*b*x)^(1/2)*(4*sin(a/2 + (b*x)/2)^2 - 6*sin((3*a)/2 + (3*b*x
)/2)^2 + 2*sin((5*a)/2 + (5*b*x)/2)^2))/(3*b*(2*sin(3*a + 3*b*x)^2 - 12*si
n(2*a + 2*b*x)^2 + 30*sin(a + b*x)^2))
```

3.120 $\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

3.120.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{4\csc(a+bx)\sqrt{\sin(2a+2bx)}}{5b} - \frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b}$$

output
```
-4/5*csc(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-1/5*csc(b*x+a)^3*sin(2*b*x+2*a)^(1/
2)/b
```

3.120.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc(a+bx)\left(4+\csc^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{5b}$$

input
```
Integrate[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/5*(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/b
```

3.120.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4788, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{4}{5}\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{1}{\sin(a+bx)\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{4780}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}-\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)}{5b}$$

input
```
Int[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-4*Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(5*b) - (Csc[a + b*x]^3*Sqrt[Sin[
2*a + 2*b*x]])/(5*b)
```

3.120.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.120.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 13.76 (sec) , antiderivative size = 482, normalized size of antiderivative = 8.76

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(16\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\operatorname{EllipticE}\left(\sqrt{t\right.\right.}{}$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/tan(1/2*a+1/2*x*
b)^3*(16*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/
2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2
)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2
-8*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b)
+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*Elli
pticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-(tan(
1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^6+(tan(1
/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^4+8*(tan(
1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4+(tan(1/2*a
+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2-8*(tan(1/2*
a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2-(tan(1/2*a+1/2
*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2))/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2
*x*b))^(1/2)/b
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sqrt{2}\left(4\cos(bx+a)^2-5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+4\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{5\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*(
cos(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

3.120.8 Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 24.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{8\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{5\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2)),x)
```

output
```
-(8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2)*(exp(a*4i + b*x*4i)*1i - exp(a*2i + b*x*2i)*3i + 1i))/(5*b*(e
xp(a*2i + b*x*2i) - 1)^3)
```

3.121 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.121.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=-\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}+\frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

output
```
-16/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-1/7*csc(b*x+a)^3/b/sin(2*b*x+2*a)
^(1/2)+32/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx =\frac{(5-12\cos(2(a+bx))+4\cos(4(a+bx)))\csc^4(a+bx)\sec(a+bx)\sqrt{\sin(2(a+bx))}}{42b}$$

input
```
Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x
]*Sqrt[Sin[2*(a + b*x)]])/(42*b)
```

3.121.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4788, 3042, 4796, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{ 4788}$$

$$\frac{8}{7}\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8}{7}\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{3/2}}dx-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{ 4796}$$

$$\frac{16}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{16}{7}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{ 4791}$$

$$\frac{16}{7}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{16}{7}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{ 4780}$$

$$\frac{16}{7}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(16*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*
Sqrt[Sin[2*a + 2*b*x]])))/7 - Csc[a + b*x]^3/(7*b*Sqrt[Sin[2*a + 2*b*x]])
```

3.121.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p]
&& IntegerQ[2*p]
```

3.121.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 60.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.74

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(-3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{8}+16\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\text{Elliptic}}{336\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right.}}$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/336*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2
*x*b)^2-1)/tan(1/2*a+1/2*x*b)^3*(-3*tan(1/2*a+1/2*x*b)^8+16*(tan(1/2*a+1/2
*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)
*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^3-
2*tan(1/2*a+1/2*x*b)^6+2*tan(1/2*a+1/2*x*b)^2+3)/(tan(1/2*a+1/2*x*b)*(tan(
1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)
/b
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.28

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{32\,\cos(bx+a)^5 - 64\,\cos(bx+a)^3 + \sqrt{2}\left(32\,\cos(bx+a)^4 - 56\,\cos(bx+a)^2 + 21\right)\sqrt{\cos(bx+a)\sin(bx}}{42\left(b\cos(bx+a)^5 - 2\,b\cos(bx+a)^3 + b\cos(bx+a)\right)}$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/42*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
 56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)
)/(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.121.7 Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

3.121.8 Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 24.17 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.73

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{10\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{21\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^2} + \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,12\mathrm{i}}{7\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^3} - \frac{8\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{7\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^4} - \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\left(\frac{10\mathrm{i}}{21\,b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,32\mathrm{i}}{21\,b}\right)\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2)),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*12i)/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (10*exp(a*1i + b*x*1
i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(21*b*
(exp(a*2i + b*x*2i)*1i - 1i)^2) - (8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x
*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i
 - 1i)^4) - (exp(a*1i + b*x*1i)*(10i/(21*b) - (exp(a*2i + b*x*2i)*32i)/(21
*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((ex
p(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
```

3.122 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.122.1 Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

output
```
-8/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)-1/9*csc(b*x+a)^3/b/sin(2*b*x+2*a)^
(3/2)+32/45*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-64/45*cos(b*x+a)/b/sin(2*b*x
+2*a)^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin(2(a+bx))}(113\csc(a+bx)+17\csc^3(a+bx)+5\csc^5(a+bx)-15\sec(a+bx)\tan(a+bx))}{180b}$$

input
```
Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/180*(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*C
sc[a + b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/b
```

3.122.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4788, 3042, 4796, 3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{4}{3}\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{5/2}}dx-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4796}$$

$$\frac{8}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{8}{3}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{8}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{8}{3}\left(\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(8*((4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*
Sqrt[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))))/
3 - Csc[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(3/2))
```

3.122.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.122.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 139.42 (sec) , antiderivative size = 560, normalized size of antiderivative = 5.23

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(5\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{10}+192\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\right.}{}$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2880*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/tan(1/2*a+1/2
*x*b)^5*(5*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1
/2*x*b)^10+192*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/
2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b)
)^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*
x*b)^4-96*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1
/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/
2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^
4-7*tan(1/2*a+1/2*x*b)^8*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/
2)+2*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b
)^6+96*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^
6+2*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)
^4-96*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4
-7*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^
2+5*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2))/(tan(1/2*a+1/2*x*
b)^3-tan(1/2*a+1/2*x*b))^(1/2)/b
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(128\cos(bx+a)^6-288\cos(bx+a)^4+180\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}+128\,(\ldots}{180\left(b\cos(bx+a)^6-2\,b\cos(bx+a)^4+b\cos(bx+a)^2\right)\ldots}$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/180*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x + a
)^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b*
x + a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x
+ a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.122.7 Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

3.122.8 Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 26.08 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.58

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{2\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{15\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3} - \frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,16i}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4} + \frac{8\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^5} + \frac{64\,e^{a\,3i+b\,x\,3i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{45\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)} - \frac{e^{a\,1i+b\,x\,1i}\left(\frac{98i}{45\,b}+\frac{e^{a\,2i+b\,x\,2i}\,38i}{45\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)^2\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(5/2)),x)
```

output
```
(8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2))/(9*b*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (exp(a*1i + b*x*1i)*((
exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*16i)/(9*b*(e
xp(a*2i + b*x*2i)*1i - 1i)^4) - (2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2
i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(15*b*(exp(a*2i + b*x*2i)*1i
- 1i)^3) + (64*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i
 + b*x*2i)*1i)/2)^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i
)*1i - 1i)) - (exp(a*1i + b*x*1i)*(98i/(45*b) + (exp(a*2i + b*x*2i)*38i)/(
45*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((
exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i + b*x*2i)*1i - 1i)^2)
```

3.123 $\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx$

3.123.1 Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx$$
$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{4+m}{2},\frac{6+m}{2},\sin^2(a+bx)\right)\sin^3(a+bx)\sin^m(2a+2bx)\tan(a+b}{b(4+m)}$$

output
```
(cos(b*x+a)^2)^(-1/2*m+1/2)*hypergeom([2+1/2*m, -1/2*m+1/2],[3+1/2*m],sin(
b*x+a)^2)*sin(b*x+a)^3*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(4+m)
```

3.123.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 6.70 (sec) , antiderivative size = 602, normalized size of antiderivative = 7.17

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx$$
$$= \frac{}{b(2+m)\left(-2(4+m)\text{AppellF1}\left(1+\frac{m}{2},-m,4+2m,2+\frac{m}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\cos^2\left(\right.\right.}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

output
```
(32*(4 + m)*(AppellF1[1 + m/2, -m, 3 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2] - AppellF1[1 + m/2, -m, 4 + 2*m, 2 + m/2, Tan[(a + b*x
)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^6*Sin[(a + b*x)/2]^4*Sin[2*
(a + b*x)]^m)/(b*(2 + m)*(-2*(4 + m)*AppellF1[1 + m/2, -m, 4 + 2*m, 2 + m/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 2*(m*Appe
llF1[2 + m/2, 1 - m, 3 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/
2]^2] - m*AppellF1[2 + m/2, 1 - m, 4 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2] + 3*AppellF1[2 + m/2, -m, 4 + 2*m, 3 + m/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*m*AppellF1[2 + m/2, -m, 4 + 2*m, 3 + m/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[2 + m/2, -m, 5 +
2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[2 +
m/2, -m, 5 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 +
 Cos[a + b*x]) + (4 + m)*AppellF1[1 + m/2, -m, 3 + 2*m, 2 + m/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x])))
```

3.123.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^m dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^m(a+bx)\sin^{m+3}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos(a+bx)^m\sin(a+bx)^{m+3}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin^3(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+4}{2},\frac{m+6}{2},\sin^2(a+bx)\right)}{b(m+4)}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

output
```
((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
 m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*
(4 + m))
```

3.123.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.123.4 Maple [F]

$$\int \sin\left(xb+a\right)^{3} \sin\left(2xb+2a\right)^{m} dx$$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

output
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

3.123.5 Fricas [F]

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m \sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**m,x)
```

output
```
Timed out
```

3.123.7 Maxima [F]

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m \sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^3, x)
```

3.123.8 Giac [F]

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m \sin(bx+a)^3\;dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^3, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(a+b\,x)^3 \sin(2\,a+2\,b\,x)^m\,dx$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^m,x)
```

output
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^m, x)
```

3.124 $\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx$

3.124.1 Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx$$
$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(a+bx)\right)\sin^2(a+bx)\sin^m(2a+2bx)\tan(a+b}{b(3+m)}$$

output
```
(cos(b*x+a)^2)^(-1/2*m+1/2)*hypergeom([3/2+1/2*m, -1/2*m+1/2],[5/2+1/2*m],
sin(b*x+a)^2)*sin(b*x+a)^2*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(3+m)
```

3.124.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(2+m,\frac{3+m}{2},\frac{5+m}{2},-\tan^2(a+bx)\right)\sec^2(a+bx)^m\sin^m(2(a+bx))\tan^3(a+bx)}{b(3+m)}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

output
```
(Hypergeometric2F1[2 + m, (3 + m)/2, (5 + m)/2, -Tan[a + b*x]^2]*(Sec[a +
b*x]^2)^m*Sin[2*(a + b*x)]^m*Tan[a + b*x]^3)/(b*(3 + m))
```

3.124.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^m dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^m(a+bx)\sin^{m+2}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos(a+bx)^m\sin(a+bx)^{m+2}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin^2(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+3}{2},\frac{m+5}{2},\sin^2(a+bx)\right)}{b(m+3)}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

output
```
((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
 m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*
(3 + m))
```

3.124.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.124.4 Maple [F]

$$\int \sin (xb+a)^2 \sin (2xb+2a)^m \, dx$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

output
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

3.124.5 Fricas [F]

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin (2\,bx+2\,a)^m \sin (bx+a)^2 \,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^m, x)
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

output
```
Timed out
```

3.124.7 Maxima [F]

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^m \sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^2, x)
```

3.124.8 Giac [F]

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^m \sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^2, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(a+b\,x)^2\sin(2\,a+2\,b\,x)^m\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^m,x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^m, x)
```

3.125 $\int \sin(a+bx)\sin^m(2a+2bx)\,dx$

3.125.1 Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx = \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{2+m}{2},\frac{4+m}{2},\sin^2(a+bx)\right)\sin(a+bx)\sin^m(2a+2bx)\tan(a+bx)}{b(2+m)}$$

output
```
(cos(b*x+a)^2)^(-1/2*m+1/2)*hypergeom([1+1/2*m, -1/2*m+1/2],[2+1/2*m],sin(
b*x+a)^2)*sin(b*x+a)*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(2+m)
```

3.125.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.43 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.39

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx = \frac{8(4+m)\,\text{AppellF1}\ldots}{b(2+m)\left(2\left(m\,\text{AppellF1}\left(2+\frac{m}{2},1-m,2+2m,3+\frac{m}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2(1+\ldots\right.\right.}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

output
```
(8*(4 + m)*AppellF1[1 + m/2, -m, 2 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^4*Sin[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m
)/(b*(2 + m)*(2*(m*AppellF1[2 + m/2, 1 - m, 2 + 2*m, 3 + m/2, Tan[(a + b*x
)/2]^2, -Tan[(a + b*x)/2]^2] + 2*(1 + m)*AppellF1[2 + m/2, -m, 3 + 2*m, 3
+ m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (4
+ m)*AppellF1[1 + m/2, -m, 2 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2]*(1 + Cos[a + b*x])))
```

3.125.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^m dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^m(a+bx)\sin^{m+1}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos(a+bx)^m\sin(a+bx)^{m+1}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+2}{2},\frac{m+4}{2},\sin^2(a+bx)\right)}{b(m+2)}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

output
```
((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 +
 m)/2, Sin[a + b*x]^2]*Sin[a + b*x]*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(2
 + m))
```

3.125.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.125.4 Maple [F]

$$\int \sin(xb+a)\sin(2xb+2a)^m\,dx$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^m,x)
```

output
```
int(sin(b*x+a)*sin(2*b*x+2*a)^m,x)
```

3.125.5 Fricas [F]

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx=\int \sin\left(2\,bx+2\,a\right)^m\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx=\text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**m,x)
```

output
```
Timed out
```

3.125.7 Maxima [F]

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx=\int \sin\left(2\,bx+2\,a\right)^m\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

3.125.8 Giac [F]

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx=\int \sin\left(2\,bx+2\,a\right)^m\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx=\int \sin\left(a+b\,x\right)\,\sin\left(2\,a+2\,b\,x\right)^m\,dx$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^m,x)
```

output
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^m, x)
```

3.126 $\int \csc(a+bx)\sin^m(2a+2bx)\,dx$

3.126.1 Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx = \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{2+m}{2},\sin^2(a+bx)\right)\sec(a+bx)\sin^m(2a+2bx)}{bm}$$

output
```
(cos(b*x+a)^2)^(-1/2*m+1/2)*hypergeom([1/2*m, -1/2*m+1/2],[1+1/2*m],sin(b*
x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/m
```

3.126.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.40 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.53

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx = \frac{2(2+m)\,\text{AppellF1}\left(\frac{m}{2},\right.}{bm\left((2+m)\,\text{AppellF1}\left(\frac{m}{2},-m,2m,\frac{2+m}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(1+\cos(a+bx))-4m\left(\right.\right.}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

output
```
(2*(2 + m)*AppellF1[m/2, -m, 2*m, (2 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2]*Cos[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(b*m*((2 + m)*AppellF1[
m/2, -m, 2*m, (2 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos
[a + b*x]) - 4*m*(AppellF1[(2 + m)/2, 1 - m, 2*m, (4 + m)/2, Tan[(a + b*x)
/2]^2, -Tan[(a + b*x)/2]^2] + 2*AppellF1[(2 + m)/2, -m, 1 + 2*m, (4 + m)/2
, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sin[(a + b*x)/2]^2))
```

3.126.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^m}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^m(a+bx)\sin^{m-1}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos(a+bx)^m\sin(a+bx)^{m-1}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{m+2}{2},\sin^2(a+bx)\right)}{bm}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

output
```
((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2,
 Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*m)
```

3.126.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.126.4 Maple [F]

$$\int \csc\left(xb+a\right)\sin\left(2xb+2a\right)^{m}dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^m,x)
```

output
```
int(csc(b*x+a)*sin(2*b*x+2*a)^m,x)
```

3.126.5 Fricas [F]

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx=\int \sin\left(2\,bx+2\,a\right)^{m}\csc\left(bx+a\right)\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

3.126.6 Sympy [F]

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx = \int \sin^m(2a+2bx)\csc(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**m,x)
```

output
```
Integral(sin(2*a + 2*b*x)**m*csc(a + b*x), x)
```

3.126.7 Maxima [F]

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m\csc(bx+a)\;dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

3.126.8 Giac [F]

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m\csc(bx+a)\;dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx = \int \frac{\sin\left(2\,a+2\,b\,x\right)^m}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^m/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^m/sin(a + b*x), x)
```

3.127 $\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx$

3.127.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = -\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(a+bx)\right)\sec(a+bx)\sin^m}{b(1-m)}$$

output
```
-(cos(b*x+a)^2)^(-1/2*m+1/2)*csc(b*x+a)*hypergeom([-1/2*m+1/2, -1/2+1/2*m]
,[1/2+1/2*m],sin(b*x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/(1-m)
```

3.127.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \frac{\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+m),m,\frac{1+m}{2},-\tan^2(a+bx)\right)\sec^2(a+bx)^m\sin^m(2(a+bx))}{b(-1+m)}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

output
```
(Cot[a + b*x]*Hypergeometric2F1[(-1 + m)/2, m, (1 + m)/2, -Tan[a + b*x]^2]
*(Sec[a + b*x]^2)^m*Sin[2*(a + b*x)]^m)/(b*(-1 + m))
```

3.127.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(2a+2bx)^m}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \; 4798$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^m(a+bx)\sin^{m-2}(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos(a+bx)^m\sin(a+bx)^{m-2}dx$$

$$\Big\downarrow \; 3057$$

$$-\frac{\csc(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m-1}{2},\frac{m+1}{2},\sin^2(a+bx)\right)}{b(1-m)}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

output
```
-(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]*Hypergeometric2F1[(1 - m)/2,
(-1 + m)/2, (1 + m)/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b
*(1 - m)))
```

3.127.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.127.4 Maple [F]

$$\int \csc (xb+a)^2 \sin (2xb+2a)^m \, dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

output
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

3.127.5 Fricas [F]

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin (2\,bx+2\,a)^m \csc (bx+a)^2 \,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

3.127.6 Sympy [F]

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin^m(2a+2bx)\csc^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

output
```
Integral(sin(2*a + 2*b*x)**m*csc(a + b*x)**2, x)
```

3.127.7 Maxima [F]

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m\csc(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

3.127.8 Giac [F]

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m\csc(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^m}{\sin(a+b\,x)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^m/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^m/sin(a + b*x)^2, x)
```

3.128 $\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx$

3.128.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx =$$
$$-\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1}{2}(-2+m),\frac{m}{2},\sin^2(a+bx)\right)\sec(a+bx)\sin^m(}{b(2-m)}$$

output
```
-(cos(b*x+a)^2)^(-1/2*m+1/2)*csc(b*x+a)^2*hypergeom([-1+1/2*m, -1/2*m+1/2]
,[1/2*m],sin(b*x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/(2-m)
```

3.128.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 17.64 (sec) , antiderivative size = 2308, normalized size of antiderivative = 27.15

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx = \text{Result too large to show}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

output
```
(AppellF1[-1 + m/2, -m, 2*m, m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
*Cos[(a + b*x)/2]^2*Cot[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(2*b*(-2 + m)*(
2*(AppellF1[m/2, 1 - m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2] + 2*AppellF1[m/2, -m, 1 + 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2])*(-1 + Cos[a + b*x]) + AppellF1[-1 + m/2, -m, 2*m, m/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))) + ((4 + m)*AppellF
1[1 + m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[
a + b*x]*Sin[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(2*b*(2 + m)*((4 + m)*Appe
llF1[1 + m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(
1 + Sec[a + b*x]) - 4*m*(AppellF1[2 + m/2, 1 - m, 2*m, 3 + m/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*AppellF1[2 + m/2, -m, 1 + 2*m, 3 + m/2,
 Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sec[a + b*x]*Sin[(a + b*x)/2]^2
)) + ((4 + m)*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2]*Sin[a + b*x]^2*Sin[2*(a + b*x)]^m)/(4*b*(2 + m)*(2*(m
*AppellF1[2 + m/2, 1 - m, 1 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] + (1 + 2*m)*AppellF1[2 + m/2, -m, 2 + 2*m, 3 + m/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (4 + m)*AppellF1[1 +
m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + C
os[a + b*x]))) + (2^(-3 + m)*Cot[(a + b*x)/2]*(Sec[(a + b*x)/2]^2)^(2*m)*(
Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Sin[2*(a...
```

3.128.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^m}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^m(a+bx)\sin^{m-3}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int\cos(a+bx)^m\sin(a+bx)^{m-3}dx$$

$$\Big\downarrow \text{3057}$$

$$-\frac{\csc^2(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m-2}{2},\frac{m}{2},\sin^2(a+bx)\right)}{b(2-m)}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

output
```
-(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]^2*Hypergeometric2F1[(1 - m)/2
, (-2 + m)/2, m/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(2
- m)))
```

3.128.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.128.4 Maple [F]

$$\int \csc (xb + a)^3 \sin (2xb + 2a)^m \, dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

output
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

3.128.5 Fricas [F]

$$\int \csc^3(a + bx) \sin^m(2a + 2bx) \, dx = \int \sin (2\,bx + 2\,a)^m \csc (bx + a)^3 \, dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

3.128.6 Sympy [F]

$$\int \csc^3(a + bx) \sin^m(2a + 2bx) \, dx = \int \sin^m (2a + 2bx) \csc^3 (a + bx) \, dx$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**m,x)
```

output
```
Integral(sin(2*a + 2*b*x)**m*csc(a + b*x)**3, x)
```

3.128.7 Maxima [F]

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m \csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

3.128.8 Giac [F]

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m \csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^m}{\sin(a+b\,x)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^m/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^m/sin(a + b*x)^3, x)
```

3.129 $\int \cos(a+bx)\sin^7(2a+2bx)\,dx$

3.129.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx = -\frac{128\cos^9(a+bx)}{9b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{128\cos^{15}(a+bx)}{15b}$$

output
```
-128/9*cos(b*x+a)^9/b+384/11*cos(b*x+a)^11/b-384/13*cos(b*x+a)^13/b+128/15
*cos(b*x+a)^15/b
```

3.129.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx = \frac{4\cos^9(a+bx)(-8330+10755\cos(2(a+bx))-3366\cos(4(a+bx))+429\cos(6(a+bx)))}{6435b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(4*Cos[a + b*x]^9*(-8330 + 10755*Cos[2*(a + b*x)] - 3366*Cos[4*(a + b*x)]
+ 429*Cos[6*(a + b*x)]))/(6435*b)
```

3.129.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^7\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$128\int \cos^8(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$128\int \cos(a+bx)^8\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{128\int \cos^8(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{128\int \left(-\cos^{14}(a+bx)+3\cos^{12}(a+bx)-3\cos^{10}(a+bx)+\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{128\left(-\frac{1}{15}\cos^{15}(a+bx)+\frac{3}{13}\cos^{13}(a+bx)-\frac{3}{11}\cos^{11}(a+bx)+\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(-128*(Cos[a + b*x]^9/9 - (3*Cos[a + b*x]^11)/11 + (3*Cos[a + b*x]^13)/13
- Cos[a + b*x]^15/15))/b
```

3.129.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.129.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(53) = 106$.

Time = 4.97 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

method	result
default	$-\frac{35\cos(xb+a)}{128b}-\frac{35\cos(3xb+3a)}{384b}+\frac{21\cos(5xb+5a)}{640b}+\frac{3\cos(7xb+7a)}{128b}-\frac{7\cos(9xb+9a)}{1152b}-\frac{7\cos(11xb+11a)}{1408b}+\frac{\cos(1}{1}$
risch	$-\frac{35\cos(xb+a)}{128b}-\frac{35\cos(3xb+3a)}{384b}+\frac{21\cos(5xb+5a)}{640b}+\frac{3\cos(7xb+7a)}{128b}-\frac{7\cos(9xb+9a)}{1152b}-\frac{7\cos(11xb+11a)}{1408b}+\frac{\cos(1}{1}$
parallelrisch	$\frac{\left(1024\tan(xb+a)^{12}+6400\tan(xb+a)^{10}+16768\tan(xb+a)^{8}+126592\tan(xb+a)^{6}+79616\tan(xb+a)^{4}+27648\tan(xb+a)^{2}+409}{}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
-35/128*cos(b*x+a)/b-35/384*cos(3*b*x+3*a)/b+21/640*cos(5*b*x+5*a)/b+3/128
*cos(7*b*x+7*a)/b-7/1152*cos(9*b*x+9*a)/b-7/1408*cos(11*b*x+11*a)/b+1/1664
*cos(13*b*x+13*a)/b+1/1920*cos(15*b*x+15*a)/b
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx$$
$$=\frac{128\left(429\cos(bx+a)^{15}-1485\cos(bx+a)^{13}+1755\cos(bx+a)^{11}-715\cos(bx+a)^{9}\right)}{6435\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
128/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^1
1 - 715*cos(b*x + a)^9)/b
```

3.129.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(53) = 106$.

Time = 25.61 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.43

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx$$
$$=\begin{cases}-\frac{1241\sin(a+bx)\sin^7(2a+2bx)}{6435b}-\frac{376\sin(a+bx)\sin^5(2a+2bx)\cos^2(2a+2bx)}{715b}-\frac{640\sin(a+bx)\sin^3(2a+2bx)\cos^4(2a+2bx)}{1287b}-\frac{1024\sin}{} \\ x\sin^7(2a)\cos(a)\end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**7,x)
```

output
```
Piecewise((-1241*sin(a + b*x)*sin(2*a + 2*b*x)**7/(6435*b) - 376*sin(a + b
*x)*sin(2*a + 2*b*x)**5*cos(2*a + 2*b*x)**2/(715*b) - 640*sin(a + b*x)*sin
(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**4/(1287*b) - 1024*sin(a + b*x)*sin(2*a
+ 2*b*x)*cos(2*a + 2*b*x)**6/(6435*b) - 3838*sin(2*a + 2*b*x)**6*cos(a + b
*x)*cos(2*a + 2*b*x)/(6435*b) - 1648*sin(2*a + 2*b*x)**4*cos(a + b*x)*cos(
2*a + 2*b*x)**3/(1287*b) - 768*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a +
2*b*x)**5/(715*b) - 2048*cos(a + b*x)*cos(2*a + 2*b*x)**7/(6435*b), Ne(b,
0)), (x*sin(2*a)**7*cos(a), True))
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx = \frac{429\cos(15\,bx+15\,a)+495\cos(13\,bx+13\,a)-4095\cos(11\,bx+11\,a)-5005\cos(9\,bx+9\,a)+19305}{823680\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
1/823680*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*
x + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b
*x + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx = \frac{128\left(429\cos(bx+a)^{15}-1485\cos(bx+a)^{13}+1755\cos(bx+a)^{11}-715\cos(bx+a)^{9}\right)}{6435\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
128/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^1
1 - 715*cos(b*x + a)^9)/b
```

3.129.9 Mupad [B] (verification not implemented)

Time = 19.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx = -\frac{-\frac{128\cos(a+b\,x)^{15}}{15}+\frac{384\cos(a+b\,x)^{13}}{13}-\frac{384\cos(a+b\,x)^{11}}{11}+\frac{128\cos(a+b\,x)^{9}}{9}}{b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^7,x)
```

output
```
-((128*cos(a + b*x)^9)/9 - (384*cos(a + b*x)^11)/11 + (384*cos(a + b*x)^13
)/13 - (128*cos(a + b*x)^15)/15)/b
```

3.129. $\int \cos(a+bx)\sin^7(2a+2bx)\,dx$

3.130 $\int \cos(a+bx)\sin^6(2a+2bx)\,dx$

3.130.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = \frac{64\sin^7(a+bx)}{7b} - \frac{64\sin^9(a+bx)}{3b} + \frac{192\sin^{11}(a+bx)}{11b} - \frac{64\sin^{13}(a+bx)}{13b}$$

output
```
64/7*sin(b*x+a)^7/b-64/3*sin(b*x+a)^9/b+192/11*sin(b*x+a)^11/b-64/13*sin(b
*x+a)^13/b
```

3.130.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = \frac{2(5230+6377\cos(2(a+bx))+1890\cos(4(a+bx))+231\cos(6(a+bx)))\sin^7(a+bx)}{3003b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(2*(5230 + 6377*Cos[2*(a + b*x)] + 1890*Cos[4*(a + b*x)] + 231*Cos[6*(a +
b*x)])*Sin[a + b*x]^7)/(3003*b)
```

3.130.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^6\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$64\int \cos^7(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^7\sin(a+bx)^6dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{64\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{64\int \left(-\sin^{12}(a+bx)+3\sin^{10}(a+bx)-3\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{64\left(-\frac{1}{13}\sin^{13}(a+bx)+\frac{3}{11}\sin^{11}(a+bx)-\frac{1}{3}\sin^9(a+bx)+\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(64*(Sin[a + b*x]^7/7 - Sin[a + b*x]^9/3 + (3*Sin[a + b*x]^11)/11 - Sin[a
+ b*x]^13/13))/b
```

3.130.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.130.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

method	result
default	$\frac{5\sin(xb+a)}{16b}-\frac{5\sin(3xb+3a)}{64b}-\frac{3\sin(5xb+5a)}{64b}+\frac{3\sin(7xb+7a)}{224b}+\frac{\sin(9xb+9a)}{96b}-\frac{\sin(11xb+11a)}{704b}-\frac{\sin(13xb+13a)}{832b}$
risch	$\frac{5\sin(xb+a)}{16b}-\frac{5\sin(3xb+3a)}{64b}-\frac{3\sin(5xb+5a)}{64b}+\frac{3\sin(7xb+7a)}{224b}+\frac{\sin(9xb+9a)}{96b}-\frac{\sin(11xb+11a)}{704b}-\frac{\sin(13xb+13a)}{832b}$
parallelrisch	$\frac{\frac{128\left(-8\tan(xb+a)^{11}-44\tan(xb+a)^{9}-99\tan(xb+a)^{7}+99\tan(xb+a)^{5}+44\tan(xb+a)^{3}+8\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{3003}+\frac{2048\left(\tan(xb+a)^{8}+5\tan\right.}{}}{}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
5/16*sin(b*x+a)/b-5/64*sin(3*b*x+3*a)/b-3/64/b*sin(5*b*x+5*a)+3/224/b*sin(
7*b*x+7*a)+1/96/b*sin(9*b*x+9*a)-1/704/b*sin(11*b*x+11*a)-1/832/b*sin(13*b
*x+13*a)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx =$$

$$-\frac{64\left(231\cos\left(bx+a\right)^{12}-567\cos\left(bx+a\right)^{10}+371\cos\left(bx+a\right)^{8}-5\cos\left(bx+a\right)^{6}-6\cos\left(bx+a\right)^{4}-8\right.}{3003\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
-64/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8 -
 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a)
/b
```

3.130.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(53) = 106.

Time = 11.06 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.82

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx$$

$$=\begin{cases}\frac{835\sin\left(a+bx\right)\sin^6\left(2a+2bx\right)}{3003b}+\frac{2776\sin\left(a+bx\right)\sin^4\left(2a+2bx\right)\cos^2\left(2a+2bx\right)}{3003b}+\frac{2944\sin\left(a+bx\right)\sin^2\left(2a+2bx\right)\cos^4\left(2a+2bx\right)}{3003b}+\frac{1024\sin\left(\right.}{} \\ x\sin^6\left(2a\right)\cos\left(a\right)\end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**6,x)
```

output
```
Piecewise((835*sin(a + b*x)*sin(2*a + 2*b*x)**6/(3003*b) + 2776*sin(a + b*
x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)**2/(3003*b) + 2944*sin(a + b*x)*si
n(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**4/(3003*b) + 1024*sin(a + b*x)*cos(2*a
 + 2*b*x)**6/(3003*b) - 1084*sin(2*a + 2*b*x)**5*cos(a + b*x)*cos(2*a + 2*
b*x)/(3003*b) - 64*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**3/(1
43*b) - 512*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**5/(3003*b), Ne
(b, 0)), (x*sin(2*a)**6*cos(a), True))
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = -\frac{231\,\sin\left(13\,bx+13\,a\right)+273\,\sin\left(11\,bx+11\,a\right)-2002\,\sin\left(9\,bx+9\,a\right)-2574\,\sin\left(7\,bx+7\,a\right)+9009\,\mathrm{si}}{192192\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
-1/192192*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*
x + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x
 + 3*a) - 60060*sin(b*x + a))/b
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = -\frac{231\,\sin\left(13\,bx+13\,a\right)+273\,\sin\left(11\,bx+11\,a\right)-2002\,\sin\left(9\,bx+9\,a\right)-2574\,\sin\left(7\,bx+7\,a\right)+9009\,\mathrm{si}}{192192\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
-1/192192*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*
x + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x
 + 3*a) - 60060*sin(b*x + a))/b
```

3.130.9 Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = \frac{-\frac{64\sin(a+b\,x)^{13}}{13}+\frac{192\sin(a+b\,x)^{11}}{11}-\frac{64\sin(a+b\,x)^{9}}{3}+\frac{64\sin(a+b\,x)^{7}}{7}}{b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^6,x)
```

output
```
((64*sin(a + b*x)^7)/7 - (64*sin(a + b*x)^9)/3 + (192*sin(a + b*x)^11)/11
- (64*sin(a + b*x)^13)/13)/b
```

3.130. $\int \cos(a+bx)\sin^6(2a+2bx)\,dx$

3.131 $\int \cos(a+bx)\sin^5(2a+2bx)\,dx$

3.131.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^{11}(a+bx)}{11b}$$

output
```
-32/7*cos(b*x+a)^7/b+64/9*cos(b*x+a)^9/b-32/11*cos(b*x+a)^11/b
```

3.131.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\cos^7(a+bx)(-365+364\cos(2(a+bx))-63\cos(4(a+bx)))}{693b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(69
3*b)
```

3.131.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(2a+2bx)^5\cos(a+bx)dx$$

$$\Big\downarrow \; 4775$$

$$32\int \cos^6(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$32\int \cos(a+bx)^6\sin(a+bx)^5dx$$

$$\Big\downarrow \; 3045$$

$$-\frac{32\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{32\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{32\left(\frac{1}{11}\cos^{11}(a+bx)-\frac{2}{9}\cos^9(a+bx)+\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-32*(Cos[a + b*x]^7/7 - (2*Cos[a + b*x]^9)/9 + Cos[a + b*x]^11/11))/b
```

3.131.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.131.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.

Time = 1.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

method	result
default	$-\frac{5\cos(xb+a)}{16b}-\frac{5\cos(3xb+3a)}{48b}+\frac{\cos(5xb+5a)}{32b}+\frac{5\cos(7xb+7a)}{224b}-\frac{\cos(9xb+9a)}{288b}-\frac{\cos(11xb+11a)}{352b}$
risch	$-\frac{5\cos(xb+a)}{16b}-\frac{5\cos(3xb+3a)}{48b}+\frac{\cos(5xb+5a)}{32b}+\frac{5\cos(7xb+7a)}{224b}-\frac{\cos(9xb+9a)}{288b}-\frac{\cos(11xb+11a)}{352b}$
parallelrisch	$\frac{\left(128\tan(xb+a)^{8}+544\tan(xb+a)^{6}+4576\tan(xb+a)^{4}+2432\tan(xb+a)^{2}+512\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}+\left(-512\tan(xb+a)^{9}-2304\tan(\right.}{6}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
-5/16*cos(b*x+a)/b-5/48*cos(3*b*x+3*a)/b+1/32*cos(5*b*x+5*a)/b+5/224*cos(7
*b*x+7*a)/b-1/288*cos(9*b*x+9*a)/b-1/352*cos(11*b*x+11*a)/b
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(63\cos\left(bx+a\right)^{11}-154\cos\left(bx+a\right)^{9}+99\cos\left(bx+a\right)^{7}\right)}{693\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-32/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

3.131.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(39) = 78$.

Time = 4.84 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.33

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = \begin{cases} -\frac{151\sin\left(a+bx\right)\sin^5\left(2a+2bx\right)}{693b} - \frac{272\sin\left(a+bx\right)\sin^3\left(2a+2bx\right)\cos^2\left(2a+2bx\right)}{693b} - \frac{128\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^4\left(2a+2bx\right)}{693b} - \frac{422\sin^4\left(2\right.}{} \\ x\sin^5\left(2a\right)\cos\left(a\right) \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-151*sin(a + b*x)*sin(2*a + 2*b*x)**5/(693*b) - 272*sin(a + b*x
)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/(693*b) - 128*sin(a + b*x)*sin(2
*a + 2*b*x)*cos(2*a + 2*b*x)**4/(693*b) - 422*sin(2*a + 2*b*x)**4*cos(a +
b*x)*cos(2*a + 2*b*x)/(693*b) - 608*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2
*a + 2*b*x)**3/(693*b) - 256*cos(a + b*x)*cos(2*a + 2*b*x)**5/(693*b), Ne(
b, 0)), (x*sin(2*a)**5*cos(a), True))
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx =$$
$$-\frac{63\,\cos\left(11\,bx+11\,a\right)+77\,\cos\left(9\,bx+9\,a\right)-495\,\cos\left(7\,bx+7\,a\right)-693\,\cos\left(5\,bx+5\,a\right)+2310\,\cos\left(3\,bx\right.}{22176\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/22176*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*
a) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx$$
$$=-\frac{32\left(63\,\cos\left(bx+a\right)^{11}-154\,\cos\left(bx+a\right)^{9}+99\,\cos\left(bx+a\right)^{7}\right)}{693\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
-32/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

3.131.9 Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx$$
$$=-\frac{32\left(63\cos\left(a+b\,x\right)^{11}-154\cos\left(a+b\,x\right)^{9}+99\cos\left(a+b\,x\right)^{7}\right)}{693\,b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^5,x)
```

output
```
-(32*(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11))/(693*b
)
```

3.132 $\int \cos(a+bx)\sin^4(2a+2bx)\,dx$

3.132.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx=\frac{16\sin^5(a+bx)}{5b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^9(a+bx)}{9b}$$

output
```
16/5*sin(b*x+a)^5/b-32/7*sin(b*x+a)^7/b+16/9*sin(b*x+a)^9/b
```

3.132.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx =\frac{2(249+220\cos(2(a+bx))+35\cos(4(a+bx)))\sin^5(a+bx)}{315b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(2*(249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(315
*b)
```

3.132.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^4\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$16\int \cos^5(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^5\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{16\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{16\int \left(\sin^8(a+bx)-2\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(\frac{1}{9}\sin^9(a+bx)-\frac{2}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(16*(Sin[a + b*x]^5/5 - (2*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/9))/b
```

3.132.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.132.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

method	result
default	$\frac{3\sin(xb+a)}{8b}-\frac{\sin(3xb+3a)}{12b}-\frac{\sin(5xb+5a)}{20b}+\frac{\sin(7xb+7a)}{112b}+\frac{\sin(9xb+9a)}{144b}$
risch	$\frac{3\sin(xb+a)}{8b}-\frac{\sin(3xb+3a)}{12b}-\frac{\sin(5xb+5a)}{20b}+\frac{\sin(7xb+7a)}{112b}+\frac{\sin(9xb+9a)}{144b}$
parallelrisch	$\frac{\left(-128\tan(xb+a)^7-448\tan(xb+a)^5+448\tan(xb+a)^3+128\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2+\left(256\tan(xb+a)^8+896\tan(xb+a)^6+1\right.}{315b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right.}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)+1/112/b*sin(7
*b*x+7*a)+1/144/b*sin(9*b*x+9*a)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\left(35\cos\left(bx+a\right)^8-50\cos\left(bx+a\right)^6+3\cos\left(bx+a\right)^4+4\cos\left(bx+a\right)^2+8\right)\sin\left(bx+a\right)}{315\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
16/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b
*x + a)^2 + 8)*sin(b*x + a)/b
```

3.132.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(39) = 78.

Time = 2.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.52

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \begin{cases} \frac{107\sin\left(a+bx\right)\sin^4\left(2a+2bx\right)}{315b}+\frac{16\sin\left(a+bx\right)\sin^2\left(2a+2bx\right)\cos^2\left(2a+2bx\right)}{21b}+\frac{128\sin\left(a+bx\right)\cos^4\left(2a+2bx\right)}{315b}-\frac{104\sin^3\left(2a+2bx\right)\cos\left(a+bx\right.}{315b} \\ x\sin^4\left(2a\right)\cos\left(a\right) \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((107*sin(a + b*x)*sin(2*a + 2*b*x)**4/(315*b) + 16*sin(a + b*x)*
sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/(21*b) + 128*sin(a + b*x)*cos(2*a
+ 2*b*x)**4/(315*b) - 104*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x
)/(315*b) - 64*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**3/(315*b),
Ne(b, 0)), (x*sin(2*a)**4*cos(a), True))
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{35\,\sin\left(9\,bx+9\,a\right)+45\,\sin\left(7\,bx+7\,a\right)-252\,\sin\left(5\,bx+5\,a\right)-420\,\sin\left(3\,bx+3\,a\right)+1890\,\sin\left(bx+a\right)}{5040\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/5040*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
 420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{35\,\sin\left(9\,bx+9\,a\right)+45\,\sin\left(7\,bx+7\,a\right)-252\,\sin\left(5\,bx+5\,a\right)-420\,\sin\left(3\,bx+3\,a\right)+1890\,\sin\left(bx+a\right)}{5040\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/5040*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
 420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\,\left(35\sin\left(a+b\,x\right)^9-90\sin\left(a+b\,x\right)^7+63\sin\left(a+b\,x\right)^5\right)}{315\,b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^4,x)
```

output
```
(16*(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9))/(315*b)
```

3.133 $\int \cos(a+bx)\sin^3(2a+2bx)\,dx$

3.133.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}$$

output
```
-8/5*cos(b*x+a)^5/b+8/7*cos(b*x+a)^7/b
```

3.133.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = \frac{4\cos^5(a+bx)(-9+5\cos(2(a+bx)))}{35b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(4*Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(35*b)
```

3.133.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^3\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$8\int \cos^4(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^4\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{8\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{8\int \left(\cos^4(a+bx)-\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8\left(\frac{1}{5}\cos^5(a+bx)-\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-8*(Cos[a + b*x]^5/5 - Cos[a + b*x]^7/7))/b
```

3.133.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.133.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

method	result
default	$-\frac{3\cos(xb+a)}{8b}-\frac{\cos(3xb+3a)}{8b}+\frac{\cos(5xb+5a)}{40b}+\frac{\cos(7xb+7a)}{56b}$
risch	$-\frac{3\cos(xb+a)}{8b}-\frac{\cos(3xb+3a)}{8b}+\frac{\cos(5xb+5a)}{40b}+\frac{\cos(7xb+7a)}{56b}$
parallelrisch	$\frac{\frac{8\left(\tan(xb+a)^4+11\tan(xb+a)^2+4\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{35}+\frac{16\left(-2\tan(xb+a)^5-5\tan(xb+a)^3-2\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{35}+\frac{32\tan(xb+a)^6}{35}+\frac{88\tan(}{3}}{b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan(xb+a)^2\right)^3}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b+1/40*cos(5*b*x+5*a)/b+1/56*cos(7*b*
x+7*a)/b
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(5\,\cos\left(bx+a\right)^7-7\,\cos\left(bx+a\right)^5\right)}{35\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
8/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

3.133.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(26) = 52.

Time = 0.77 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.13

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{9\sin\left(a+bx\right)\sin^3\left(2a+2bx\right)}{35b} - \frac{8\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^2\left(2a+2bx\right)}{35b} - \frac{22\sin^2\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{35b} - \frac{16\cos\left(a+bx\right)\cos}{35b} \\ x\sin^3\left(2a\right)\cos\left(a\right) \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-9*sin(a + b*x)*sin(2*a + 2*b*x)**3/(35*b) - 8*sin(a + b*x)*sin
(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(35*b) - 22*sin(2*a + 2*b*x)**2*cos(a +
b*x)*cos(2*a + 2*b*x)/(35*b) - 16*cos(a + b*x)*cos(2*a + 2*b*x)**3/(35*b),
 Ne(b, 0)), (x*sin(2*a)**3*cos(a), True))
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = \frac{5\cos(7\,bx+7\,a)+7\cos(5\,bx+5\,a)-35\cos(3\,bx+3\,a)-105\cos(bx+a)}{280\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/280*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105
*cos(b*x + a))/b
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(5\cos(bx+a)^7-7\cos(bx+a)^5\right)}{35\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
8/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

3.133.9 Mupad [B] (verification not implemented)

Time = 19.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(7\cos(a+b\,x)^5-5\cos(a+b\,x)^7\right)}{35\,b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^3,x)
```

output
```
-(8*(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7))/(35*b)
```

3.134 $\int \cos(a+bx)\sin^2(2a+2bx)\,dx$

3.134.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx=\frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}$$

output
```
4/3*sin(b*x+a)^3/b-4/5*sin(b*x+a)^5/b
```

3.134.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx=\frac{2(7+3\cos(2(a+bx)))\sin^3(a+bx)}{15b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(2*(7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(15*b)
```

3.134.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(2a+2bx)^2\cos(a+bx)dx$$

$$\Big\downarrow \text{ 4775}$$

$$4\int \cos^3(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$4\int \cos(a+bx)^3\sin(a+bx)^2dx$$

$$\Big\downarrow \text{ 3044}$$

$$\frac{4\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{4\int \left(\sin^2(a+bx)-\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4\left(\frac{1}{3}\sin^3(a+bx)-\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(4*(Sin[a + b*x]^3/3 - Sin[a + b*x]^5/5))/b
```

3.134.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.134.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sin(xb+a)}{2b}-\frac{\sin(3xb+3a)}{12b}-\frac{\sin(5xb+5a)}{20b}$
risch	$\frac{\sin(xb+a)}{2b}-\frac{\sin(3xb+3a)}{12b}-\frac{\sin(5xb+5a)}{20b}$
parallelrisch	$\frac{\frac{8\left(-\tan(xb+a)^{3}+\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{15}+\frac{8\left(2\tan(xb+a)^{4}+3\tan(xb+a)^{2}+2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{15}+\frac{8\tan(xb+a)^{3}}{15}-\frac{8\tan(xb+a)}{15}}{b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)\left(1+\tan(xb+a)^{2}\right)^{2}}$
norman	$\frac{\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{15b}-\frac{8\tan(xb+a)}{15b}+\frac{8\tan(xb+a)^{3}}{15b}+\frac{8\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)^{2}}{5b}+\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)^{4}}{15b}+\frac{8\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\tan(xb+a)}{15b}-8\,\mathrm{ta}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)\left(1+\tan(xb+a)^{2}\right)^{2}}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(3\cos(bx+a)^4-\cos(bx+a)^2-2\right)\sin(bx+a)}{15\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-4/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

3.134.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(26) = 52.

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.90

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \begin{cases} \frac{7\sin(a+bx)\sin^2(2a+2bx)}{15b}+\frac{8\sin(a+bx)\cos^2(2a+2bx)}{15b}-\frac{4\sin(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{15b} & \text{for } b\neq 0 \\ x\sin^2(2a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((7*sin(a + b*x)*sin(2*a + 2*b*x)**2/(15*b) + 8*sin(a + b*x)*cos(
2*a + 2*b*x)**2/(15*b) - 4*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)/
(15*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a), True))
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx = -\frac{3\sin(5\,bx+5\,a)+5\sin(3\,bx+3\,a)-30\sin(bx+a)}{60\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/60*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx = -\frac{3\sin(5\,bx+5\,a)+5\sin(3\,bx+3\,a)-30\sin(bx+a)}{60\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/60*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx = \frac{4\left(5\sin(a+b\,x)^3-3\sin(a+b\,x)^5\right)}{15\,b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^2,x)
```

output
```
(4*(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5))/(15*b)
```

3.135 $\int \cos(a+bx)\sin(2a+2bx)\,dx$

3.135.1 Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \cos(a+bx)\sin(2a+2bx)\,dx=-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

output
```
-1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

3.135.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int \cos(a+bx)\sin(2a+2bx)\,dx=-\frac{2\cos^3(a+bx)}{3b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Cos[a + b*x]^3)/(3*b)
```

3.135.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)\cos(a+bx)dx$$

$$\Big\downarrow \text{4772}$$

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
-1/2*Cos[a + b*x]/b - Cos[3*a + 3*b*x]/(6*b)
```

3.135.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.135.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(3xb+3a)}{6b}$	27
risch	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(3xb+3a)}{6b}$	27
parallelrisch	$\frac{4\tan(xb+a)^2-4\tan(xb+a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{3b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan(xb+a)^2\right)}$	74
norman	$\frac{-\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)}{3b}+\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{3b}+\frac{4\tan(xb+a)^2}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan(xb+a)^2\right)}$	79

input
```
int(cos(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = -\frac{2\cos(bx+a)^3}{3b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2/3*cos(b*x + a)^3/b
```

3.135.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = \begin{cases} -\frac{\sin(a+bx)\sin(2a+2bx)}{3b}-\frac{2\cos(a+bx)\cos(2a+2bx)}{3b} & \text{for } b\neq 0 \\ x\sin(2a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-sin(a + b*x)*sin(2*a + 2*b*x)/(3*b) - 2*cos(a + b*x)*cos(2*a +
 2*b*x)/(3*b), Ne(b, 0)), (x*sin(2*a)*cos(a), True))
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(3\,bx+3\,a)}{6\,b} - \frac{\cos(bx+a)}{2\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/6*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = -\frac{2\,\cos(bx+a)^3}{3\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-2/3*cos(b*x + a)^3/b
```

3.135.9 Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = \begin{cases} x\,\left(2\,\sin(a)-2\sin(a)^3\right) & \text{if } b=0 \\ -\frac{3\,\cos(a+b\,x)+\cos(3\,a+3\,b\,x)}{6\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x),x)
```

output
```
piecewise(b == 0, x*(2*sin(a) - 2*sin(a)^3), b ~= 0, -(3*cos(a + b*x) + co
s(3*a + 3*b*x))/(6*b))
```

3.135. $\int \cos(a+bx)\sin(2a+2bx)\,dx$

3.136 $\int \cos(a+bx)\csc(2a+2bx)\,dx$

3.136.1 Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \cos(a+bx)\csc(2a+2bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}$$

output
```
-1/2*arctanh(cos(b*x+a))/b
```

3.136.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \cos(a+bx)\csc(2a+2bx)\,dx=\frac{1}{2}\left(-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\right)$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

output
```
(-(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b)/2
```

3.136.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4775, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{2}\int \csc(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc(a+bx)dx$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*ArcTanh[Cos[a + b*x]]/b
```

3.136.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.136.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

method	result	size
default	$\frac{\ln(\csc(xb+a)-\cot(xb+a))}{2b}$	22
risch	$-\frac{\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{2b}+\frac{\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{2b}$	36

input
```
int(cos(b*x+a)/sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = -\frac{\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-1/4*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a),x)
```

output
```
Timed out
```

3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 6.00

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = -\frac{\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) - \log\left(\cos(bx)^2 \right.}{4b}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b
*x)*sin(a) + sin(a)^2) - log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = -\frac{\log\left(\cos(bx+a)+1\right) - \log\left(-\cos(bx+a)+1\right)}{4b}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/4*(log(cos(b*x + a) + 1) - log(-cos(b*x + a) + 1))/b
```

3.136.9 Mupad [B] (verification not implemented)

Time = 21.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \cos(a+bx)\csc(2a+2bx)\,dx=-\frac{\operatorname{atanh}(\cos(a+b\,x))}{2\,b}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x),x)
```

output
```
-atanh(cos(a + b*x))/(2*b)
```

3.137 $\int \cos(a+bx)\csc^2(2a+2bx)\,dx$

3.137.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{4b} - \frac{\csc(a+bx)}{4b}$$

output
```
1/4*arctanh(sin(b*x+a))/b-1/4*csc(b*x+a)/b
```

3.137.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{4b}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
```

3.137.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4775, 3042, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{4}\int \csc^2(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)^2\sec(a+bx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{4b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\csc(a+bx)-\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)}{4b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\csc(a+bx)-\text{arctanh}(\csc(a+bx))}{4b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*(-ArcTanh[Csc[a + b*x]] + Csc[a + b*x])/b
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.137.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\frac{1}{\sin(xb+a)}+\ln(\sec(xb+a)+\tan(xb+a))}{4b}$	31
risch	$-\frac{i e^{i(xb+a)}}{2b\left(e^{2i(xb+a)}-1\right)}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{4b}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{4b}$	66

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \frac{\log(\sin(bx+a)+1)\sin(bx+a) - \log(-\sin(bx+a)+1)\sin(bx+a) - 2}{8\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/8*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
 a) - 2)/(b*sin(b*x + a))
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

3.137.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(24) = 48.

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 8.32

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\left(\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,bx+2\,a\right)^2-2\,\cos\left(2\,bx+2\,a\right)+1\right)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\sin(}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\sin(}\right.}{8\left(b\cos\left(2\,bx+2\,a\right)^2+b\sin\left(\right.\right.}$$

input

```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output

```
-1/8*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)
^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*
cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a
)^2)) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a)
+ 4*sin(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2
*b*x + 2*a) + b)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\frac{2}{\sin(bx+a)}-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(-\sin\left(bx+a\right)+1\right)}{8\,b}$$

input

```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output

```
-1/8*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx=\frac{\operatorname{atanh}(\sin(a+b\,x))}{4\,b}-\frac{1}{4\,b\,\sin(a+b\,x)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^2,x)
```

output
```
atanh(sin(a + b*x))/(4*b) - 1/(4*b*sin(a + b*x))
```

3.138 $\int \cos(a+bx)\csc^3(2a+2bx)\,dx$

3.138.1 Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = -\frac{3\text{arctanh}(\cos(a+bx))}{16b}+\frac{3\sec(a+bx)}{16b} - \frac{\csc^2(a+bx)\sec(a+bx)}{16b}$$

output
```
-3/16*arctanh(cos(b*x+a))/b+3/16*sec(b*x+a)/b-1/16*csc(b*x+a)^2*sec(b*x+a)
/b
```

3.138.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(49) = 98.

Time = 0.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.92

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = \frac{\csc^4(a+bx)\left(2-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3(a}{16b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\right.}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

output
```
(Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a
+ b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] +
 Cos[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(
16*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

3.138.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4775, 3042, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{8}\int \csc^3(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^3\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)}{8b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{8b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\operatorname{arctanh}(\sec(a+bx))-\sec(a+bx))}{8b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

output
```
((-3*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Se
c[a + b*x]^2)))/(8*b)
```

3.138.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.138.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{-\frac{1}{2\sin(xb+a)^2\cos(xb+a)}+\frac{3}{2\cos(xb+a)}+\frac{3\ln(\csc(xb+a)-\cot(xb+a))}{2}}{8b}$	53
risch	$\frac{3\,\mathrm{e}^{5i(xb+a)}-2\,\mathrm{e}^{3i(xb+a)}+3\,\mathrm{e}^{i(xb+a)}}{8b\left(\mathrm{e}^{2i(xb+a)}-1\right)^2\left(\mathrm{e}^{2i(xb+a)}+1\right)}-\frac{3\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{16b}+\frac{3\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{16b}$	101

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/2/sin(b*x+a)^2/cos(b*x+a)+3/2/cos(b*x+a)+3/2*ln(csc(b*x+a)-cot(b
*x+a)))
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(43) = 86$.

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.96

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx$$
$$=\frac{6\cos(bx+a)^2-3\left(\cos(bx+a)^3-\cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+3\left(\cos(bx+a)^3-\cos(bx+a\right.}{32\left(b\cos(bx+a)^3-b\cos(bx+a)\right)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
1/32*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
 + a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1
/2) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

3.138.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(43) = 86$.

Time = 0.21 (sec) , antiderivative size = 974, normalized size of antiderivative = 19.88

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/32*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b
*x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a)
+ 4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x
+ 2*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(co
s(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)
^2 - 2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(
2*b*x + 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a)
- sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*co
s(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*
(2*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x
+ 6*a)^2 - 2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- cos(2*b*x + 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x +
 6*a) - sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2
- 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2
) + 4*(3*sin(5*b*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x
 + 6*a) - 12*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2
*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*
sin(2*b*x + 2*a) - 12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/...
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx$$
$$= \frac{\frac{2\left(3\cos(bx+a)^2-2\right)}{\cos(bx+a)^3-\cos(bx+a)} - 3\log\left(\cos\left(bx+a\right)+1\right) + 3\log\left(-\cos\left(bx+a\right)+1\right)}{32\,b}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
1/32*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos
(b*x + a) + 1) + 3*log(-cos(b*x + a) + 1))/b
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = -\frac{3\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{16\,b} - \frac{\frac{3\cos(a+b\,x)^2}{16}-\frac{1}{8}}{b\,\left(\cos\left(a+b\,x\right)-\cos\left(a+b\,x\right)^3\right)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^3,x)
```

output
```
- (3*atanh(cos(a + b*x)))/(16*b) - ((3*cos(a + b*x)^2)/16 - 1/8)/(b*(cos(a
 + b*x) - cos(a + b*x)^3))
```

3.139 $\int \cos(a+bx)\csc^4(2a+2bx)\,dx$

3.139.1 Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = \frac{5\text{arctanh}(\sin(a+bx))}{32b} - \frac{5\csc(a+bx)}{32b} - \frac{5\csc^3(a+bx)}{96b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b}$$

output
```
5/32*arctanh(sin(b*x+a))/b-5/32*csc(b*x+a)/b-5/96*csc(b*x+a)^3/b+1/32*csc(
b*x+a)^3*sec(b*x+a)^2/b
```

3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.47

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\sin^2(a+bx)\right)}{48b}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/48*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

3.139.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4775, 3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \; 4775$$

$$\frac{1}{16}\int \csc^4(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{16}\int \csc(a+bx)^4\sec(a+bx)^3dx$$

$$\Big\downarrow \; 3101$$

$$-\frac{\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{16b}$$

$$\Big\downarrow \; 252$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{16b}$$

$$\Big\downarrow \; 254$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{16b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\csc(a+bx))-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)}{16b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/16*(Csc[a + b*x]^5/(2*(1 - Csc[a + b*x]^2)) - (5*(ArcTanh[Csc[a + b*x]]
 - Csc[a + b*x] - Csc[a + b*x]^3/3))/2)/b
```

3.139.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.139.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{-\frac{1}{3\sin(xb+a)^{3}\cos(xb+a)^{2}}+\frac{5}{6\sin(xb+a)\cos(xb+a)^{2}}-\frac{5}{2\sin(xb+a)}+\frac{5\ln(\sec(xb+a)+\tan(xb+a))}{2}}{16b}$	69
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(xb+a)}-20\,\mathrm{e}^{7i(xb+a)}-22\,\mathrm{e}^{5i(xb+a)}-20\,\mathrm{e}^{3i(xb+a)}+15\,\mathrm{e}^{i(xb+a)}\right)}{48b\left(\mathrm{e}^{2i(xb+a)}-1\right)^{3}\left(\mathrm{e}^{2i(xb+a)}+1\right)^{2}}-\frac{5\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{32b}+\frac{5\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{32b}$	126

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)^2+5/6/sin(b*x+a)/cos(b*x+a)^2-5/2/sin
(b*x+a)+5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(58) = 116.

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.97

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx =$$

$$-\frac{30\cos(bx+a)^4-15\left(\cos(bx+a)^4-\cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a)+15\left(\cos(bx+\right.}{192\left(b\cos(bx+a)^4-b\cos(bx+a)^2\right.}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-1/192*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b
*x + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(
b*x + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b
*cos(b*x + a)^2)*sin(b*x + a))
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a + bx) \csc^4(2a + 2bx) \, dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

3.139.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(58) = 116$.

Time $= 0.38$ (sec) , antiderivative size $= 1780$, normalized size of antiderivative $= 26.97$

$$\int \cos(a + bx) \csc^4(2a + 2bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/192*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a)
- 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*
x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos
(9*b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*
x + 3*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*
sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x +
5*a) + 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin
(4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a
) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b
*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a)
- cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(
2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x +
 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*
(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x
 + 2*a)^2 + 2*(sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a)
- sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6
*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - si
n(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6
*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b...
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx$$
$$= -\frac{\frac{6\,\sin(bx+a)}{\sin(bx+a)^2-1} + \frac{4\left(6\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 15\,\log\left(\sin\left(bx+a\right)+1\right) + 15\,\log\left(-\sin\left(bx+a\right)+1\right)}{192\,b}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/192*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin
(b*x + a)^3 - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

3.139.9 Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = \frac{5\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{32\,b} - \frac{-\frac{5\sin(a+b\,x)^4}{32} + \frac{5\sin(a+b\,x)^2}{48} + \frac{1}{48}}{b\,\left(\sin\left(a+b\,x\right)^3 - \sin\left(a+b\,x\right)^5\right)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^4,x)
```

output
```
(5*atanh(sin(a + b*x)))/(32*b) - ((5*sin(a + b*x)^2)/48 - (5*sin(a + b*x)^
4)/32 + 1/48)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

3.140 $\int \cos(a+bx)\csc^5(2a+2bx)\,dx$

3.140.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = -\frac{35\text{arctanh}(\cos(a+bx))}{256b}+\frac{35\sec(a+bx)}{256b}+\frac{35\sec^3(a+bx)}{768b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b}$$

output
```
-35/256*arctanh(cos(b*x+a))/b+35/256*sec(b*x+a)/b+35/768*sec(b*x+a)^3/b-7/
256*csc(b*x+a)^2*sec(b*x+a)^3/b-1/128*csc(b*x+a)^4*sec(b*x+a)^3/b
```

3.140.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. $2(89) = 178$.

Time = 0.73 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.01

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = -\frac{\csc^{10}(a+bx)\left(-204+658\cos(2(a+bx))-228\cos(3(a+bx))+140\cos(4(a+bx))-76\cos(5(a+bx\right.}{}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

output
```
-1/768*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)
] + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76
*Cos[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5
*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)
/2]] + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b
*x)/2]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x
)]*Log[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

3.140.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4775, 3042, 3102, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{32}\int \csc^5(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^5\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{32b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{32b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\left(\operatorname{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-1/4*Sec[a + b*x]^7/(1 - Sec[a + b*x]^2)^2 + (7*(Sec[a + b*x]^5/(2*(1 - S
ec[a + b*x]^2)) - (5*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x] - Sec[a + b*x]^
3/3))/2))/4)/(32*b)
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.140. $\int \cos(a+bx)\csc^5(2a+2bx)\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.140.4 Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

method	result
default	$\frac{-\frac{1}{4\sin(xb+a)^4\cos(xb+a)^3}+\frac{7}{12\sin(xb+a)^2\cos(xb+a)^3}-\frac{35}{24\sin(xb+a)^2\cos(xb+a)}+\frac{35}{8\cos(xb+a)}+\frac{35\ln(\csc(xb+a)-\cot(xb+a))}{8}}{32b}$
risch	$\frac{105\,\mathrm{e}^{13i(xb+a)}-70\,\mathrm{e}^{11i(xb+a)}-329\,\mathrm{e}^{9i(xb+a)}+204\,\mathrm{e}^{7i(xb+a)}-329\,\mathrm{e}^{5i(xb+a)}-70\,\mathrm{e}^{3i(xb+a)}+105\,\mathrm{e}^{i(xb+a)}}{384b(\mathrm{e}^{2i(xb+a)}-1)^4(\mathrm{e}^{2i(xb+a)}+1)^3}+\frac{35\ln(\mathrm{e}^{i(xb+a)}-1)}{256b}-\frac{35}{}$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^3+7/12/sin(b*x+a)^2/cos(b*x+a)^3-35/2
4/sin(b*x+a)^2/cos(b*x+a)+35/8/cos(b*x+a)+35/8*ln(csc(b*x+a)-cot(b*x+a)))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx$$
$$=\frac{210\cos(bx+a)^6-350\cos(bx+a)^4+112\cos(bx+a)^2-105\left(\cos(bx+a)^7-2\cos(bx+a)^5+\cos(bx\right.}{1536\left(b\cos(bx+a)^7\right.}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
1/1536*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105
*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a)
 + 1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/
2*cos(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos
(b*x + a)^3)
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

3.140.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3846 vs. $2(79) = 158$.

Time = 0.33 (sec) , antiderivative size = 3846, normalized size of antiderivative = 43.21

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/1536*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x
+ 9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a)
 + 105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(
10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4
*a) + cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a)
+ 329*cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*
cos(3*b*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*
x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) +
 cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204
*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b
*x + a))*cos(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a
) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*
cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*
x + a))*cos(8*b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) -
cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(
3*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a
) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*c
os(b*x + a))*cos(4*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a
) - 420*cos(2*b*x + 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos
(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x...
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{6\left(11\cos(bx+a)^3-13\cos(bx+a)\right)}{\left(\cos(bx+a)^2-1\right)^2}+\frac{16\left(9\cos(bx+a)^2+1\right)}{\cos(bx+a)^3}-105\log\left(\cos\left(bx+a\right)+1\right)+105\log\left(-\cos\left(bx+a\right)+1\right)}{1536\,b}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
1/1536*(6*(11*cos(b*x + a)^3 - 13*cos(b*x + a))/(cos(b*x + a)^2 - 1)^2 + 1
6*(9*cos(b*x + a)^2 + 1)/cos(b*x + a)^3 - 105*log(cos(b*x + a) + 1) + 105*
log(-cos(b*x + a) + 1))/b
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{35\cos(a+b\,x)^6}{256}-\frac{175\cos(a+b\,x)^4}{768}+\frac{7\cos(a+b\,x)^2}{96}+\frac{1}{96}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)} - \frac{35\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^5,x)
```

output
```
((7*cos(a + b*x)^2)/96 - (175*cos(a + b*x)^4)/768 + (35*cos(a + b*x)^6)/25
6 + 1/96)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*a
tanh(cos(a + b*x)))/(256*b)
```

3.141 $\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx$

3.141.1 Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx=-\frac{4\cos^8(a+bx)}{b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{8\cos^{12}(a+bx)}{3b}$$

output
```
-4*cos(b*x+a)^8/b+32/5*cos(b*x+a)^10/b-8/3*cos(b*x+a)^12/b
```

3.141.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx=$$
$$-\frac{600\cos(2(a+bx))+75\cos(4(a+bx))-100\cos(6(a+bx))-30\cos(8(a+bx))+12\cos(10(a+bx))}{3840b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
-1/3840*(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x)]
 - 30*Cos[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/b
```

3.141.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4775, 3042, 3045, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(2a+2bx)^5\cos(a+bx)^2dx$$

$$\Big\downarrow \; 4775$$

$$32\int \cos^7(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$32\int \cos(a+bx)^7\sin(a+bx)^5dx$$

$$\Big\downarrow \; 3045$$

$$-\frac{32\int \cos^7(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 243$$

$$-\frac{16\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos^2(a+bx)}{b}$$

$$\Big\downarrow \; 49$$

$$-\frac{16\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos^2(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{16\left(\frac{1}{6}\cos^{12}(a+bx)-\frac{2}{5}\cos^{10}(a+bx)+\frac{1}{4}\cos^8(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-16*(Cos[a + b*x]^8/4 - (2*Cos[a + b*x]^10)/5 + Cos[a + b*x]^12/6))/b
```

3.141.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.141.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

method	result	size
parallelrisch	$\frac{-600\cos(2xb+2a)-1662-5\cos(12xb+12a)-12\cos(10xb+10a)+30\cos(8xb+8a)+100\cos(6xb+6a)-75\cos(4xb+4a)}{3840b}$	74
default	$-\frac{5\cos(2xb+2a)}{32b}-\frac{5\cos(4xb+4a)}{256b}+\frac{5\cos(6xb+6a)}{192b}+\frac{\cos(8xb+8a)}{128b}-\frac{\cos(10xb+10a)}{320b}-\frac{\cos(12xb+12a)}{768b}$	86
risch	$-\frac{5\cos(2xb+2a)}{32b}-\frac{5\cos(4xb+4a)}{256b}+\frac{5\cos(6xb+6a)}{192b}+\frac{\cos(8xb+8a)}{128b}-\frac{\cos(10xb+10a)}{320b}-\frac{\cos(12xb+12a)}{768b}$	86

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3840*(-600*cos(2*b*x+2*a)-1662-5*cos(12*b*x+12*a)-12*cos(10*b*x+10*a)+30
*cos(8*b*x+8*a)+100*cos(6*b*x+6*a)-75*cos(4*b*x+4*a))/b
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = -\frac{4\left(10\,\cos\left(bx+a\right)^{12}-24\,\cos\left(bx+a\right)^{10}+15\,\cos\left(bx+a\right)^{8}\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-4/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

3.141.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(37) = 74.

Time = 11.30 (sec) , antiderivative size = 597, normalized size of antiderivative = 13.57

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = \begin{cases} -\frac{5x\sin^2(a+bx)\sin^5(2a+2bx)}{32} - \frac{5x\sin^2(a+bx)\sin^3(2a+2bx)\cos^2(2a+2bx)}{16} - \frac{5x\sin^2(a+bx)\sin(2a+2bx)\cos^4(2a+2bx)}{32} - \frac{5x\sin(a+ \\ x\sin^5(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-5*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**5/32 - 5*x*sin(a + b*x)*
*2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/16 - 5*x*sin(a + b*x)**2*sin(2*
a + 2*b*x)*cos(2*a + 2*b*x)**4/32 - 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**4*c
os(a + b*x)*cos(2*a + 2*b*x)/16 - 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos
(a + b*x)*cos(2*a + 2*b*x)**3/8 - 5*x*sin(a + b*x)*cos(a + b*x)*cos(2*a +
2*b*x)**5/16 + 5*x*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/32 + 5*x*sin(2*a +
2*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 + 5*x*sin(2*a + 2*b*x)*co
s(a + b*x)**2*cos(2*a + 2*b*x)**4/32 - 125*sin(a + b*x)**2*sin(2*a + 2*b*x
)**4*cos(2*a + 2*b*x)/(384*b) - 2*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(
2*a + 2*b*x)**3/(3*b) - 217*sin(a + b*x)**2*cos(2*a + 2*b*x)**5/(640*b) +
95*sin(a + b*x)*sin(2*a + 2*b*x)**5*cos(a + b*x)/(192*b) + 13*sin(a + b*x)
*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(12*b) + 109*sin(a +
 b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(192*b) - 67*sin(2
*a + 2*b*x)**4*cos(a + b*x)**2*cos(2*a + 2*b*x)/(384*b) + 139*cos(a + b*x)
**2*cos(2*a + 2*b*x)**5/(1920*b), Ne(b, 0)), (x*sin(2*a)**5*cos(a)**2, Tru
e))
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx =$$
$$-\frac{5\,\cos\left(12\,bx+12\,a\right)+12\,\cos\left(10\,bx+10\,a\right)-30\,\cos\left(8\,bx+8\,a\right)-100\,\cos\left(6\,bx+6\,a\right)+75\,\cos\left(4\,bx+\right.}{3840\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/3840*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a
) - 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx$$
$$=-\frac{4\left(10\,\cos\left(bx+a\right)^{12}-24\,\cos\left(bx+a\right)^{10}+15\,\cos\left(bx+a\right)^{8}\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
-4/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

3.141.9 Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = -\frac{4\cos\left(a+b\,x\right)^8\,\left(10\cos\left(a+b\,x\right)^4-24\cos\left(a+b\,x\right)^2+15\right)}{15\,b}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^5,x)
```

output
```
-(4*cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(15*b)
```

3.142 $\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$

3.142.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3x}{16}-\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b}-\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b}+\frac{\sin^5(2a+2bx)}{20b}$$

output
```
3/16*x-3/32*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/16*cos(2*b*x+2*a)*sin(2*b*x+
2*a)^3/b+1/20*sin(2*b*x+2*a)^5/b
```

3.142.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{120bx+20\sin(2(a+bx))-40\sin(4(a+bx))-10\sin(6(a+bx))+5\sin(8(a+bx))+2\sin(10(a+bx))}{640b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
(120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
 + 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(640*b)
```

3.142.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 4773, 3042, 3044, 15, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^4\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4773}$$

$$\frac{1}{2}\int \sin^4(2a+2bx)dx+\frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx+\frac{1}{2}\int \cos(2a+2bx)\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^4(2a+2bx)d\sin(2a+2bx)}{4b}+\frac{1}{2}\int \sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx+\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin^2(2a+2bx)dx-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)+\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin(2a+2bx)^2dx-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)+\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)+\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin^5(2a+2bx)}{20b}+\frac{1}{2}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
Sin[2*a + 2*b*x]^5/(20*b) + (-1/8*(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^3)/b
+ (3*(x/2 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b)))/4)/2
```

3.142.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4773
```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Simp[1/2   Int[(g*Sin[c + d*x])^p, x], x] + Simp[1/2   Int[Cos[c + d*
x]*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d
, 0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

3.142.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{120xb+20\sin(2xb+2a)-40\sin(4xb+4a)-10\sin(6xb+6a)+5\sin(8xb+8a)+2\sin(10xb+10a)}{640b}$	66
default	$\frac{3x}{16}+\frac{\sin(2xb+2a)}{32b}-\frac{\sin(4xb+4a)}{16b}-\frac{\sin(6xb+6a)}{64b}+\frac{\sin(8xb+8a)}{128b}+\frac{\sin(10xb+10a)}{320b}$	75
risch	$\frac{3x}{16}+\frac{\sin(2xb+2a)}{32b}-\frac{\sin(4xb+4a)}{16b}-\frac{\sin(6xb+6a)}{64b}+\frac{\sin(8xb+8a)}{128b}+\frac{\sin(10xb+10a)}{320b}$	75

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/640*(120*x*b+20*sin(2*b*x+2*a)-40*sin(4*b*x+4*a)-10*sin(6*b*x+6*a)+5*sin
(8*b*x+8*a)+2*sin(10*b*x+10*a))/b
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{15\,bx+\left(128\,\cos\left(bx+a\right)^9-176\,\cos\left(bx+a\right)^7+8\,\cos\left(bx+a\right)^5+10\,\cos\left(bx+a\right)^3+15\,\cos\left(bx+a\right)\right)\sin}{80\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/80*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

3.142.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(70) = 140.

Time = 4.91 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.71

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^2(a+bx)\sin^4(2a+2bx)}{16}+\frac{3x\sin^2(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{8}+\frac{3x\sin^2(a+bx)\cos^4(2a+2bx)}{16}+\frac{3x\sin^4(2a+2bx)\cos^2(a+}{16} \\ x\sin^4(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**4/16 + 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*sin(a + b*x)**2*cos(2*a
+ 2*b*x)**4/16 + 3*x*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/16 + 3*x*sin(2*a
+ 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*cos(a + b*x)**2*co
s(2*a + 2*b*x)**4/16 + 7*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b
*x)/(160*b) + 19*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(480
*b) + sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)/(10*b) + 2*sin(a + b*x
)*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(5*b) + 4*sin(a + b
*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(15*b) - 57*sin(2*a + 2*b*x)**3*cos(a
 + b*x)**2*cos(2*a + 2*b*x)/(160*b) - 109*sin(2*a + 2*b*x)*cos(a + b*x)**2
*cos(2*a + 2*b*x)**3/(480*b), Ne(b, 0)), (x*sin(2*a)**4*cos(a)**2, True))
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{120\,bx + 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) - 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) + 20\,\sin\left(2\,bx + \right.}{640\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/640*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x
+ 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{120\,bx + 120\,a + 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) - 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) + 20\,\mathrm{s}}{640\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/640*(120*b*x + 120*a + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*si
n(6*b*x + 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

3.142.9 Mupad [B] (verification not implemented)

Time = 21.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3\,x}{16} + \frac{\frac{3\tan(a+b\,x)^9}{16}+\frac{7\tan(a+b\,x)^7}{8}+\frac{8\tan(a+b\,x)^5}{5}-\frac{7\tan(a+b\,x)^3}{8}-\frac{3\tan(a+b\,x)}{16}}{b\,\left(\tan\left(a+b\,x\right)^{10}+5\tan\left(a+b\,x\right)^8+10\tan\left(a+b\,x\right)^6+10\tan\left(a+b\,x\right)^4+5\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^4,x)
```

output
```
(3*x)/16 + ((8*tan(a + b*x)^5)/5 - (7*tan(a + b*x)^3)/8 - (3*tan(a + b*x))
/16 + (7*tan(a + b*x)^7)/8 + (3*tan(a + b*x)^9)/16)/(b*(5*tan(a + b*x)^2 +
 10*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^1
0 + 1))
```

3.143 $\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx$

3.143.1 Optimal result

Integrand size = 20, antiderivative size = 28

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{4\cos^6(a+bx)}{3b}+\frac{\cos^8(a+bx)}{b}$$

output
```
-4/3*cos(b*x+a)^6/b+cos(b*x+a)^8/b
```

3.143.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{384b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8
*(a + b*x)])/(384*b)
```

3.143.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^3\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4775}$$

$$8\int \cos^5(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^5\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{8\int \cos^5(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{8\int \left(\cos^5(a+bx)-\cos^7(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8\left(\frac{1}{6}\cos^6(a+bx)-\frac{1}{8}\cos^8(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-8*(Cos[a + b*x]^6/6 - Cos[a + b*x]^8/8))/b
```

3.143.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.143.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

method	result	size
parallelrisch	$\frac{-72\cos(2xb+2a)-199+3\cos(8xb+8a)+8\cos(6xb+6a)-12\cos(4xb+4a)}{384b}$	52
default	$-\frac{3\cos(2xb+2a)}{16b}-\frac{\cos(4xb+4a)}{32b}+\frac{\cos(6xb+6a)}{48b}+\frac{\cos(8xb+8a)}{128b}$	58
risch	$-\frac{3\cos(2xb+2a)}{16b}-\frac{\cos(4xb+4a)}{32b}+\frac{\cos(6xb+6a)}{48b}+\frac{\cos(8xb+8a)}{128b}$	58

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/384*(-72*cos(2*b*x+2*a)-199+3*cos(8*b*x+8*a)+8*cos(6*b*x+6*a)-12*cos(4*b
*x+4*a))/b
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{3\cos(bx+a)^8 - 4\cos(bx+a)^6}{3b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
1/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

3.143.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(22) = 44.

Time = 2.05 (sec) , antiderivative size = 362, normalized size of antiderivative = 12.93

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16} - \frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16} - \frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8} - 3 \\ x\sin^3(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 - 3*x*sin(a + b*x)*
*2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 - 3*x*sin(a + b*x)*sin(2*a + 2*
b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 - 3*x*sin(a + b*x)*cos(a + b*x)*co
s(2*a + 2*b*x)**3/8 + 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 + 3*x*sin
(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - sin(a + b*x)**2*sin
(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(2*b) - 49*sin(a + b*x)**2*cos(2*a + 2*b
*x)**3/(96*b) + 13*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16*b) +
7*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b) + 1
7*cos(a + b*x)**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(2*a)**3*co
s(a)**2, True))
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{3\cos(8\,bx+8\,a)+8\cos(6\,bx+6\,a)-12\cos(4\,bx+4\,a)-72\cos(2\,bx+2\,a)}{384\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/384*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*
cos(2*b*x + 2*a))/b
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{3\cos(bx+a)^8-4\cos(bx+a)^6}{3\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
1/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{\frac{4\cos(a+b\,x)^6}{3}-\cos(a+b\,x)^8}{b}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^3,x)
```

output
```
-((4*cos(a + b*x)^6)/3 - cos(a + b*x)^8)/b
```

3.144 $\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx$

3.144.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx=\frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\sin^3(2a+2bx)}{12b}$$

output
```
1/4*x-1/8*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b+1/12*sin(2*b*x+2*a)^3/b
```

3.144.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = -\frac{-12bx-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))}{48b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
-1/48*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)
])/b
```

3.144.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4773, 3042, 3044, 15, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^2\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4773}$$

$$\frac{1}{2}\int \sin^2(2a+2bx)dx+\frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx+\frac{1}{2}\int \cos(2a+2bx)\sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^2(2a+2bx)d\sin(2a+2bx)}{4b}+\frac{1}{2}\int \sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx+\frac{\sin^3(2a+2bx)}{12b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)+\frac{\sin^3(2a+2bx)}{12b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin^3(2a+2bx)}{12b}+\frac{1}{2}\left(\frac{x}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
Sin[2*a + 2*b*x]^3/(12*b) + (x/2 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*
b))/2
```

3.144.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4773
```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Simp[1/2   Int[(g*Sin[c + d*x])^p, x], x] + Simp[1/2   Int[Cos[c + d*
x]*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d
, 0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

3.144.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{12xb+3\sin(2xb+2a)-3\sin(4xb+4a)-\sin(6xb+6a)}{48b}$	44
default	$\frac{x}{4}+\frac{\sin(2xb+2a)}{16b}-\frac{\sin(4xb+4a)}{16b}-\frac{\sin(6xb+6a)}{48b}$	47
risch	$\frac{x}{4}+\frac{\sin(2xb+2a)}{16b}-\frac{\sin(4xb+4a)}{16b}-\frac{\sin(6xb+6a)}{48b}$	47

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(12*x*b+3*sin(2*b*x+2*a)-3*sin(4*b*x+4*a)-sin(6*b*x+6*a))/b
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{3\,bx-\left(8\,\cos\left(bx+a\right)^5-2\,\cos\left(bx+a\right)^3-3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{12\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b
*x + a))/b
```

3.144.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(41) = 82.

Time = 0.82 (sec) , antiderivative size = 231, normalized size of antiderivative = 4.71

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = \begin{cases} \frac{x\sin^2\left(a+bx\right)\sin^2\left(2a+2bx\right)}{4}+\frac{x\sin^2\left(a+bx\right)\cos^2\left(2a+2bx\right)}{4}+\frac{x\sin^2\left(2a+2bx\right)\cos^2\left(a+bx\right)}{4}+\frac{x\cos^2\left(a+bx\right)\cos^2\left(2a+2bx\right)}{4}+\frac{\sin^2\left(a+bx\right.}{} \\ x\sin^2\left(2a\right)\cos^2\left(a\right) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos
(2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b
*x)**2*cos(2*a + 2*b*x)**2/4 + sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a +
2*b*x)/(24*b) + sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) + sin(
a + b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) - 7*sin(2*a + 2*b*x)*cos(a
 + b*x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a)**2, T
rue))
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{12\,bx - \sin\left(6\,bx+6\,a\right) - 3\,\sin\left(4\,bx+4\,a\right) + 3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/48*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))
/b
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{12\,bx + 12\,a - \sin\left(6\,bx+6\,a\right) - 3\,\sin\left(4\,bx+4\,a\right) + 3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/48*(12*b*x + 12*a - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x
+ 2*a))/b
```

3.144.9 Mupad [B] (verification not implemented)

Time = 19.74 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{x}{4} - \frac{\frac{\sin(4\,a+4\,b\,x)}{16} - \frac{\sin(2\,a+2\,b\,x)}{16} + \frac{\sin(6\,a+6\,b\,x)}{48}}{b}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^2,x)
```

output
```
x/4 - (sin(4*a + 4*b*x)/16 - sin(2*a + 2*b*x)/16 + sin(6*a + 6*b*x)/48)/b
```

3.145 $\int \cos^2(a+bx)\sin(2a+2bx)\,dx$

3.145.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx=-\frac{\cos^4(a+bx)}{2b}$$

output
```
-1/2*cos(b*x+a)^4/b
```

3.145.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx=-\frac{\cos^4(a+bx)}{2b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
-1/2*Cos[a + b*x]^4/b
```

3.145.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4775, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4775}$$

$$2\int \cos^3(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)^3\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{2\int \cos^3(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\cos^4(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
-1/2*Cos[a + b*x]^4/b
```

3.145.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.145.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

method	result
default	$-\frac{\cos(2xb+2a)}{4b}-\frac{\cos(4xb+4a)}{16b}$
risch	$-\frac{\cos(2xb+2a)}{4b}-\frac{\cos(4xb+4a)}{16b}$
parallelrisch	$\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4\tan(xb+a)xb+\left(-2\tan(xb+a)^2xb+2xb-6\tan(xb+a)\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3+\left(-6\tan(xb+a)xb+4\tan(xb+a)^2-4\right)\tan}{2b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^2\left(1+\tan(xb+a)^2\right)}$
norman	$\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3+x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)^2+\frac{3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)}{b}-x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+\frac{x\tan(xb+a)}{2}-3x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan(xb+}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right.}$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(2*b*x+2*a)/b-1/16*cos(4*b*x+4*a)/b
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(bx+a)^4}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-1/2*cos(b*x + a)^4/b
```

3.145.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(12) = 24.

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 8.87

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = \begin{cases} -\frac{x\sin^2(a+bx)\sin(2a+2bx)}{4} - \frac{x\sin(a+bx)\cos(a+bx)\cos(2a+2bx)}{2} + \frac{x\sin(2a+2bx)\cos^2(a+bx)}{4} - \frac{\sin^2(a+bx)\cos(2a+2bx)}{2b} + \frac{3\sin \\ x\sin(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 - x*sin(a + b*x)*cos(a +
b*x)*cos(2*a + 2*b*x)/2 + x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - sin(a + b
*x)**2*cos(2*a + 2*b*x)/(2*b) + 3*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*
x)/(4*b), Ne(b, 0)), (x*sin(2*a)*cos(a)**2, True))
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(4\,bx+4\,a)+4\,\cos(2\,bx+2\,a)}{16\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/16*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(bx+a)^4}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/2*cos(b*x + a)^4/b
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(a+b\,x)^4}{2\,b}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x),x)
```

output
```
-cos(a + b*x)^4/(2*b)
```

3.146 $\int \cos^2(a+bx)\csc(2a+2bx)\,dx$

3.146.1 Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx=\frac{\log(\sin(a+bx))}{2b}$$

output
```
1/2*ln(sin(b*x+a))/b
```

3.146.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx=\frac{1}{2}\left(\frac{\log(\cos(a+bx))}{b}+\frac{\log(\tan(a+bx))}{b}\right)$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

output
```
(Log[Cos[a + b*x]]/b + Log[Tan[a + b*x]]/b)/2
```

3.146.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4775, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)}dx$$
$$\Big\downarrow \; 4775$$
$$\frac{1}{2}\int \cot(a+bx)dx$$
$$\Big\downarrow \; 3042$$
$$\frac{1}{2}\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \; 25$$
$$-\frac{1}{2}\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$
$$\Big\downarrow \; 3956$$
$$\frac{\log(-\sin(a+bx))}{2b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

output
```
Log[-Sin[a + b*x]]/(2*b)
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.146.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(\sin(xb+a))}{2b}$	13
risch	$-\frac{ix}{2}-\frac{ia}{b}+\frac{\ln(e^{2i(xb+a)}-1)}{2b}$	30

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sin(b*x+a))/b
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx = \frac{\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)}{2\,b}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/2*log(1/2*sin(b*x + a))/b
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a),x)
```

output
```
Timed out
```

3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(12) = 24.

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 5.86

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx$$
$$= \frac{\log\left(\cos\left(bx\right)^2 + 2\,\cos\left(bx\right)\cos\left(a\right) + \cos\left(a\right)^2 + \sin\left(bx\right)^2 - 2\,\sin\left(bx\right)\sin\left(a\right) + \sin\left(a\right)^2\right) + \log\left(\cos\left(bx\right)^2 - 2\right.}{4\,b}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + si
n(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx=\frac{\log\left(|\sin\left(bx+a\right)|\right)}{2\,b}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
1/2*log(abs(sin(b*x + a)))/b
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx=\frac{\ln\left(\sin\left(a+b\,x\right)^2\right)}{4\,b}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x),x)
```

output
```
log(sin(a + b*x)^2)/(4*b)
```

3.147 $\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx$

3.147.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\cot(a+bx)}{4b}$$

output
```
-1/4*cot(b*x+a)/b
```

3.147.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\cot(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*Cot[a + b*x]/b
```

3.147.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4775, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \; 4775$$

$$\frac{1}{4}\int \csc^2(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \csc(a+bx)^2dx$$

$$\Big\downarrow \; 4254$$

$$-\frac{\int 1d\cot(a+bx)}{4b}$$

$$\Big\downarrow \; 24$$

$$-\frac{\cot(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*Cot[a + b*x]/b
```

3.147.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.147.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\cot(xb+a)}{4b}$	12
risch	$-\frac{i}{2b\left(\mathrm{e}^{2i(xb+a)}-1\right)}$	20

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*cot(b*x+a)/b
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\cos(bx+a)}{4\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-1/4*cos(b*x + a)/(b*sin(b*x + a))
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\sin(2\,bx+2\,a)}{2\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2-2\,b\cos(2\,bx+2\,a)+b\right)}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*c
os(2*b*x + 2*a) + b)
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{1}{4\,b\tan(bx+a)}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/4/(b*tan(b*x + a))
```

3.147.9 Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\cot(a+b\,x)}{4\,b}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^2,x)
```

output
```
-cot(a + b*x)/(4*b)
```

3.148 $\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx$

3.148.1 Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\cot^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}$$

output
```
-1/16*cot(b*x+a)^2/b+1/8*ln(tan(b*x+a))/b
```

3.148.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{1}{8}\left(-\frac{\csc^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}\right)$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-1/2*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/b + Log[Sin[a + b*x]]/b)/8
```

3.148.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{8}\int \csc^3(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^3\sec(a+bx)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^3(a+bx)+\cot(a+bx)\right)d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(\tan(a+bx))-\frac{1}{2}\cot^2(a+bx)}{8b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-1/2*Cot[a + b*x]^2 + Log[Tan[a + b*x]])/(8*b)
```

3.148.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.148.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{-\frac{1}{2\sin(xb+a)^2}+\ln(\tan(xb+a))}{8b}$	24
risch	$\frac{e^{2i(xb+a)}}{4b\left(e^{2i(xb+a)}-1\right)^2}+\frac{\ln\left(e^{2i(xb+a)}-1\right)}{8b}-\frac{\ln\left(e^{2i(xb+a)}+1\right)}{8b}$	63
parallelrisch	$-\frac{\sec(xb+a)^2\csc(xb+a)^2\left(\ln\left(\sqrt{\tan(xb+a)}\right)\cos(4xb+4a)+\cos(2xb+2a)+\cos(4xb+4a)-\ln\left(\sqrt{\tan(xb+a)}\right)\right)}{32b}$	71

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\left(\cos(bx+a)^2-1\right)\log\left(\cos(bx+a)^2\right)-\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{16\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/16*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log
(-1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(26) = 52$.

Time = 0.22 (sec) , antiderivative size = 656, normalized size of antiderivative = 21.87

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{4\cos(4bx+4a)\cos(2bx+2a)-8\cos(2bx+2a)^2+\left(2\left(2\cos(2bx+2a)-1\right)\cos(4bx+4a)-\cos(\ldots}{}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*c
os(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x +
 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin
(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*
cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)
 - (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*c
os(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos
(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*
(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b
*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4
*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*
cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b
*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*
sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*
b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{\frac{1}{\cos(bx+a)^2-1} + \log\left(-\cos(bx+a)^2+1\right) - 2\log\left(|\cos(bx+a)|\right)}{16\,b}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
1/16*(1/(cos(b*x + a)^2 - 1) + log(-cos(b*x + a)^2 + 1) - 2*log(abs(cos(b*
x + a))))/b
```

3.148.9 Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\frac{\ln(\cos(a+b\,x))}{8}-\frac{\ln\left(\sin(a+b\,x)^2\right)}{16}+\frac{1}{16\sin(a+b\,x)^2}}{b}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^3,x)
```

output
```
-(log(cos(a + b*x))/8 - log(sin(a + b*x)^2)/16 + 1/(16*sin(a + b*x)^2))/b
```

3.149 $\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$

3.149.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx=-\frac{\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}$$

output
```
-1/8*cot(b*x+a)/b-1/48*cot(b*x+a)^3/b+1/16*tan(b*x+a)/b
```

3.149.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx=-\frac{5\cot(a+bx)}{48b}-\frac{\cot(a+bx)\csc^2(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-5*Cot[a + b*x])/(48*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(48*b) + Tan[a +
b*x]/(16*b)
```

3.149.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{16}\int \csc^4(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^4\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^4(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^4(a+bx)+2\cot^2(a+bx)+1\right)d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan(a+bx)-\frac{1}{3}\cot^3(a+bx)-2\cot(a+bx)}{16b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-2*Cot[a + b*x] - Cot[a + b*x]^3/3 + Tan[a + b*x])/(16*b)
```

3.149.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.149.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{i\left(2\,\mathrm{e}^{2i(xb+a)}-1\right)}{3b\left(\mathrm{e}^{2i(xb+a)}-1\right)^{3}\left(\mathrm{e}^{2i(xb+a)}+1\right)}$	46
default	$\frac{-\frac{1}{3\cos(xb+a)\sin(xb+a)^{3}}+\frac{4}{3\sin(xb+a)\cos(xb+a)}-\frac{8\cot(xb+a)}{3}}{16b}$	51

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*I*(2*exp(2*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^3/(exp(2*I*(b*x+a))+1)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{8\cos(bx+a)^4-12\cos(bx+a)^2+3}{48\left(b\cos(bx+a)^3-b\cos(bx+a)\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-1/48*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*co
s(b*x + a))*sin(b*x + a))
```

3.149.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(36) = 72.

Time = 0.23 (sec) , antiderivative size = 308, normalized size of antiderivative = 7.33

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$$

$$= \frac{(2\ \cdots}{3\left(b\cos(8\,bx+8\,a)^2+4\,b\cos(6\,bx+6\,a)^2+4\,b\cos(2\,bx+2\,a)^2+b\sin(8\,bx+8\,a)^2+4\,b\sin(6\,bx+6\,a)^2\cdots\right.}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1
)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6
*a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b
*cos(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*
sin(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*
b*x + 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x
 + 2*a) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*
a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\frac{6\tan(bx+a)^2+1}{\tan(bx+a)^3} - 3\tan(bx+a)}{48\,b}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/48*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

3.149.9 Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = \frac{\tan(a+b\,x)}{16\,b} - \frac{\frac{\tan(a+b\,x)^2}{8}+\frac{1}{48}}{b\tan\left(a+b\,x\right)^3}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^4,x)
```

output
```
tan(a + b*x)/(16*b) - (tan(a + b*x)^2/8 + 1/48)/(b*tan(a + b*x)^3)
```

3.150 $\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$

3.150.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = -\frac{3\cot^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{\tan^2(a+bx)}{64b}$$

output
```
-3/64*cot(b*x+a)^2/b-1/128*cot(b*x+a)^4/b+3/32*ln(tan(b*x+a))/b+1/64*tan(b
*x+a)^2/b
```

3.150.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = -\frac{4\csc^2(a+bx)+\csc^4(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-2\sec^2(a+bx)}{128b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

output
```
-1/128*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[
Sin[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

3.150.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4775, 3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{32}\int \csc^5(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^5\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^5(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{32b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^3(a+bx)+3\cot^2(a+bx)+3\cot(a+bx)+1\right)d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^2(a+bx)-\frac{1}{2}\cot^2(a+bx)-3\cot(a+bx)+3\log\left(\tan^2(a+bx)\right)}{64b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-3*Cot[a + b*x] - Cot[a + b*x]^2/2 + 3*Log[Tan[a + b*x]^2] + Tan[a + b*x]
^2)/(64*b)
```

3.150.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.150. $\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$

3.150.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

method	result
default	$\frac{-\frac{1}{4 \sin(xb+a)^4 \cos(xb+a)^2}+\frac{3}{4 \sin(xb+a)^2 \cos(xb+a)^2}-\frac{3}{2 \sin(xb+a)^2}+3 \ln(\tan(xb+a))}{32b}$
parallelrisch	$-\frac{\sec(xb+a)^4 \csc(xb+a)^4 (36 \ln(\tan(xb+a)) \cos(4xb+4a)-9 \ln(\tan(xb+a)) \cos(8xb+8a)+66 \cos(2xb+2a)-18 \cos(6xb+6a)+6}{12288b}$
risch	$\frac{3\,\mathrm{e}^{10i(xb+a)}-6\,\mathrm{e}^{8i(xb+a)}-2\,\mathrm{e}^{6i(xb+a)}-6\,\mathrm{e}^{4i(xb+a)}+3\,\mathrm{e}^{2i(xb+a)}}{16b\left(\mathrm{e}^{2i(xb+a)}-1\right)^4\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}+\frac{3\ln\left(\mathrm{e}^{2i(xb+a)}-1\right)}{32b}-\frac{3\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{32b}$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^2+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/s
in(b*x+a)^2+3*ln(tan(b*x+a)))
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(52) = 104.

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$$
$$= \frac{6\,\cos(bx+a)^4 - 9\,\cos(bx+a)^2 - 6\left(\cos(bx+a)^6 - 2\,\cos(bx+a)^4 + \cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right)}{128\left(b\cos(bx+a)^6 - 2\,b\cos(bx+a)^4\right.}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
1/128*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x
 + a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(
b*x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*
x + a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

3.150.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3188 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 3188, normalized size of antiderivative = 53.13

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/64*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) -
6*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b
*x + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a)
 + 3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a
) - 12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*co
s(8*b*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*
b*x + 6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x
 + 4*a) + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b
*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*
cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8
*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) -
 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) -
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x +
8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) -
16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(
4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*
x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
 + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*si
n(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2...
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$$
$$= \frac{\frac{6\cos(bx+a)^4-9\cos(bx+a)^2+2}{\left(\cos(bx+a)^2-1\right)^2\cos(bx+a)^2} + 6\log\left(-\cos(bx+a)^2+1\right) - 12\log\left(|\cos(bx+a)|\right)}{128\,b}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
1/128*((6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 + 2)/((cos(b*x + a)^2 - 1)^2*c
os(b*x + a)^2) + 6*log(-cos(b*x + a)^2 + 1) - 12*log(abs(cos(b*x + a))))/b
```

3.150.9 Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = \frac{3\ln\left(\sin(a+b\,x)^2\right)}{64\,b} - \frac{3\ln\left(\cos(a+b\,x)\right)}{32\,b} + \frac{\frac{3\cos(a+b\,x)^4}{64} - \frac{9\cos(a+b\,x)^2}{128} + \frac{1}{64}}{b\,\left(\cos(a+b\,x)^6 - 2\cos(a+b\,x)^4 + \cos(a+b\,x)^2\right)}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^5,x)
```

output
```
(3*log(sin(a + b*x)^2))/(64*b) - (3*log(cos(a + b*x)))/(32*b) + ((3*cos(a
+ b*x)^4)/64 - (9*cos(a + b*x)^2)/128 + 1/64)/(b*(cos(a + b*x)^2 - 2*cos(a
 + b*x)^4 + cos(a + b*x)^6))
```

3.151 $\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx$

3.151.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx=-\frac{32\cos^9(a+bx)}{9b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^{13}(a+bx)}{13b}$$

output
```
-32/9*cos(b*x+a)^9/b+64/11*cos(b*x+a)^11/b-32/13*cos(b*x+a)^13/b
```

3.151.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$=\frac{4\cos^9(a+bx)(-505+540\cos(2(a+bx))-99\cos(4(a+bx)))}{1287b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*Cos[a + b*x]^9*(-505 + 540*Cos[2*(a + b*x)] - 99*Cos[4*(a + b*x)]))/(12
87*b)
```

3.151.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^5\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4775}$$

$$32\int \cos^8(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^8\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{32\int \cos^8(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{32\int \left(\cos^{12}(a+bx)-2\cos^{10}(a+bx)+\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{32\left(\frac{1}{13}\cos^{13}(a+bx)-\frac{2}{11}\cos^{11}(a+bx)+\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-32*(Cos[a + b*x]^9/9 - (2*Cos[a + b*x]^11)/11 + Cos[a + b*x]^13/13))/b
```

3.151.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.151.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(40) = 80$.

Time = 5.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

method	result
parallelrisch	$\frac{475136-180180\cos(xb+a)+18018\cos(7xb+7a)+9009\cos(5xb+5a)-2457\cos(11xb+11a)+2002\cos(9xb+9a)-693\cos(13xb+}{1153152b}$
default	$-\frac{5\cos(xb+a)}{32b}-\frac{25\cos(3xb+3a)}{384b}+\frac{\cos(5xb+5a)}{128b}+\frac{\cos(7xb+7a)}{64b}+\frac{\cos(9xb+9a)}{576b}-\frac{3\cos(11xb+11a)}{1408b}-\frac{\cos(13xb+13}{1664b}$
risch	$-\frac{5\cos(xb+a)}{32b}-\frac{25\cos(3xb+3a)}{384b}+\frac{\cos(5xb+5a)}{128b}+\frac{\cos(7xb+7a)}{64b}+\frac{\cos(9xb+9a)}{576b}-\frac{3\cos(11xb+11a)}{1408b}-\frac{\cos(13xb+13}{1664b}$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/1153152*(475136-180180*cos(b*x+a)+18018*cos(7*b*x+7*a)+9009*cos(5*b*x+5*
a)-2457*cos(11*b*x+11*a)+2002*cos(9*b*x+9*a)-693*cos(13*b*x+13*a)-75075*co
s(3*b*x+3*a))/b
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(99\,\cos\left(bx+a\right)^{13}-234\,\cos\left(bx+a\right)^{11}+143\,\cos\left(bx+a\right)^{9}\right)}{1287\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-32/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

3.151.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(39) = 78.

Time = 25.73 (sec) , antiderivative size = 447, normalized size of antiderivative = 9.72

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = \begin{cases} -\frac{2234\sin^3(a+bx)\sin^5(2a+2bx)}{9009b} - \frac{4544\sin^3(a+bx)\sin^3(2a+2bx)\cos^2(2a+2bx)}{9009b} - \frac{256\sin^3(a+bx)\sin(2a+2bx)\cos^4(2a+2bx)}{1001b} - \frac{1388 \\ x\sin^5(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-2234*sin(a + b*x)**3*sin(2*a + 2*b*x)**5/(9009*b) - 4544*sin(a
 + b*x)**3*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/(9009*b) - 256*sin(a +
b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**4/(1001*b) - 1388*sin(a + b*x)*
*2*sin(2*a + 2*b*x)**4*cos(a + b*x)*cos(2*a + 2*b*x)/(3003*b) - 2944*sin(a
 + b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**3/(3003*b) -
 512*sin(a + b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**5/(1001*b) + 271*sin(a
 + b*x)*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/(3003*b) + 48*sin(a + b*x)*sin
(2*a + 2*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(143*b) + 640*sin(a +
 b*x)*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**4/(3003*b) - 1366
*sin(2*a + 2*b*x)**4*cos(a + b*x)**3*cos(2*a + 2*b*x)/(3003*b) - 4960*sin(
2*a + 2*b*x)**2*cos(a + b*x)**3*cos(2*a + 2*b*x)**3/(9009*b) - 256*cos(a +
 b*x)**3*cos(2*a + 2*b*x)**5/(1287*b), Ne(b, 0)), (x*sin(2*a)**5*cos(a)**3
, True))
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx =$$
$$-\frac{99\cos(13bx+13a)+351\cos(11bx+11a)-286\cos(9bx+9a)-2574\cos(7bx+7a)-1287\cos\ldots}{164736\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/164736*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x
+ 9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x +
 3*a) + 25740*cos(b*x + a))/b
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(99\cos(bx+a)^{13}-234\cos(bx+a)^{11}+143\cos(bx+a)^{9}\right)}{1287\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
-32/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

3.151.9 Mupad [B] (verification not implemented)

Time = 20.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= -\frac{32\,\left(99\cos\left(a+b\,x\right)^{13}-234\cos\left(a+b\,x\right)^{11}+143\cos\left(a+b\,x\right)^{9}\right)}{1287\,b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^5,x)
```

output
```
-(32*(143*cos(a + b*x)^9 - 234*cos(a + b*x)^11 + 99*cos(a + b*x)^13))/(128
7*b)
```

3.152 $\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$

3.152.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\sin^5(a+bx)}{5b} - \frac{48\sin^7(a+bx)}{7b} + \frac{16\sin^9(a+bx)}{3b} - \frac{16\sin^{11}(a+bx)}{11b}$$

output
```
16/5*sin(b*x+a)^5/b-48/7*sin(b*x+a)^7/b+16/3*sin(b*x+a)^9/b-16/11*sin(b*x+
a)^11/b
```

3.152.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{(3042+3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx)))\sin^5(a+bx)}{2310b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x
)])*Sin[a + b*x]^5)/(2310*b)
```

3.152.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^4\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4775}$$

$$16\int \cos^7(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^7\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{16\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{16\int \left(-\sin^{10}(a+bx)+3\sin^8(a+bx)-3\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(-\frac{1}{11}\sin^{11}(a+bx)+\frac{1}{3}\sin^9(a+bx)-\frac{3}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(16*(Sin[a + b*x]^5/5 - (3*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/3 - Sin[a +
b*x]^11/11))/b
```

3.152.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.152.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$\frac{16170\sin(xb+a)-2310\sin(3xb+3a)-2541\sin(5xb+5a)-165\sin(7xb+7a)+385\sin(9xb+9a)+105\sin(11xb+11a)}{73920b}$	70
default	$\frac{7\sin(xb+a)}{32b}-\frac{\sin(3xb+3a)}{32b}-\frac{11\sin(5xb+5a)}{320b}-\frac{\sin(7xb+7a)}{448b}+\frac{\sin(9xb+9a)}{192b}+\frac{\sin(11xb+11a)}{704b}$	83
risch	$\frac{7\sin(xb+a)}{32b}-\frac{\sin(3xb+3a)}{32b}-\frac{11\sin(5xb+5a)}{320b}-\frac{\sin(7xb+7a)}{448b}+\frac{\sin(9xb+9a)}{192b}+\frac{\sin(11xb+11a)}{704b}$	83

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/73920*(16170*sin(b*x+a)-2310*sin(3*b*x+3*a)-2541*sin(5*b*x+5*a)-165*sin(
7*b*x+7*a)+385*sin(9*b*x+9*a)+105*sin(11*b*x+11*a))/b
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\frac{16\left(105\cos(bx+a)^{10}-140\cos(bx+a)^{8}+5\cos(bx+a)^{6}+6\cos(bx+a)^{4}+8\cos(bx+a)^{2}+16\right)\sin}{1155\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
16/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*c
os(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

3.152.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(53) = 106.

Time = 11.33 (sec) , antiderivative size = 366, normalized size of antiderivative = 6.00

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\begin{cases}\frac{46\sin^3(a+bx)\sin^4(2a+2bx)}{165b}+\frac{192\sin^3(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{385b}+\frac{256\sin^3(a+bx)\cos^4(2a+2bx)}{1155b}+\frac{272\sin^2(a+bx)\sin^3(2a}{} \\ x\sin^4(2a)\cos^3(a)\end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((46*sin(a + b*x)**3*sin(2*a + 2*b*x)**4/(165*b) + 192*sin(a + b*
x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/(385*b) + 256*sin(a + b*x)**
3*cos(2*a + 2*b*x)**4/(1155*b) + 272*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*c
os(a + b*x)*cos(2*a + 2*b*x)/(1155*b) + 256*sin(a + b*x)**2*sin(2*a + 2*b*
x)*cos(a + b*x)*cos(2*a + 2*b*x)**3/(1155*b) + 211*sin(a + b*x)*sin(2*a +
2*b*x)**4*cos(a + b*x)**2/(1155*b) + 304*sin(a + b*x)*sin(2*a + 2*b*x)**2*
cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(385*b) + 128*sin(a + b*x)*cos(a + b*x
)**2*cos(2*a + 2*b*x)**4/(231*b) - 472*sin(2*a + 2*b*x)**3*cos(a + b*x)**3
*cos(2*a + 2*b*x)/(1155*b) - 64*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a +
 2*b*x)**3/(231*b), Ne(b, 0)), (x*sin(2*a)**4*cos(a)**3, True))
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\frac{105\,\sin(11\,bx+11\,a)+385\,\sin(9\,bx+9\,a)-165\,\sin(7\,bx+7\,a)-2541\,\sin(5\,bx+5\,a)-2310\,\sin(3\,bx}{73920\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/73920*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7
*a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/
b
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\frac{105\,\sin(11\,bx+11\,a)+385\,\sin(9\,bx+9\,a)-165\,\sin(7\,bx+7\,a)-2541\,\sin(5\,bx+5\,a)-2310\,\sin(3\,bx}{73920\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/73920*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7
*a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/
b
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx=\frac{-\frac{16\sin(a+b\,x)^{11}}{11}+\frac{16\sin(a+b\,x)^9}{3}-\frac{48\sin(a+b\,x)^7}{7}+\frac{16\sin(a+b\,x)^5}{5}}{b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^4,x)
```

output
```
((16*sin(a + b*x)^5)/5 - (48*sin(a + b*x)^7)/7 + (16*sin(a + b*x)^9)/3 - (
16*sin(a + b*x)^11)/11)/b
```

3.152. $\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$

3.153 $\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx$

3.153.1 Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\cos^7(a+bx)}{7b}+\frac{8\cos^9(a+bx)}{9b}$$

output
```
-8/7*cos(b*x+a)^7/b+8/9*cos(b*x+a)^9/b
```

3.153.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{4\cos^7(a+bx)(-11+7\cos(2(a+bx)))}{63b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(4*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

3.153.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^3\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4775}$$

$$8\int \cos^6(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^6\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{8\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{8\int \left(\cos^6(a+bx)-\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8\left(\frac{1}{7}\cos^7(a+bx)-\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-8*(Cos[a + b*x]^7/7 - Cos[a + b*x]^9/9))/b
```

3.153.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.153.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

method	result	size
parallelrisch	$\frac{4864-1890\cos(xb+a)+135\cos(7xb+7a)+35\cos(9xb+9a)-840\cos(3xb+3a)}{10080b}$	49
default	$-\frac{3\cos(xb+a)}{16b}-\frac{\cos(3xb+3a)}{12b}+\frac{3\cos(7xb+7a)}{224b}+\frac{\cos(9xb+9a)}{288b}$	55
risch	$-\frac{3\cos(xb+a)}{16b}-\frac{\cos(3xb+3a)}{12b}+\frac{3\cos(7xb+7a)}{224b}+\frac{\cos(9xb+9a)}{288b}$	55

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10080*(4864-1890*cos(b*x+a)+135*cos(7*b*x+7*a)+35*cos(9*b*x+9*a)-840*cos
(3*b*x+3*a))/b
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(7\cos\left(bx+a\right)^9-9\cos\left(bx+a\right)^7\right)}{63\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
8/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

3.153.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(26) = 52.

Time = 4.92 (sec) , antiderivative size = 284, normalized size of antiderivative = 9.16

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{94\sin^3(a+bx)\sin^3(2a+2bx)}{315b} - \frac{32\sin^3(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{105b} - \frac{4\sin^2(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{7b} - \\ x\sin^3(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-94*sin(a + b*x)**3*sin(2*a + 2*b*x)**3/(315*b) - 32*sin(a + b*
x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(105*b) - 4*sin(a + b*x)**2*sin
(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/(7*b) - 64*sin(a + b*x)**2*
cos(a + b*x)*cos(2*a + 2*b*x)**3/(105*b) + 13*sin(a + b*x)*sin(2*a + 2*b*x
)**3*cos(a + b*x)**2/(105*b) + 8*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x
)**2*cos(2*a + 2*b*x)**2/(35*b) - 46*sin(2*a + 2*b*x)**2*cos(a + b*x)**3*c
os(2*a + 2*b*x)/(105*b) - 16*cos(a + b*x)**3*cos(2*a + 2*b*x)**3/(63*b), N
e(b, 0)), (x*sin(2*a)**3*cos(a)**3, True))
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{7\cos(9\,bx+9\,a)+27\cos(7\,bx+7\,a)-168\cos(3\,bx+3\,a)-378\cos(bx+a)}{2016\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/2016*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) -
378*cos(b*x + a))/b
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(7\cos(bx+a)^9-9\cos(bx+a)^7\right)}{63\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
8/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

3.153.9 Mupad [B] (verification not implemented)

Time = 19.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(9\cos(a+b\,x)^7-7\cos(a+b\,x)^9\right)}{63\,b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^3,x)
```

output
```
-(8*(9*cos(a + b*x)^7 - 7*cos(a + b*x)^9))/(63*b)
```

3.154 $\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx$

3.154.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx=\frac{4\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^7(a+bx)}{7b}$$

output
```
4/3*sin(b*x+a)^3/b-8/5*sin(b*x+a)^5/b+4/7*sin(b*x+a)^7/b
```

3.154.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx =\frac{(157+108\cos(2(a+bx))+15\cos(4(a+bx)))\sin^3(a+bx)}{210b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(210*b
)
```

3.154.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^2\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4775}$$

$$4\int \cos^5(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^5\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{4\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{4\int \left(\sin^6(a+bx)-2\sin^4(a+bx)+\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(\frac{1}{7}\sin^7(a+bx)-\frac{2}{5}\sin^5(a+bx)+\frac{1}{3}\sin^3(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(4*(Sin[a + b*x]^3/3 - (2*Sin[a + b*x]^5)/5 + Sin[a + b*x]^7/7))/b
```

3.154.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.154.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$\frac{525\sin(xb+a)-35\sin(3xb+3a)-63\sin(5xb+5a)-15\sin(7xb+7a)}{1680b}$	48
default	$\frac{5\sin(xb+a)}{16b}-\frac{\sin(3xb+3a)}{48b}-\frac{3\sin(5xb+5a)}{80b}-\frac{\sin(7xb+7a)}{112b}$	55
risch	$\frac{5\sin(xb+a)}{16b}-\frac{\sin(3xb+3a)}{48b}-\frac{3\sin(5xb+5a)}{80b}-\frac{\sin(7xb+7a)}{112b}$	55

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1680*(525*sin(b*x+a)-35*sin(3*b*x+3*a)-63*sin(5*b*x+5*a)-15*sin(7*b*x+7*
a))/b
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(15\cos(bx+a)^6-3\cos(bx+a)^4-4\cos(bx+a)^2-8\right)\sin(bx+a)}{105\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-4/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b
*x + a)/b
```

3.154.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(39) = 78.

Time = 2.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.39

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = \begin{cases} \frac{38\sin^3(a+bx)\sin^2(2a+2bx)}{105b} + \frac{32\sin^3(a+bx)\cos^2(2a+2bx)}{105b} + \frac{8\sin^2(a+bx)\sin(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{35b} + \frac{11\sin(a+bx)\sin^2(\\ x\sin^2(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((38*sin(a + b*x)**3*sin(2*a + 2*b*x)**2/(105*b) + 32*sin(a + b*x
)**3*cos(2*a + 2*b*x)**2/(105*b) + 8*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(
a + b*x)*cos(2*a + 2*b*x)/(35*b) + 11*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos
(a + b*x)**2/(35*b) + 24*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/
(35*b) - 12*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)/(35*b), Ne(b
, 0)), (x*sin(2*a)**2*cos(a)**3, True))
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{15\,\sin\left(7\,bx+7\,a\right)+63\,\sin\left(5\,bx+5\,a\right)+35\,\sin\left(3\,bx+3\,a\right)-525\,\sin\left(bx+a\right)}{1680\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/1680*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
 525*sin(b*x + a))/b
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{15\,\sin\left(7\,bx+7\,a\right)+63\,\sin\left(5\,bx+5\,a\right)+35\,\sin\left(3\,bx+3\,a\right)-525\,\sin\left(bx+a\right)}{1680\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/1680*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
 525*sin(b*x + a))/b
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{4\,\left(15\sin\left(a+b\,x\right)^7-42\sin\left(a+b\,x\right)^5+35\sin\left(a+b\,x\right)^3\right)}{105\,b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

output
```
(4*(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7))/(105*b)
```

3.155 $\int \cos^3(a+bx)\sin(2a+2bx)\,dx$

3.155.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx=-\frac{2\cos^5(a+bx)}{5b}$$

output
```
-2/5*cos(b*x+a)^5/b
```

3.155.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx=-\frac{2\cos^5(a+bx)}{5b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Cos[a + b*x]^5)/(5*b)
```

3.155.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4775, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4775}$$

$$2\int \cos^4(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)^4\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{2\int \cos^4(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\cos^5(a+bx)}{5b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Cos[a + b*x]^5)/(5*b)
```

3.155.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.155.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.

Time $= 0.45$ (sec) , antiderivative size $= 41$, normalized size of antiderivative $= 2.73$

method	result
default	$-\frac{\cos(xb+a)}{4b}-\frac{\cos(3xb+3a)}{8b}-\frac{\cos(5xb+5a)}{40b}$
risch	$-\frac{\cos(xb+a)}{4b}-\frac{\cos(3xb+3a)}{8b}-\frac{\cos(5xb+5a)}{40b}$
parallelrisch	$\frac{-\frac{4}{5}+4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{5}\tan(xb+a)+4\left(-\tan(xb+a)^{2}+1\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}-8\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}\tan(xb+a)-\frac{12\tan(xb+a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{5}+\frac{4}{}}{b\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{3}\left(1+\tan(xb+a)^{2}\right)}$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b-1/40*cos(5*b*x+5*a)/b
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\cos(bx+a)^5}{5b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2/5*cos(b*x + a)^5/b
```

3.155.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(14) = 28$.

Time = 0.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 7.80

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = \begin{cases} -\frac{2\sin^3(a+bx)\sin(2a+2bx)}{5b} - \frac{4\sin^2(a+bx)\cos(a+bx)\cos(2a+2bx)}{5b} + \frac{\sin(a+bx)\sin(2a+2bx)\cos^2(a+bx)}{5b} - \frac{2\cos^3(a+bx)\cos(2a+2b}{5b} \\ x\sin(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-2*sin(a + b*x)**3*sin(2*a + 2*b*x)/(5*b) - 4*sin(a + b*x)**2*c
os(a + b*x)*cos(2*a + 2*b*x)/(5*b) + sin(a + b*x)*sin(2*a + 2*b*x)*cos(a +
 b*x)**2/(5*b) - 2*cos(a + b*x)**3*cos(2*a + 2*b*x)/(5*b), Ne(b, 0)), (x*s
in(2*a)*cos(a)**3, True))
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(5bx+5a) + 5\cos(3bx+3a) + 10\cos(bx+a)}{40b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/40*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\,\cos\left(bx+a\right)^5}{5\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-2/5*cos(b*x + a)^5/b
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\cos\left(a+b\,x\right)^5}{5\,b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x),x)
```

output
```
-(2*cos(a + b*x)^5)/(5*b)
```

3.156 $\int \cos^3(a+bx)\csc(2a+2bx)\,dx$

3.156.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{\cos(a+bx)}{2b}$$

output
```
-1/2*arctanh(cos(b*x+a))/b+1/2*cos(b*x+a)/b
```

3.156.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx=\frac{1}{2}\left(\frac{\cos(a+bx)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

output
```
(Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)/2
```

3.156.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 4775, 3042, 25, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{2}\int \cos(a+bx)\cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -\sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cos^2(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\int \frac{1}{1-\cos^2(a+bx)}d\cos(a+bx)-\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}(\cos(a+bx))-\cos(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*(ArcTanh[Cos[a + b*x]] - Cos[a + b*x])/b
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.156.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\cos(xb+a)+\ln(\csc(xb+a)-\cot(xb+a))}{2b}$	29
risch	$\frac{e^{i(xb+a)}}{4b}+\frac{e^{-i(xb+a)}}{4b}+\frac{\ln(e^{i(xb+a)}-1)}{2b}-\frac{\ln(e^{i(xb+a)}+1)}{2b}$	64

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \frac{2\cos(bx+a) - \log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/4*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a)
+ 1/2))/b
```

3.156.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a),x)
```

output
```
Timed out
```

3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.29

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \frac{2\cos(bx+a) - \log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + }{4b}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/4*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(
b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a
) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \frac{2\cos(bx+a) - \log(\cos(bx+a)+1) + \log(-\cos(bx+a)+1)}{4b}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
1/4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(-cos(b*x + a) + 1))/b
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \frac{\frac{\cos(a+b\,x)}{2} - \frac{\mathrm{atanh}(\cos(a+b\,x))}{2}}{b}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x),x)
```

output
```
(cos(a + b*x)/2 - atanh(cos(a + b*x))/2)/b
```

3.157 $\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx$

3.157.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\csc(a+bx)}{4b}$$

output
```
-1/4*csc(b*x+a)/b
```

3.157.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\csc(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*Csc[a + b*x]/b
```

3.157.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4775, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \; 4775$$

$$\frac{1}{4}\int \cot(a+bx)\csc(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int -\sec\left(a+bx-\frac{\pi}{2}\right)\tan\left(a+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{4}\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)\tan\left(\frac{1}{2}(2a-\pi)+bx\right)dx$$

$$\Big\downarrow \; 3086$$

$$-\frac{\int 1d\csc(a+bx)}{4b}$$

$$\Big\downarrow \; 24$$

$$-\frac{\csc(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*Csc[a + b*x]/b
```

3.157.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.157.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{4\sin(xb+a)b}$	14
risch	$-\frac{i e^{i(xb+a)}}{2b\left(e^{2i(xb+a)}-1\right)}$	29

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/sin(b*x+a)/b
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{1}{4\,b\sin{(bx+a)}}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-1/4/(b*sin(b*x + a))
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(11) = 22$.

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 6.46

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\cos{(bx+a)}\sin{(2\,bx+2\,a)} - \cos{(2\,bx+2\,a)}\sin{(bx+a)} + \sin{(bx+a)}}{2\left(b\cos{(2\,bx+2\,a)}^2 + b\sin{(2\,bx+2\,a)}^2 - 2\,b\cos{(2\,bx+2\,a)} + b\right)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/2*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(
b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2
*a) + b)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{1}{4\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/4/(b*sin(b*x + a))
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{1}{4\,b\,\sin(a+b\,x)}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^2,x)
```

output
```
-1/(4*b*sin(a + b*x))
```

3.158 $\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx$

3.158.1 Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}$$

output
```
-1/16*arctanh(cos(b*x+a))/b-1/16*cot(b*x+a)*csc(b*x+a)/b
```

3.158.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx=\frac{1}{8}\left(-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-1/8*Csc[(a + b*x)/2]^2/b - Log[Cos[(a + b*x)/2]]/(2*b) + Log[Sin[(a + b*
x)/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b))/8
```

3.158.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4775, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{8}\int \csc^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \csc(a+bx)dx-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \csc(a+bx)dx-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{8}\left(-\frac{\text{arctanh}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-1/2*ArcTanh[Cos[a + b*x]]/b - (Cot[a + b*x]*Csc[a + b*x])/(2*b))/8
```

3.158.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.158.4 Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{-\frac{\csc(xb+a)\cot(xb+a)}{2}+\frac{\ln(\csc(xb+a)-\cot(xb+a))}{2}}{8b}$	39
risch	$\frac{e^{3i(xb+a)}+e^{i(xb+a)}}{8b\left(e^{2i(xb+a)}-1\right)^{2}}-\frac{\ln\left(e^{i(xb+a)}+1\right)}{16b}+\frac{\ln\left(e^{i(xb+a)}-1\right)}{16b}$	73

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/2*csc(b*x+a)*cot(b*x+a)+1/2*ln(csc(b*x+a)-cot(b*x+a)))
```

3.158.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = -\frac{(\cos(bx+a)^2-1)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-(\cos(bx+a)^2-1)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-2\cos(b}{32\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/32*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2
- 1)*log(-1/2*cos(b*x + a) + 1/2) - 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(30) = 60$.

Time = 0.21 (sec) , antiderivative size = 558, normalized size of antiderivative = 16.41

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx$$
$$= \frac{4(\cos(3bx+3a)+\cos(bx+a))\cos(4bx+4a)-4(2\cos(2bx+2a)-1)\cos(3bx+3a)-8\cos(2bx}{}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/32*(4*(cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) - 4*(2*cos(2*b*
x + 2*a) - 1)*cos(3*b*x + 3*a) - 8*cos(2*b*x + 2*a)*cos(b*x + a) + (2*(2*c
os(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x +
 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin
(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(
a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*b
*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + c
os(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(sin(3*b*x + 3*a)
 + sin(b*x + a))*sin(4*b*x + 4*a) - 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) -
8*sin(2*b*x + 2*a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(4*b*x + 4*a)^2 +
4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*
b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1} - \log(\cos(bx+a)+1) + \log(-\cos(bx+a)+1)}{32\,b}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
1/32*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(-c
os(b*x + a) + 1))/b
```

3.158.9 Mupad [B] (verification not implemented)

Time = 19.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{\cos(a+b\,x)}{16\,b\,\left(\cos(a+b\,x)^2-1\right)} - \frac{\mathrm{atanh}(\cos(a+b\,x))}{16\,b}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^3,x)
```

output
```
cos(a + b*x)/(16*b*(cos(a + b*x)^2 - 1)) - atanh(cos(a + b*x))/(16*b)
```

3.159 $\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx$

3.159.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{16b}-\frac{\csc(a+bx)}{16b}-\frac{\csc^3(a+bx)}{48b}$$

output
```
1/16*arctanh(sin(b*x+a))/b-1/16*csc(b*x+a)/b-1/48*csc(b*x+a)^3/b
```

3.159.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(a+bx)\right)}{48b}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/48*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

3.159.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4775, 3042, 3101, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{ 4775}$$

$$\frac{1}{16}\int \csc^4(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{16}\int \csc(a+bx)^4\sec(a+bx)dx$$

$$\Big\downarrow \text{ 3101}$$

$$-\frac{\int -\frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\text{arctanh}(\csc(a+bx))+\frac{1}{3}\csc^3(a+bx)+\csc(a+bx)}{16b}$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/16*(-ArcTanh[Csc[a + b*x]] + Csc[a + b*x] + Csc[a + b*x]^3/3)/b
```

3.159.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.159.4 Maple [A] (verified)

Time = 10.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\frac{1}{3 \sin(xb+a)^3}-\frac{1}{\sin(xb+a)}+\ln(\sec(xb+a)+\tan(xb+a))}{16b}$	41
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(xb+a)}-10\,\mathrm{e}^{3i(xb+a)}+3\,\mathrm{e}^{i(xb+a)}\right)}{24b\left(\mathrm{e}^{2i(xb+a)}-1\right)^3}-\frac{\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{16b}+\frac{\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{16b}$	91

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(-1/3/sin(b*x+a)^3-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{3\left(\cos(bx+a)^2-1\right)\log(\sin(bx+a)+1)\sin(bx+a)-3\left(\cos(bx+a)^2-1\right)\log(-\sin(bx+a)+1)\sin}{96\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/96*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b
*x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)
/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(37) = 74$.

Time = 0.32 (sec) , antiderivative size = 834, normalized size of antiderivative = 19.40

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/96*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*
b*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 1
2*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*
b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2
 + 6*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*
cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6
*a) - sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x
 + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(
b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(
b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(
3*cos(5*b*x + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a
) - 12*(3*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12
*(10*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x
 + 2*a) - 1)*sin(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36
*cos(b*x + a)*sin(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin
(b*x + a))/(b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x
+ 2*a)^2 + b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x
+ 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a)
 - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a)...
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(-\sin\left(bx+a\right)+1\right)}{96\,b}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/96*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) +
 3*log(-sin(b*x + a) + 1))/b
```

3.159.9 Mupad [B] (verification not implemented)

Time = 19.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{\operatorname{atanh}(\sin{(a+b\,x)})}{16\,b}-\frac{\frac{\sin(a+b\,x)^2}{16}+\frac{1}{48}}{b\sin{(a+b\,x)}^3}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^4,x)
```

output
```
atanh(sin(a + b*x))/(16*b) - (sin(a + b*x)^2/16 + 1/48)/(b*sin(a + b*x)^3)
```

3.160 $\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$

3.160.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = -\frac{15\operatorname{arctanh}(\cos(a+bx))}{256b}+\frac{15\sec(a+bx)}{256b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{256b}-\frac{\csc^4(a+bx)\sec(a+bx)}{128b}$$

output
```
-15/256*arctanh(cos(b*x+a))/b+15/256*sec(b*x+a)/b-5/256*csc(b*x+a)^2*sec(b
*x+a)/b-1/128*csc(b*x+a)^4*sec(b*x+a)/b
```

3.160.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(70) = 140.

Time = 0.88 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.79

$$\begin{aligned}\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = &-\frac{7\csc^2\left(\frac{1}{2}(a+bx)\right)}{1024b}-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{2048b}\\&-\frac{15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{256b}+\frac{15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{256b}\\&+\frac{7\sec^2\left(\frac{1}{2}(a+bx)\right)}{1024b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{2048b}\\&+\frac{\sin\left(\frac{1}{2}(a+bx)\right)}{32b\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}\\&-\frac{\sin\left(\frac{1}{2}(a+bx)\right)}{32b\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}\end{aligned}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-7*Csc[(a + b*x)/2]^2)/(1024*b) - Csc[(a + b*x)/2]^4/(2048*b) - (15*Log[C
os[(a + b*x)/2]])/(256*b) + (15*Log[Sin[(a + b*x)/2]])/(256*b) + (7*Sec[(a
 + b*x)/2]^2)/(1024*b) + Sec[(a + b*x)/2]^4/(2048*b) + Sin[(a + b*x)/2]/(3
2*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - Sin[(a + b*x)/2]/(32*b*(Cos[(
a + b*x)/2] + Sin[(a + b*x)/2]))
```

3.160.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4775, 3042, 3102, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{32}\int \csc^5(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^5\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{32b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{32b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\sec(a+bx))-\sec(a+bx))\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-1/4*Sec[a + b*x]^5/(1 - Sec[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Sec[a + b*x
]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Sec[a + b*x]^2))))/4)/(32*b
)
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.160.4 Maple [A] (verified)

Time = 19.83 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{-\frac{1}{4\cos(xb+a)\sin(xb+a)^4}-\frac{5}{8\sin(xb+a)^2\cos(xb+a)}+\frac{15}{8\cos(xb+a)}+\frac{15\ln(\csc(xb+a)-\cot(xb+a))}{8}}{32b}$	71
risch	$\frac{15\,\mathrm{e}^{9i(xb+a)}-40\,\mathrm{e}^{7i(xb+a)}+18\,\mathrm{e}^{5i(xb+a)}-40\,\mathrm{e}^{3i(xb+a)}+15\,\mathrm{e}^{i(xb+a)}}{128b\left(\mathrm{e}^{2i(xb+a)}-1\right)^4\left(\mathrm{e}^{2i(xb+a)}+1\right)}-\frac{15\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{256b}+\frac{15\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{256b}$	123

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(-1/4/cos(b*x+a)/sin(b*x+a)^4-5/8/sin(b*x+a)^2/cos(b*x+a)+15/8/cos(
b*x+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(62) = 124.

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.89

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$$
$$= \frac{30\cos(bx+a)^4 - 50\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - 2\cos(bx+a)^3 + \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+\right.}{512\left(b\cos(bx+a)^5 - 2b\cos(bx+\right.}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
1/512*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(
b*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^
5 - 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b
*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. 2(62) = 124.

Time = 0.27 (sec) , antiderivative size = 2237, normalized size of antiderivative = 31.96

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/512*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a)
- 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*
b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a)
- 1)*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40
*cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x +
 6*a) + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*
(18*cos(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x +
6*a) + 72*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) -
 40*(8*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*
b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*
(2*(3*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2
*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*
x + 6*a) + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) -
 9*cos(8*b*x + 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*co
s(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b
*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x +
 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(
10*b*x + 10*a) - sin(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*
x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4
*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b...
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{2\left(7\cos(bx+a)^3-9\cos(bx+a)\right)}{\left(\cos(bx+a)^2-1\right)^2}+\frac{16}{\cos(bx+a)}-15\log\left(\cos\left(bx+a\right)+1\right)+15\log\left(-\cos\left(bx+a\right)+1\right)}{512\,b}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
1/512*(2*(7*cos(b*x + a)^3 - 9*cos(b*x + a))/(cos(b*x + a)^2 - 1)^2 + 16/c
os(b*x + a) - 15*log(cos(b*x + a) + 1) + 15*log(-cos(b*x + a) + 1))/b
```

3.160.9 Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{15\cos(a+b\,x)^4}{256}-\frac{25\cos(a+b\,x)^2}{256}+\frac{1}{32}}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)} - \frac{15\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^5,x)
```

output
```
((15*cos(a + b*x)^4)/256 - (25*cos(a + b*x)^2)/256 + 1/32)/(b*(cos(a + b*x
) - 2*cos(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(256*b)
```

3.161 $\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.161.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\begin{aligned}\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = &-\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{32b}\\ &-\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b}\\ &+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}\\ &-\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}\\ &+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\end{aligned}$$

output
```
-5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-5/24*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/6*sin(b*x+a)*si
n(2*b*x+2*a)^(5/2)/b+5/16*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.161.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{-5\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin(2}}{32b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] -
 3*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(32*b)
```

3.161.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4789, 3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \sin(2a+2bx)^{5/2}\cos(a+bx)dx$$
$$\Big\downarrow \ 4789$$
$$\frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$
$$\Big\downarrow \ 3042$$
$$\frac{5}{6}\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$
$$\Big\downarrow \ 4790$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{4789}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{4794}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+\right.\right.$$
$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b) + (5*((3*((-1/2*ArcSin[Cos[a +
 b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a +
 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 - (Co
s[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/6
```

3.161. $\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.161.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.161.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 71.55 (sec) , antiderivative size = 221760772, normalized size of antiderivative = 1630593.91

method	result	size
default	Expression too large to display	221760772

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(118) = 236$.

Time = 0.28 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.13

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx =$$

$$-\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-12\cos\left(bx+a\right)^{2}-15\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\sin\left(bx+a\right)-30\arctan\left(-\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/384*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*
x + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*
sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(
cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)
*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x +
 a) - sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x +
a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x
+ a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)
)/b
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.161.7 Maxima [F]

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

3.161.8 Giac [F]

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(a+b\,x)\,\sin(2\,a+2\,b\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(5/2), x)
```

3.162 $\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.162.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{16b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b} - \frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{8b} + \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

output
```
-3/16*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+1/4*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-3/8*cos(b*x+a)*sin
(2*b*x+2*a)^(1/2)/b
```

3.162.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{3\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(2\cos(}{16b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[S
in[2*(a + b*x)]])/(16*b)
```

3.162.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^{3/2}\cos(a+bx)dx$$

$$\Big\downarrow \text{4789}$$

$$\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

$$\Big\downarrow \text{4790}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

$$\Big\downarrow \text{4793}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a+2b}}{}\right.$$

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)))/4 + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

3.162.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.162.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 16.62 (sec) , antiderivative size = 85939054, normalized size of antiderivative = 781264.13

method	result	size
default	Expression too large to display	85939054

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(96) = 192.

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.55

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx =$$

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^3-\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(b}{\cos(bx+a)^2+2\,c}\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b
*x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*lo
g(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1
)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(
b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.162.7 Maxima [F]

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

3.162.8 Giac [F]

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(a+b\,x)\,\sin(2\,a+2\,b\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(3/2), x)
```

3.163 $\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.163.1 Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} + \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}$$

output
```
-1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+1/2*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.163.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx)) + \log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right) - 2\sin(a+bx}{4b}$$

input
```
Integrate[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/4*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

3.163.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(2a+2bx)}\cos(a+bx)dx$$

$$\Big\downarrow \text{4789}$$

$$\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}$$

$$\Big\downarrow \text{4794}$$

$$\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}$$

input
```
Int[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b
*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*
x]])/(2*b)
```

3.163.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.163.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.02 (sec) , antiderivative size = 5537888, normalized size of antiderivative = 65927.24

method	result	size
default	Expression too large to display	5537888

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(74) = 148.

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.17

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) + 2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\,\mathrm{s}}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + co
s(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1
)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) -
 sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4
*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b
*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x
+ a)*sin(b*x + a) + 1))/b
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.163.7 Maxima [F]

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(bx+a)\sqrt{\sin(2bx+2a)}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

3.163.8 Giac [F]

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(bx+a)\sqrt{\sin(2bx+2a)}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2), x)
```

3.164 $\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

3.164.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

output
```
-1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b
```

3.164.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{2b}$$

input
```
Integrate[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] +
Sqrt[Sin[2*(a + b*x)]]])/(2*b)
```

3.164.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4793}$$

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}$$

input
```
Int[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*
x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

3.164.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.164.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.07 (sec) , antiderivative size = 18441891, normalized size of antiderivative = 317963.64

method	result	size
default	Expression too large to display	18441891

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(52) = 104.

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.17

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-c}{\cos(bx+a)-\sin(}\right.}{}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))
- cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*co
s(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*
x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)
^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.164.7 Maxima [F]

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

3.164.8 Giac [F]

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(1/2), x)
```

3.165 $\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.165.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
-cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-(Cos[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]]))
```

3.165.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4779}$$

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-(Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]]))
```

3.165.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

3.165.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 6.42 (sec) , antiderivative size = 57905011, normalized size of antiderivative = 2412708.79

method	result	size
default	Expression too large to display	57905011

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}+\sin(bx+a)}{2\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x +
 a))
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.165.7 Maxima [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

3.165.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2029 vs. $2(22) = 44$.

Time = 13.87 (sec) , antiderivative size = 2029, normalized size of antiderivative = 84.54

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*(((sqrt(2)*tan(1/2*a)^26 + 5*sqrt(2)*tan(1/2*a)^24 -
 10*sqrt(2)*tan(1/2*a)^22 - 154*sqrt(2)*tan(1/2*a)^20 - 605*sqrt(2)*tan(1/
2*a)^18 - 1353*sqrt(2)*tan(1/2*a)^16 - 1980*sqrt(2)*tan(1/2*a)^14 - 1980*s
qrt(2)*tan(1/2*a)^12 - 1353*sqrt(2)*tan(1/2*a)^10 - 605*sqrt(2)*tan(1/2*a)
^8 - 154*sqrt(2)*tan(1/2*a)^6 - 10*sqrt(2)*tan(1/2*a)^4 + 5*sqrt(2)*tan(1/
2*a)^2 + sqrt(2))*tan(1/2*b*x)/(tan(1/2*a)^24 + 12*tan(1/2*a)^22 + 66*tan(
1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792*tan(1/2*a)^14 + 92
4*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 220*tan(1/2*a)^6
+ 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1) - 8*(sqrt(2)*tan(1/2*a)^25 + 10*s
qrt(2)*tan(1/2*a)^23 + 44*sqrt(2)*tan(1/2*a)^21 + 110*sqrt(2)*tan(1/2*a)^1
9 + 165*sqrt(2)*tan(1/2*a)^17 + 132*sqrt(2)*tan(1/2*a)^15 - 132*sqrt(2)*ta
n(1/2*a)^11 - 165*sqrt(2)*tan(1/2*a)^9 - 110*sqrt(2)*tan(1/2*a)^7 - 44*sqr
t(2)*tan(1/2*a)^5 - 10*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))/(tan(1/2
*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan
(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 4
95*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 ...
```

3.165.9 Mupad [B] (verification not implemented)

Time = 20.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{2\,b\,\sin\left(a+b\,x\right)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
-sin(2*a + 2*b*x)^(1/2)/(2*b*sin(a + b*x))
```

3.166 $\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.166.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+2/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/
2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\left(-\frac{1}{12}\cot(a+bx)\csc(a+bx)+\frac{1}{4}\sec(a+bx)\right)\sqrt{\sin(2(a+bx))}}{b}$$

input
```
Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
((-1/12*(Cot[a + b*x]*Csc[a + b*x]) + Sec[a + b*x]/4)*Sqrt[Sin[2*(a + b*x)
]])/b
```

3.166.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*Sqrt[
Sin[2*a + 2*b*x]])
```

3.166.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.166.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 14.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.66

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+}\right.\right.}{24b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}}$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/
2*x*b)^2-1)/tan(1/2*a+1/2*x*b)*(2*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2
*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*
x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)-tan(1/2*a+1/2*x*b)^4+1)/(tan
(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(
1/2*a+1/2*x*b))^(1/2)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{4\,\cos\left(bx+a\right)^{3}+\sqrt{2}\left(4\,\cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-4\,\cos\left(bx+a\right)}{12\,\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*
sin(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.166.7 Maxima [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

3.166.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7875 vs. $2(45) = 90$.

Time = 53.65 (sec) , antiderivative size = 7875, normalized size of antiderivative = 148.58

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
1/48*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*(((((((sqrt(2)*tan(1/2*a)^57 + 18*sqrt(2)*tan(1/2*a)
^55 + 132*sqrt(2)*tan(1/2*a)^53 + 374*sqrt(2)*tan(1/2*a)^51 - 1375*sqrt(2)
*tan(1/2*a)^49 - 19620*sqrt(2)*tan(1/2*a)^47 - 108560*sqrt(2)*tan(1/2*a)^4
5 - 399740*sqrt(2)*tan(1/2*a)^43 - 1096755*sqrt(2)*tan(1/2*a)^41 - 2340250
*sqrt(2)*tan(1/2*a)^39 - 3941740*sqrt(2)*tan(1/2*a)^37 - 5204670*sqrt(2)*t
an(1/2*a)^35 - 5163155*sqrt(2)*tan(1/2*a)^33 - 3268760*sqrt(2)*tan(1/2*a)^
31 + 3268760*sqrt(2)*tan(1/2*a)^27 + 5163155*sqrt(2)*tan(1/2*a)^25 + 52046
70*sqrt(2)*tan(1/2*a)^23 + 3941740*sqrt(2)*tan(1/2*a)^21 + 2340250*sqrt(2)
*tan(1/2*a)^19 + 1096755*sqrt(2)*tan(1/2*a)^17 + 399740*sqrt(2)*tan(1/2*a)
^15 + 108560*sqrt(2)*tan(1/2*a)^13 + 19620*sqrt(2)*tan(1/2*a)^11 + 1375*sq
rt(2)*tan(1/2*a)^9 - 374*sqrt(2)*tan(1/2*a)^7 - 132*sqrt(2)*tan(1/2*a)^5 -
 18*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^51
 + 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/
2*a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37
 + 389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 53
4888*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 5348...
```

3.166.9 Mupad [B] (verification not implemented)

Time = 23.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.96

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{2\sqrt{\sin(2a+2bx)}\,(3\cos(a+bx)-6\cos(3a+3bx)+4\cos(5a+5bx)-\cos(7a+7bx))}{3b\,(4\cos(2a+2bx)+4\cos(4a+4bx)-4\cos(6a+6bx)+\cos(8a+8bx)-5)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
-(2*sin(2*a + 2*b*x)^(1/2)*(3*cos(a + b*x) - 6*cos(3*a + 3*b*x) + 4*cos(5*
a + 5*b*x) - cos(7*a + 7*b*x)))/(3*b*(4*cos(2*a + 2*b*x) + 4*cos(4*a + 4*b
*x) - 4*cos(6*a + 6*b*x) + cos(8*a + 8*b*x) - 5))
```

3.166. $\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\, dx$

3.167 $\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

3.167.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)+4/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(3
/2)-8/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin(2(a+bx))}(27\csc(a+bx)+3\csc^3(a+bx)-5\sec(a+bx)\tan(a+bx))}{120b}$$

input
```
Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/120*(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec
[a + b*x]*Tan[a + b*x]))/b
```

3.167.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt
[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))
```

3.167.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.167.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 231.20 (sec) , antiderivative size = 481, normalized size of antiderivative = 6.09

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(24\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\operatorname{EllipticE}\left(\right.\right.}{}$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/160/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/tan(1/2*a+1/
2*x*b)^3*(24*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*
a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^
(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*
b)^2-12*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2
*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)
*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2+
(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^6+1
2*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4-(ta
n(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^4-12*(
tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2-(tan(1
/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2+(tan(1/
2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2))/(tan(1/2*a+1/2*x*b)^3-tan(1/
2*a+1/2*x*b))^(1/2)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.30

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(32\cos(bx+a)^4-40\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\left(\cos(bx+a)^4-\cos(bx+\right.}{120\left(b\cos(bx+a)^4-b\cos(bx+a)^2\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x +
 a)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b
*cos(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.167.7 Maxima [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos{(bx+a)}}{\sin{(2\,bx+2\,a)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

3.167.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18022 vs. $2(67) = 134$.

Time = 181.76 (sec) , antiderivative size = 18022, normalized size of antiderivative = 228.13

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
-1/960*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*
a)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1
/2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*ta
n(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan
(1/2*b*x) + tan(1/2*a))*((((((((((3*sqrt(2)*tan(1/2*a)^88 + 221*sqrt(2)*t
an(1/2*a)^86 + 4529*sqrt(2)*tan(1/2*a)^84 + 46575*sqrt(2)*tan(1/2*a)^82 +
267014*sqrt(2)*tan(1/2*a)^80 + 611706*sqrt(2)*tan(1/2*a)^78 - 3503654*sqrt
(2)*tan(1/2*a)^76 - 44470106*sqrt(2)*tan(1/2*a)^74 - 259557037*sqrt(2)*tan
(1/2*a)^72 - 1054367027*sqrt(2)*tan(1/2*a)^70 - 3278963927*sqrt(2)*tan(1/2
*a)^68 - 8089589961*sqrt(2)*tan(1/2*a)^66 - 16006283224*sqrt(2)*tan(1/2*a)
^64 - 25186632744*sqrt(2)*tan(1/2*a)^62 - 30337876456*sqrt(2)*tan(1/2*a)^6
0 - 24685712920*sqrt(2)*tan(1/2*a)^58 - 5629982106*sqrt(2)*tan(1/2*a)^56 +
 19969391706*sqrt(2)*tan(1/2*a)^54 + 37658626338*sqrt(2)*tan(1/2*a)^52 + 3
6190152990*sqrt(2)*tan(1/2*a)^50 + 18717018180*sqrt(2)*tan(1/2*a)^48 + 204
0819900*sqrt(2)*tan(1/2*a)^46 + 2040819900*sqrt(2)*tan(1/2*a)^44 + 1871701
8180*sqrt(2)*tan(1/2*a)^42 + 36190152990*sqrt(2)*tan(1/2*a)^40 + 376586263
38*sqrt(2)*tan(1/2*a)^38 + 19969391706*sqrt(2)*tan(1/2*a)^36 - 5629982106*
sqrt(2)*tan(1/2*a)^34 - 24685712920*sqrt(2)*tan(1/2*a)^32 - 30337876456*sq
rt(2)*tan(1/2*a)^30 - 25186632744*sqrt(2)*tan(1/2*a)^28 - 16006283224*sqrt
(2)*tan(1/2*a)^26 - 8089589961*sqrt(2)*tan(1/2*a)^24 - 3278963927*sqrt(...
```

3.167.9 Mupad [B] (verification not implemented)

Time = 23.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}\right)}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^2}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
(4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*
x*6i)*2i - exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(
exp(a*2i + b*x*2i) + 1)^2)
```

3.168 $\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

3.168.1 Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/7*cos(b*x+a)/b/sin(2*b*x+2*a)^(7/2)+6/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(5
/2)-8/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+16/35*sin(b*x+a)/b/sin(2*b*x+2*
a)^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{(5-10\cos(2(a+bx))-4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^4(a+bx)\sec^3(a+bx)\sqrt{\sin(2(a+bx))}}{560b}$$

input
```
Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
 + b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

3.168.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4791, 3042, 4792, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$\frac{6}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{7/2}}dx-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{6}{7}\left(\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(6*((4*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3
*b*Sqrt[Sin[2*a + 2*b*x]])))/5 + Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))
))/7 - Cos[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2))
```

3.168.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.168.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 157.90 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.11

$$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{8}+40\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-t}\right.}{2688b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}-\right.\right.}}$$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

output
```
1/2688/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1
/2*x*b)^2-1)/tan(1/2*a+1/2*x*b)^3*(3*tan(1/2*a+1/2*x*b)^8+40*(tan(1/2*a+1/
2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2
)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^3
-26*tan(1/2*a+1/2*x*b)^6+26*tan(1/2*a+1/2*x*b)^2-3)/(tan(1/2*a+1/2*x*b)*(t
an(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1
/2)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{128\cos(bx+a)^{7}-256\cos(bx+a)^{5}+128\cos(bx+a)^{3}+\sqrt{2}\left(128\cos(bx+a)^{6}-224\cos(bx+a)^{4}+\right.}{560\left(b\cos(bx+a)^{7}-2\,b\cos(bx+a)^{5}+b\cos(bx+a)^{3}\right)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
1/560*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt
(2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt
(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*co
s(b*x + a)^3)
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

3.168.7 Maxima [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

3.168.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.168.9 Mupad [B] (verification not implemented)

Time = 24.20 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.33

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{7\,b\,(e^{a\,2i+b\,x\,2i}\,1i-i)^4} + \frac{e^{a\,3i+b\,x\,3i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,16i}{35\,b\,(e^{a\,2i+b\,x\,2i}+1)\,(e^{a\,2i+b\,x\,2i}\,1i-i)} - \frac{e^{a\,1i+b\,x\,1i}\left(\frac{1}{7\,b}-\frac{8\,e^{a\,2i+b\,x\,2i}}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{(e^{a\,2i+b\,x\,2i}+1)^2\,(e^{a\,2i+b\,x\,2i}\,1i-i)^2} + \frac{e^{a\,1i+b\,x\,1i}\left(\frac{16i}{35\,b}+\frac{e^{a\,2i+b\,x\,2i}\,44i}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{(e^{a\,2i+b\,x\,2i}+1)^3\,(e^{a\,2i+b\,x\,2i}\,1i-i)^3}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(9/2),x)
```

output
```
(exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*16i)/(35*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i)
) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)
*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*
(1/(7*b) - (8*exp(a*2i + b*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (
exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i + b
*x*2i)*1i - 1i)^2) + (exp(a*1i + b*x*1i)*(16i/(35*b) + (exp(a*2i + b*x*2i)
*44i)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1
/2))/((exp(a*2i + b*x*2i) + 1)^3*(exp(a*2i + b*x*2i)*1i - 1i)^3)
```

3.169 $\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

3.169.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{5\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{42b} - \frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} + \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

output
```
-5/42*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi
+b*x),2^(1/2))/b-1/14*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b+1/18*sin(2*b*x
+2*a)^(9/2)/b-5/42*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

3.169.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{240\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}+70\sin(2(a+bx))-156\sin(4(a+bx))-35\sin(6(a+bx}{2016b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 70*Sin[2*(a + b
*x)] - 156*Sin[4*(a + b*x)] - 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] +
7*Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

3.169.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4785, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^{7/2}\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{7/2}dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{5}{7}\int \sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{7}\int \sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{1}{2}\left(\frac{5}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
Sin[2*a + 2*b*x]^(9/2)/(18*b) + ((5*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) -
(Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b)))/7 - (Cos[2*a + 2*b*x]*Si
n[2*a + 2*b*x]^(5/2))/(7*b))/2
```

3.169.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.169. $\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.169.4 Maple [F(-1)]

Timed out.

$$\int \cos (xb+a)^2 \sin (2xb+2a)^{\frac{7}{2}} dx$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

3.169.5 Fricas [F]

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \cos (bx+a)^2 \sin (2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sin(2*b*x +
 2*a)^(3/2), x)
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.169.7 Maxima [F]

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

3.169.8 Giac [F]

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \cos(a+b\,x)^2 \sin(2\,a+2\,b\,x)^{7/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(7/2),x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(7/2), x)
```

3.170 $\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

3.170.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} - \frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b} + \frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

output
```
-3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))/b-1/10*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b+1/14*sin(2*b*x
+2*a)^(7/2)/b
```

3.170.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{84E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}(15\sin(2(a+bx))-14\sin(4(a+bx))-5\sin(6(a+bx)))}{280b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b
*x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(280*b)
```

3.170.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4785, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^{5/2}\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{5/2}dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{1}{2}\left(\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
Sin[2*a + 2*b*x]^(7/2)/(14*b) + ((3*EllipticE[a - Pi/4 + b*x, 2])/(5*b) -
(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(3/2))/(5*b))/2
```

3.170.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.170.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 152.69 (sec) , antiderivative size = 364390662, normalized size of antiderivative = 5281024.09

method	result	size
default	Expression too large to display	364390662

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.170.5 Fricas [F]

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^2\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sqrt(sin(2*
b*x + 2*a)), x)
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.170.7 Maxima [F]

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^2\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

3.170.8 Giac [F]

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx=\int \cos\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx=\int \cos\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(5/2), x)
```

3.171 $\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.171.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

output
```
-1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b+1/10*sin(2*b*x+2*a)^(5/2)/b-1/6*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^(1/2)/b
```

3.171.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\frac{20\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}+9\sin(2(a+bx))-10\sin(4(a+bx))-3\sin(6(a+bx))}{120b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x
)] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)
]])
```

3.171.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4785, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^{3/2}\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{3/2}dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{2}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b
*x]])/(3*b))/2 + Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

3.171.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.171.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 49.13 (sec) , antiderivative size = 198204147, normalized size of antiderivative = 2872523.87

method	result	size
default	Expression too large to display	198204147

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.171.5 Fricas [F]

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.171.7 Maxima [F]

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

3.171.8 Giac [F]

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^2\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(a+b\,x)^2\sin(2\,a+2\,b\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(3/2), x)
```

3.172 $\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.172.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b} + \frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

output
```
-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b+1/6*sin(2*b*x+2*a)^(3/2)/b
```

3.172.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))}{6b}$$

input
```
Integrate[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/(6*b)
```

3.172.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4785, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sin(2a+2bx)}\cos(a+bx)^2dx$$

$$\Big\downarrow \text{ 4785}$$

$$\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx+\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx+\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

input
```
Int[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

3.172.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.172.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 5.16 (sec) , antiderivative size = 26159851, normalized size of antiderivative = 653996.28

method	result	size
default	Expression too large to display	26159851

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.172.5 Fricas [F]

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos\left(bx+a\right)^2\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.172.7 Maxima [F]

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(bx+a)^2\,\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

3.172.8 Giac [F]

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(bx+a)^2\,\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos\left(a+b\,x\right)^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(1/2), x)
```

3.173 $\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}} \, dx$

3.173.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}} \, dx = \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{2b}+\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

output
```
-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b+1/2*sin(2*b*x+2*a)^(1/2)/b
```

3.173.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.56 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}} \, dx = \frac{\left(1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\sin(2(a+bx))}}{2b}$$

input
```
Integrate[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
((1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^
2])*Sqrt[Sin[2*(a + b*x)]])/(2*b)
```

3.173.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4785, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b}$$

input
```
Int[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
EllipticF[a - Pi/4 + b*x, 2]/(2*b) + Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

3.173.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

3.173.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 9.33 (sec) , antiderivative size = 67488705, normalized size of antiderivative = 1687217.62

method	result	size
default	Expression too large to display	67488705

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.173.5 Fricas [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.173.7 Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos(bx+a)^2}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

3.173.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos(bx+a)^2}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=\int \frac{\cos\left(a+b\,x\right)^2}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(1/2), x)
```

3.174 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.174.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b-cos(b*x+a)^2/b/sin(2*b*x+2*a)^(1/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\cot(a+bx)\sqrt{\sin(2(a+bx))}}{2b}$$

input
```
Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/
b
```

3.174.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4783, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{ 4783}$$

$$-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/2*EllipticE[a - Pi/4 + b*x, 2]/b - Cos[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b
*x]])
```

3.174.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

3.174.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 11.49 (sec) , antiderivative size = 107396897, normalized size of antiderivative = 2334715.15

method	result	size
default	Expression too large to display	107396897

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.174.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.39

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{-i\sqrt{2i}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+i\sqrt{-2i}E(\arcsin(\cos(bx+a)-i\sin(}{}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(-I*sqrt(2*I)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*si
n(b*x + a) + I*sqrt(-2*I)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a))
, -1)*sin(b*x + a) + I*sqrt(2*I)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*
x + a)), -1)*sin(b*x + a) - I*sqrt(-2*I)*elliptic_f(arcsin(cos(b*x + a) -
I*sin(b*x + a)), -1)*sin(b*x + a) - 2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
a))*cos(b*x + a))/(b*sin(b*x + a))
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.174.7 Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

3.174.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(a+b\,x)^2}{\sin(2\,a+2\,b\,x)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(3/2), x)
```

3.175 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.175.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
-1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/3*cos(b*x+a)^2/b/sin(2*b*x+2*a)^(3/2)
```

3.175.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.89 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\left(\csc^2(a+bx)-2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\sin(2(a+bx))}}{12b}$$

input
```
Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/12*((Csc[a + b*x]^2 - 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^
2]*Sqrt[Sec[a + b*x]^2])*Sqrt[Sin[2*(a + b*x)]])/b
```

3.175.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4783, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx - \frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx - \frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{6b} - \frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
EllipticF[a - Pi/4 + b*x, 2]/(6*b) - Cos[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^
(3/2))
```

3.175.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

3.175.4 Maple [A] (verified)

Time = 27.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.56

method	result
default	$\frac{\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)\sin(2xb+2a)-2\cos(2xb+2a)^2-2\cos(2x}{12\sin(2xb+2a)^{\frac{3}{2}}\cos(2xb+2a)b}$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin
(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^
(1/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

3.175.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.15

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2i}\left(\cos\left(bx+a\right)^2-1\right)F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+\sqrt{-2i}\left(\cos\left(bx+a\right)^2-1\right)F(\arcs}{12\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(2*I)*(cos(b*x + a)^2 - 1)*elliptic_f(arcsin(cos(b*x + a) + I*s
in(b*x + a)), -1) + sqrt(-2*I)*(cos(b*x + a)^2 - 1)*elliptic_f(arcsin(cos(
b*x + a) - I*sin(b*x + a)), -1) - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))
/(b*cos(b*x + a)^2 - b)
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.175.7 Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

3.175.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{5/2}}\,dx$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(5/2), x)
```

3.176 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

3.176.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} - \frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

output
```
3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/5*cos(b*x+a)^2/b/sin(2*b*x+2*a)^(5/2)-3/10*cos(2*b*x+2*a
)/b/sin(2*b*x+2*a)^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{-12E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{2(1-6\cos(2(a+bx))+3\cos(4(a+bx)))\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))}}{40b}$$

input
```
Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(
a + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(40*b)
```

3.176.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4783, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\int \frac{1}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{10}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3}{10}\left(-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(3*(-(EllipticE[a - Pi/4 + b*x, 2]/b) - Cos[2*a + 2*b*x]/(b*Sqrt[Sin[2*a +
 2*b*x]])))/10 - Cos[a + b*x]^2/(5*b*Sin[2*a + 2*b*x]^(5/2))
```

3.176.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

3.176.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(92) = 184$.

Time = 145.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.95

method	result
default	$\frac{\sqrt{2}\left(-\frac{8\sqrt{2}}{5\sin(2xb+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\sin(2xb+2a)^2\,\mathrm{EllipticE}\left(\sqrt{\sin(2xb+2a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(2}{5\sin(2x}\right.}{32b}$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)
^(5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x
+2*a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/
2))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a
))^(1/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+
6*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

3.176.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.45

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx =$$

$$-\frac{6\sqrt{2i}\left(i\,\cos\left(bx+a\right)^3-i\,\cos\left(bx+a\right)\right)E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)\sin\left(bx+a\right)+6\sqrt{}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/40*(6*sqrt(2*I)*(I*cos(b*x + a)^3 - I*cos(b*x + a))*elliptic_e(arcsin(c
os(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 6*sqrt(-2*I)*(-I*cos(b*x
 + a)^3 + I*cos(b*x + a))*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a))
, -1)*sin(b*x + a) + 6*sqrt(2*I)*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*elli
ptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 6*sqrt(-2
*I)*(I*cos(b*x + a)^3 - I*cos(b*x + a))*elliptic_f(arcsin(cos(b*x + a) - I
*sin(b*x + a)), -1)*sin(b*x + a) + sqrt(2)*(12*cos(b*x + a)^4 - 18*cos(b*x
 + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)))/((b*cos(b*x + a)^3 - b*cos(b
*x + a))*sin(b*x + a))
```

3.176.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.176.7 Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

3.176.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{7/2}}\,dx$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(7/2), x)
```

3.177 $\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.177.1 Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{7\arcsin(\cos(a+bx)-\sin(a+bx))}{64b} + \frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{64b} - \frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{32b} + \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} + \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

output
```
-7/64*arcsin(cos(b*x+a)-sin(b*x+a))/b+7/64*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+7/48*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/12*cos(b*x+a)*s
in(2*b*x+2*a)^(5/2)/b-7/32*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.177.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$=\frac{-7\arcsin(\cos(a+bx)-\sin(a+bx))+7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-\frac{2}{3}(10\cos(}{64b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 7*Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2
*Cos[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(64*b)
```

3.177.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4785, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\cos^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(2a+2bx)^{3/2}\cos(a+bx)^3dx$$
$$\Big\downarrow \text{4785}$$
$$\frac{7}{12}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{7}{12}\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$
$$\Big\downarrow \text{4789}$$

$$\frac{7}{12}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{12}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{4790}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{4793}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\sqrt{\sin(2a\right.\right.$$
$$\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b) + (7*((3*((-1/2*ArcSin[Cos[a
+ b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a
+ 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 + (S
in[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/12
```

3.177. $\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

3.177.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.177.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 204.95 (sec) , antiderivative size = 523275790, normalized size of antiderivative = 3847616.10

method	result	size
default	Expression too large to display	523275790

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(118) = 236$.

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.14

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{8\sqrt{2}\left(32\cos(bx+a)^5-4\cos(bx+a)^3-7\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-42\arctan\left(-\frac{\sqrt{2}}{\cdots}\right.}{\cdots}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/768*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*
sqrt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(co
s(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*s
qrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a
) - sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)
^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/
b
```

3.177.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.177.7 Maxima [F]

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^3\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

3.177.8 Giac [F]

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^3\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(a+b\,x)^3\sin(2\,a+2\,b\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(3/2), x)
```

3.178 $\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

3.178.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{32b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b} + \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b} + \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

output
```
-5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+1/8*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+5/16*sin(b*x+a)*si
n(2*b*x+2*a)^(1/2)/b
```

3.178.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{-5\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin(2}}{32b}$$

input
```
Integrate[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + S
in[3*(a + b*x)]))/(32*b)
```

3.178.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4785, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(2a+2bx)}\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{5}{8}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{8}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{4789}$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{4794}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx)\ldots}{}\right.$$
$$\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

input
```
Int[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(5*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)))/8 + (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

3.178.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.178.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 18.15 (sec) , antiderivative size = 103475646, normalized size of antiderivative = 940687.69

method	result	size
default	Expression too large to display	103475646

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(96) = 192.

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.55

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{8\sqrt{2}\left(4\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx}{\cos(bx+a)^2+2\,c}\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*si
n(b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x
+ a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(
b*x + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) +
5*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2
 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*
cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

3.178.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.178.7 Maxima [F]

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(bx+a)^3\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

3.178.8 Giac [F(-2)]

Exception generated.

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]e
xt_reduce Error: Bad Argument TypeDone
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos\left(a+b\,x\right)^3\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(1/2), x)
```

3.179 $\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

3.179.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b} + \frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

output
```
-3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+1/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

3.179.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{-3\arcsin(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+\csc(a+b}{8b}$$

input
```
Integrate[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/(8*b)
```

3.179.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4785, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{4793}$$

$$\frac{3}{4}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(3*(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a
+ b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)))/4 + (Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b)
```

3.179.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.179.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 36.61 (sec) , antiderivative size = 213968312, normalized size of antiderivative = 2547241.81

method	result	size
default	Expression too large to display	213968312

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.179.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(74) = 148$.

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.19

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a) + 6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\,\mathrm{s}}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + co
s(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1
)) - 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) -
 sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 +
 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos
(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*
x + a)*sin(b*x + a) + 1))/b
```

3.179.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

3.179.7 Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos{(bx+a)}^3}{\sqrt{\sin{(2\,bx+2\,a)}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

3.179.8 Giac [F]

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos{(bx+a)}^3}{\sqrt{\sin{(2\,bx+2\,a)}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos{(a+b\,x)}^3}{\sqrt{\sin{(2\,a+2\,b\,x)}}}\,dx$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(1/2), x)
```

3.180 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.180.1 Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} - \frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b-cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\csc(a+bx)\sqrt{}}{4b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

3.180.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4781, 3042, 4795, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4781}$$

$$-\frac{1}{4}\int \sec(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}\int \frac{\sqrt{\sin(2a+2bx)}}{\cos(a+bx)}dx-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{4795}$$

$$-\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{4794}$$

$$\frac{1}{2}\left(\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) + Log[Cos[a + b*x] + Sin[a + b*
x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b
*x]])
```

3.180.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4781
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(p + 1))), x] + Simp[e^4*((m + p - 1)/(4*g^2*(p + 1)))   Int[(e*Co
s[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d,
e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
 && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4795
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.180. $\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

3.180.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 35.44 (sec) , antiderivative size = 211562444, normalized size of antiderivative = 2580029.80

method	result	size
default	Expression too large to display	211562444

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.60

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos}}{}\right.}{}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) -
sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a
)*sin(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)
*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)
))*sin(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (
4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin
(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x
 + a) + 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin
(b*x + a))
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

3.180.7 Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

3.180.8 Giac [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(3/2), x)
```

3.181 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

3.181.1 Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
-1/3*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(3/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2(a+bx))}{24b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/24*(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/b
```

3.181.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4779}$$

$$-\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/3*Cos[a + b*x]^3/(b*Sin[2*a + 2*b*x]^(3/2))
```

3.181.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

3.181.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 68.59 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.86

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(4\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1},\right.\right.}{24\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,b}$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x
*b)^2-1)/tan(1/2*a+1/2*x*b)*(4*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+
1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)+tan(1/2*a+1/2*x*b)^4-1)/(tan(1/
2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2
*a+1/2*x*b))^(1/2)/b
```

3.181.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+\cos(bx+a)^2-1}{12\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^
2 - 1)/(b*cos(b*x + a)^2 - b)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

3.181.7 Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

3.181.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15292 vs. $2(24) = 48$.

Time = 104.03 (sec) , antiderivative size = 15292, normalized size of antiderivative = 546.14

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
1/48*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*(((((((sqrt(2)*tan(1/2*a)^57 + 50*sqrt(2)*tan(1/2*a)
^55 + 516*sqrt(2)*tan(1/2*a)^53 + 1750*sqrt(2)*tan(1/2*a)^51 - 5215*sqrt(2
)*tan(1/2*a)^49 - 77796*sqrt(2)*tan(1/2*a)^47 - 386576*sqrt(2)*tan(1/2*a)^
45 - 1186876*sqrt(2)*tan(1/2*a)^43 - 2515251*sqrt(2)*tan(1/2*a)^41 - 37599
30*sqrt(2)*tan(1/2*a)^39 - 3812844*sqrt(2)*tan(1/2*a)^37 - 2196894*sqrt(2)
*tan(1/2*a)^35 - 36499*sqrt(2)*tan(1/2*a)^33 + 824296*sqrt(2)*tan(1/2*a)^3
1 - 824296*sqrt(2)*tan(1/2*a)^27 + 36499*sqrt(2)*tan(1/2*a)^25 + 2196894*s
qrt(2)*tan(1/2*a)^23 + 3812844*sqrt(2)*tan(1/2*a)^21 + 3759930*sqrt(2)*tan
(1/2*a)^19 + 2515251*sqrt(2)*tan(1/2*a)^17 + 1186876*sqrt(2)*tan(1/2*a)^15
 + 386576*sqrt(2)*tan(1/2*a)^13 + 77796*sqrt(2)*tan(1/2*a)^11 + 5215*sqrt(
2)*tan(1/2*a)^9 - 1750*sqrt(2)*tan(1/2*a)^7 - 516*sqrt(2)*tan(1/2*a)^5 - 5
0*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^51 +
 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*
a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 +
 389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 5348
88*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888...
```

3.181.9 Mupad [B] (verification not implemented)

Time = 22.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.36

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(\frac{2\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{3}-\sin\left(\frac{3\,a}{2}+\frac{3\,b\,x}{2}\right)^2+\frac{\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2}{3}\right)}{b\,\left(30\sin\left(a+b\,x\right)^2-12\sin\left(2\,a+2\,b\,x\right)^2+2\sin\left(3\,a+3\,b\,x\right)^2\right)}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
(sin(2*a + 2*b*x)^(1/2)*((2*sin(a/2 + (b*x)/2)^2)/3 - sin((3*a)/2 + (3*b*x
)/2)^2 + sin((5*a)/2 + (5*b*x)/2)^2/3))/(b*(2*sin(3*a + 3*b*x)^2 - 12*sin(
2*a + 2*b*x)^2 + 30*sin(a + b*x)^2))
```

3.182 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

3.182.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/5*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(5/2)-1/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(
1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=-\frac{\csc(a+bx)\left(4+\csc^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{40b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/40*(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/b
```

3.182.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4783, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{1}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/5*Cos[a + b*x]^3/(b*Sin[2*a + 2*b*x]^(5/2)) - Cos[a + b*x]/(5*b*Sqrt[Si
n[2*a + 2*b*x]])
```

3.182.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

3.182.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 230.85 (sec) , antiderivative size = 482, normalized size of antiderivative = 8.76

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1}}\left(16\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{t\cdots}\right.\right.}{}$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/160*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/tan(1/2*a+1/2*x
*b)^3*(16*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1
/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/
2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^
2-8*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b
)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*Ell
ipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-(tan
(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^6+(tan(
1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^4+8*(tan
(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4+(tan(1/2*
a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2-8*(tan(1/2
*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2-(tan(1/2*a+1/
2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2))/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/
2*x*b))^(1/2)/b
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(4\cos(bx+a)^2-5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+4\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{40\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/40*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*
(cos(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

3.182.7 Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

3.182.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.182.9 Mupad [B] (verification not implemented)

Time = 23.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{5\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
-(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(exp(a*4i + b*x*4i)*1i - exp(a*2i + b*x*2i)*3i + 1i))/(5*b*(exp
(a*2i + b*x*2i) - 1)^3)
```

3.183 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

3.183.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx=-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/7*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(7/2)-2/21*cos(b*x+a)/b/sin(2*b*x+2*a)^
(3/2)+4/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx =\frac{(5-12\cos(2(a+bx))+4\cos(4(a+bx)))\csc^4(a+bx)\sec(a+bx)\sqrt{\sin(2(a+bx))}}{336b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x
]*Sqrt[Sin[2*(a + b*x)]])/(336*b)
```

3.183.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4783, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{2}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{2}{7}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{2}{7}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(2*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*S
qrt[Sin[2*a + 2*b*x]])))/7 - Cos[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2))
```

3.183. $\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$

3.183.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.183.4 Maple [F(-1)]

Timed out.

$$\int \frac{\cos(xb+a)^3}{\sin(2xb+2a)^{\frac{9}{2}}}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.28

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$$
$$= \frac{32\,\cos{(bx+a)}^5 - 64\,\cos{(bx+a)}^3 + \sqrt{2}\left(32\,\cos{(bx+a)}^4 - 56\,\cos{(bx+a)}^2 + 21\right)\sqrt{\cos{(bx+a)}\sin{(bx}}}{336\left(b\cos{(bx+a)}^5 - 2\,b\cos{(bx+a)}^3 + b\cos{(bx+a)}\right)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
1/336*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4
- 56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a
))/(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

3.183.7 Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\cos{(bx+a)}^3}{\sin{(2\,bx+2\,a)}^{\frac{9}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

3.183.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.183.9 Mupad [B] (verification not implemented)

Time = 23.99 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.73

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{5\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{84\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2} + \frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,3i}{14\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3} - \frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4} - \frac{e^{a\,1i+b\,x\,1i}\left(\frac{5i}{84\,b}-\frac{e^{a\,2i+b\,x\,2i}\,4i}{21\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(9/2),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*3i)/(14*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (5*exp(a*1i + b*x*1i
)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(84*b*(
exp(a*2i + b*x*2i)*1i - 1i)^2) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i
)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i -
1i)^4) - (exp(a*1i + b*x*1i)*(5i/(84*b) - (exp(a*2i + b*x*2i)*4i)/(21*b))*
((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2
i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
```

3.184 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

3.184.1 Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} - \frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/9*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(9/2)-1/15*cos(b*x+a)/b/sin(2*b*x+2*a)^
(5/2)+4/45*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-8/45*cos(b*x+a)/b/sin(2*b*x+2
*a)^(1/2)
```

3.184.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin(2(a+bx))}(113\csc(a+bx)+17\csc^3(a+bx)+5\csc^5(a+bx)-15\sec(a+bx)\tan(a+bx))}{1440b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

output
```
-1/1440*(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*
Csc[a + b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/b
```

3.184.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4783, 3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{11/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{1}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{1}{3}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

output
```
((4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqr
t[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)))/3 -
Cos[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2))
```

3.184.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

3.184.4 Maple [F(-1)]

Timed out.

$$\int \frac{\cos \left(xb+a\right)^{3}}{\sin \left(2xb+2a\right)^{\frac{11}{2}}}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

output
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(128\,\cos\left(bx+a\right)^{6}-288\,\cos\left(bx+a\right)^{4}+180\,\cos\left(bx+a\right)^{2}-15\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+128\,(}{1440\,\left(b\cos\left(bx+a\right)^{6}-2\,b\cos\left(bx+a\right)^{4}+b\cos\left(bx+a\right)^{2}}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

output
```
-1/1440*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x +
a)^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b
*x + a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x
 + a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

output
```
Timed out
```

3.184.7 Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

3.184.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

output
```
Timed out
```

3.184.9 Mupad [B] (verification not implemented)

Time = 25.36 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.58

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{60\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3} - \frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,2i}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4} + \frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^5} + \frac{8\,e^{a\,3i+b\,x\,3i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{45\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)} - \frac{e^{a\,1i+b\,x\,1i}\,\left(\frac{49i}{180\,b}+\frac{e^{a\,2i+b\,x\,2i}\,19i}{180\,b}\right)\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(11/2),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2))/(9*b*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (exp(a*1i + b*x*1i)*((ex
p(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*2i)/(9*b*(exp(
a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i
)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(60*b*(exp(a*2i + b*x*2i)*1i - 1i)
^3) + (8*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x
*2i)*1i)/2)^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i -
 1i)) - (exp(a*1i + b*x*1i)*(49i/(180*b) + (exp(a*2i + b*x*2i)*19i)/(180*b
))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(
a*2i + b*x*2i) + 1)^2*(exp(a*2i + b*x*2i)*1i - 1i)^2)
```

3.185 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$

3.185.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) + \frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

output
```
-1/2*arcsin(cos(x)-sin(x))+1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

3.185.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{2}\left(-\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

input
```
Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

3.185.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))$$

input
```
Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

output
```
-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

3.185.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.185.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

method	result	size
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)}}$	98

input
```
int(cos(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*
2^(1/2))
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) - \frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{8}\log\left(-32\cos(x)^4 + 4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2+16\cos(x)\sin(x)+1\right)$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x)
)/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*s
in(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sq
rt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)
) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)/sin(2*x)**(1/2),x)
```

output
```
Timed out
```

3.185.7 Maxima [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(x)/sqrt(sin(2*x)), x)
```

3.185.8 Giac [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(x)/sqrt(sin(2*x)), x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2\,x)}}\,dx$$

input
```
int(cos(x)/sin(2*x)^(1/2),x)
```

output
```
int(cos(x)/sin(2*x)^(1/2), x)
```

3.186 $\int \csc(x)\sqrt{\sin(2x)}\,dx$

3.186.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = -\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

output
```
-arcsin(cos(x)-sin(x))+ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

3.186.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = -\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

input
```
Integrate[Csc[x]*Sqrt[Sin[2*x]],x]
```

output
```
-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

3.186.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4796, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2x)} \csc(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin(2x)}}{\sin(x)} dx$$

$$\Big\downarrow \text{4796}$$

$$2 \int \frac{\cos(x)}{\sqrt{\sin(2x)}} dx$$

$$\Big\downarrow \text{3042}$$

$$2 \int \frac{\cos(x)}{\sqrt{\sin(2x)}} dx$$

$$\Big\downarrow \text{4793}$$

$$2 \left(\frac{1}{2} \log\left(\sin(x) + \sqrt{\sin(2x)} + \cos(x)\right) - \frac{1}{2} \arcsin(\cos(x) - \sin(x))\right)$$

input
```
Int[Csc[x]*Sqrt[Sin[2*x]],x]
```

output
```
2*(-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2)
```

3.186.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.186.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

method	result	size
default	$\frac{2\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)}}$	99

input
```
int(csc(x)*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1
/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x
))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/
2*2^(1/2))
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.48

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \frac{1}{2}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) - \frac{1}{2}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{4}\log\left(-32\cos(x)^4 + 4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)}+32\cos(x)^2 + 16\cos(x)\sin(x)+1\right)$$

input
```
integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x)
)/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/2*arctan(-(2*sqrt(2)*sqrt(cos(x)*s
in(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/4*log(-32*cos(x)^4 + 4*sq
rt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)
) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \text{Timed out}$$

input
```
integrate(csc(x)*sin(2*x)**(1/2),x)
```

output
```
Timed out
```

3.186.7 Maxima [F]

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \int \csc(x)\sqrt{\sin(2x)}\,dx$$

input
```
integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(x)*sqrt(sin(2*x)), x)
```

3.186.8 Giac [F]

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \int \csc(x)\sqrt{\sin(2x)}\,dx$$

input
```
integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(x)*sqrt(sin(2*x)), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \int \frac{\sqrt{\sin(2x)}}{\sin(x)}\,dx$$

input
```
int(sin(2*x)^(1/2)/sin(x),x)
```

output
```
int(sin(2*x)^(1/2)/sin(x), x)
```

3.187 $\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx$

3.187.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx = -\frac{\cos^3(a+bx)\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{4+m}{2},\frac{6+m}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+}{b(4+m)}$$

output
```
-cos(b*x+a)^3*cot(b*x+a)*hypergeom([2+1/2*m, -1/2*m+1/2],[3+1/2*m],cos(b*x
+a)^2)*(sin(b*x+a)^2)^(-1/2*m+1/2)*sin(2*b*x+2*a)^m/b/(4+m)
```

3.187.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 13.91 (sec) , antiderivative size = 2472, normalized size of antiderivative = 29.08

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

output
```
(2^(1 + m)*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m, 2*(2 + m), (3 + m)/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 1 +
2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(1
+ m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*
Cos[a + b*x]^3*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x
)/2] + Sin[(3*(a + b*x))/2]))^m*Sin[2*(a + b*x)]^m*Tan[(a + b*x)/2])/(b*(1
 + m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m*((2^m*(6*AppellF1[(1 + m)/2, -m,
 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*Appell
F1[(1 + m)/2, -m, 2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)
/2]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2] - 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(
a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(1 + 2*m)*(Cos[(
a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m)/((1 + m)*(Cos[a
 + b*x]*Sec[(a + b*x)/2]^2)^m) + (2^(1 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2
*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1
[(1 + m)/2, -m, 2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2] - 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a +...
```

3.187.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4797, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)^3\sin(2a+2bx)^m dx$$

$$\Big\downarrow \text{4797}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^{m+3}(a+bx)\sin^m(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int\cos(a+bx)^{m+3}\sin(a+bx)^m dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos^3(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+4}{2},\frac{m+6}{2},\cos^2(a+bx)\right)}{b(m+4)}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

output
```
-((Cos[a + b*x]^3*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6
+ m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(
b*(4 + m)))
```

3.187.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 4797
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p)   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.187.4 Maple [F]

$$\int \cos(xb+a)^3 \sin(2xb+2a)^m \, dx$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

output
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

3.187.5 Fricas [F]

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\, dx = \int \sin(2\,bx+2\,a)^m \cos(bx+a)^3 \, dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

3.187.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\, dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**m,x)
```

output
```
Timed out
```

3.187.7 Maxima [F]

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m \cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

3.187.8 Giac [F]

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m \cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx = \int \cos(a+b\,x)^3 \sin(2\,a+2\,b\,x)^m\,dx$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^m,x)
```

output
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^m, x)
```

3.188 $\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx$

3.188.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = -\frac{\cos^2(a+bx)\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{3+m}{2},\frac{5+m}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+}{b(3+m)}$$

output
```
-cos(b*x+a)^2*cot(b*x+a)*hypergeom([3/2+1/2*m, -1/2*m+1/2],[5/2+1/2*m],cos
(b*x+a)^2)*(sin(b*x+a)^2)^(-1/2*m+1/2)*sin(2*b*x+2*a)^m/b/(3+m)
```

3.188.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1+m}{2},2+m,\frac{3+m}{2},-\tan^2(a+bx)\right)\sec^2(a+bx)^m\sin^m(2(a+bx))\tan(a+bx)}{b(1+m)}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

output
```
(Hypergeometric2F1[(1 + m)/2, 2 + m, (3 + m)/2, -Tan[a + b*x]^2]*(Sec[a +
b*x]^2)^m*Sin[2*(a + b*x)]^m*Tan[a + b*x])/(b*(1 + m))
```

3.188.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4797, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)^2\sin(2a+2bx)^m dx$$

$$\Big\downarrow \text{4797}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^{m+2}(a+bx)\sin^m(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos(a+bx)^{m+2}\sin(a+bx)^m dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos^2(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+3}{2},\frac{m+5}{2},\cos^2(a+bx)\right)}{b(m+3)}$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

output
```
-((Cos[a + b*x]^2*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5
+ m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(
b*(3 + m)))
```

3.188.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 4797
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p)   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.188.4 Maple [F]

$$\int \cos\left(xb+a\right)^{2} \sin\left(2xb+2a\right)^{m} dx$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

output
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

3.188.5 Fricas [F]

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{m} \cos\left(bx+a\right)^{2}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

3.188.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

output
```
Timed out
```

3.188.7 Maxima [F]

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m\cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

3.188.8 Giac [F]

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m\cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \int \cos\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^m\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^m,x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^m, x)
```

3.189 $\int \cos(a+bx)\sin^m(2a+2bx)\,dx$

3.189.1 Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx = -\frac{\cos(a+bx)\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+}{b(2+m)}$$

output
```
-cos(b*x+a)*cot(b*x+a)*hypergeom([1+1/2*m, -1/2*m+1/2],[2+1/2*m],cos(b*x+a
)^2)*(sin(b*x+a)^2)^(-1/2*m+1/2)*sin(2*b*x+2*a)^m/b/(2+m)
```

3.189.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 5.28 (sec) , antiderivative size = 567, normalized size of antiderivative = 6.83

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx$$

$$= \frac{}{2b(1+m)\left(2(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},-m,2(1+m),\frac{3+m}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-(3+r\right.}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

output
```
((3 + m)*(2*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sin[2*(a + b*x)]^(1 + m))/(2*b*(1 +
 m)*(2*(3 + m)*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)
/2]^2, -Tan[(a + b*x)/2]^2] - (3 + m)*AppellF1[(1 + m)/2, -m, 1 + 2*m, (3
+ m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*(-2*m*AppellF1[(3 + m
)/2, 1 - m, 2*(1 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
 + m*AppellF1[(3 + m)/2, 1 - m, 1 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] + AppellF1[(3 + m)/2, -m, 2*(1 + m), (5 + m)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*m*AppellF1[(3 + m)/2, -m, 2*(1 + m),
 (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(3 + m)/
2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*m*
AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2])*Tan[(a + b*x)/2]^2))
```

3.189.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4797, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)\sin(2a+2bx)^m dx$$

$$\Big\downarrow \text{4797}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos^{m+1}(a+bx)\sin^m(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-m}(a+bx)\sin^m(2a+2bx)\cos^{-m}(a+bx)\int \cos(a+bx)^{m+1}\sin(a+bx)^m dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(a+bx)\right)}{b(m+2)}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

output
```
-((Cos[a + b*x]*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 +
m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(2 + m)))
```

3.189.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 4797
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p)   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

3.189.4 Maple [F]

$$\int \cos(xb + a) \sin(2xb + 2a)^m \, dx$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^m,x)
```

output
```
int(cos(b*x+a)*sin(2*b*x+2*a)^m,x)
```

3.189.5 Fricas [F]

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^m \cos\left(bx+a\right)\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

3.189.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**m,x)
```

output
```
Timed out
```

3.189.7 Maxima [F]

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^m \cos\left(bx+a\right)\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

3.189.8 Giac [F]

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^m\cos(bx+a)\;dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx = \int \cos(a+b\,x)\;\sin(2\,a+2\,b\,x)^m\,dx$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^m,x)
```

output
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^m, x)
```

3.190 $\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx$

3.190.1 Optimal result

Integrand size = 28, antiderivative size = 46

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^9(a+bx)}{9b}$$

output
```
-4/5*cos(b*x+a)^5/b+8/7*cos(b*x+a)^7/b-4/9*cos(b*x+a)^9/b
```

3.190.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{\cos^5(a+bx)(-249+220\cos(2(a+bx))-35\cos(4(a+bx)))}{630b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(630*
b)
```

3.190.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4800, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^2\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4800}$$

$$4\int \cos^4(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^4\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{4\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{4\int \left(\cos^8(a+bx)-2\cos^6(a+bx)+\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\left(\frac{1}{9}\cos^9(a+bx)-\frac{2}{7}\cos^7(a+bx)+\frac{1}{5}\cos^5(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*(Cos[a + b*x]^5/5 - (2*Cos[a + b*x]^7)/7 + Cos[a + b*x]^9/9))/b
```

3.190.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4800
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((f_.)*sin[(a_.) + (b_.)*(x_)])^(
n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_Symbol] :> Simp[2^p/(e^p*f^p)   Int[(
e*Cos[a + b*x])^(m + p)*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]
```

3.190.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

method	result	size
parallelrisch	$\frac{-2048-1890\cos(xb+a)+45\cos(7xb+7a)+252\cos(5xb+5a)-35\cos(9xb+9a)-420\cos(3xb+3a)}{20160b}$	60
default	$-\frac{3\cos(xb+a)}{32b}-\frac{\cos(3xb+3a)}{48b}+\frac{\cos(5xb+5a)}{80b}+\frac{\cos(7xb+7a)}{448b}-\frac{\cos(9xb+9a)}{576b}$	69
risch	$-\frac{3\cos(xb+a)}{32b}-\frac{\cos(3xb+3a)}{48b}+\frac{\cos(5xb+5a)}{80b}+\frac{\cos(7xb+7a)}{448b}-\frac{\cos(9xb+9a)}{576b}$	69

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/20160*(-2048-1890*cos(b*x+a)+45*cos(7*b*x+7*a)+252*cos(5*b*x+5*a)-35*cos
(9*b*x+9*a)-420*cos(3*b*x+3*a))/b
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5\right)}{315\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas"
)
```

output
```
-4/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

3.190.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(39) = 78$.

Time = 10.97 (sec) , antiderivative size = 318, normalized size of antiderivative = 6.91

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = \begin{cases} -\frac{8\sin^5(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{315b} + \frac{16\sin^4(a+bx)\sin^2(2a+2bx)\cos(a+bx)}{315b} - \frac{16\sin^4(a+bx)\cos(a+bx)\cos^2(2a+2bx)}{315b} + \frac{44\sin \\ x\sin^3(a)\sin^2(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((-8*sin(a + b*x)**5*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(315*b) +
16*sin(a + b*x)**4*sin(2*a + 2*b*x)**2*cos(a + b*x)/(315*b) - 16*sin(a + b
*x)**4*cos(a + b*x)*cos(2*a + 2*b*x)**2/(315*b) + 44*sin(a + b*x)**3*sin(2
*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)/(315*b) - 113*sin(a + b*x)**2
*sin(2*a + 2*b*x)**2*cos(a + b*x)**3/(315*b) + 8*sin(a + b*x)**2*cos(a + b
*x)**3*cos(2*a + 2*b*x)**2/(315*b) - 88*sin(a + b*x)*sin(2*a + 2*b*x)*cos(
a + b*x)**4*cos(2*a + 2*b*x)/(315*b) - 2*sin(2*a + 2*b*x)**2*cos(a + b*x)*
*5/(63*b) - 32*cos(a + b*x)**5*cos(2*a + 2*b*x)**2/(315*b), Ne(b, 0)), (x*
sin(a)**3*sin(2*a)**2*cos(a)**2, True))
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{35\cos(9bx+9a)-45\cos(7bx+7a)-252\cos(5bx+5a)+420\cos(3bx+3a)+1890\cos(bx+a)}{20160b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima"
)
```

output
```
-1/20160*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
 + 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5\right)}{315b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-4/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

3.190.9 Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(35\cos(a+bx)^9-90\cos(a+bx)^7+63\cos(a+bx)^5\right)}{315b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

output
```
-(4*(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9))/(315*b)
```

3.191 $\int \sin(a+bx)\sin^n(c+dx)\,dx$

3.191.1 Optimal result

Integrand size = 15, antiderivative size = 293

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = -\frac{2^{-1-n}e^{i(a-cn)+i(b-dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,\frac{b-dn}{2d},\frac{1}{2}\right.}{b-dn} - \frac{2^{-1-n}e^{-i(a+cn)-i(b+dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{b+d}{2d}\right.}{b+dn}$$

output
```
-2^(-1-n)*exp(I*(-c*n+a)+I*(-d*n+b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp
(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],exp(2*I*(d
*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(-d*n+b)-2^(-1-n)*exp(-I*(c*n+a)-I*(d*n
+b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, 1/
2*(-d*n-b)/d],[1+1/2*(-d*n-b)/d],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))
^n)/(d*n+b)
```

3.191.2 Mathematica [A] (warning: unable to verify)

Time = 2.76 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.62

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = \frac{i2^{-1-n}e^{-i(a-c+(b-d)x)}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^{1+n}\left((b-dn)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{b}{d}+n\right),-\right.\right.}{(b-dn)(b+}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + d*x]^n,x]
```

output
```
(I*2^(-1 - n)*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^(1 + n)*
((b - d*n)*Hypergeometric2F1[1, (2 - b/d + n)/2, -1/2*(b + d*(-2 + n))/d,
E^((2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(b + d*n)*Hypergeometric2F1[1,
(b + d*(2 + n))/(2*d), (2 + b/d - n)/2, E^((2*I)*(c + d*x))]))/(E^(I*(a -
c + (b - d)*x))*(b - d*n)*(b + d*n))
```

3.191.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^n(c+dx)\,dx$$

$$\Big\downarrow \text{ 5064}$$

$$2^{-n-1}\int \left(ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n-ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2^{-n-1}\left(-\frac{\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(-n,\frac{b-dn}{2d},\frac{1}{2}\left(\frac{b}{d}-n+2\right),e^{2i(c+dx)}\right)\exp(i\right.}{b-dn}$$

input
```
Int[Sin[a + b*x]*Sin[c + d*x]^n,x]
```

output
```
2^(-1 - n)*(-((E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c
+ d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 +
 b/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(b - d
*n))) - (E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(I/E^(I*(c + d
*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(b + d*n)/d, 1 - (b
 + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(b +
 d*n)))
```

3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

3.191.4 Maple [F]

$$\int \sin(xb+a)\sin(dx+c)^n\,dx$$

input
```
int(sin(b*x+a)*sin(d*x+c)^n,x)
```

output
```
int(sin(b*x+a)*sin(d*x+c)^n,x)
```

3.191.5 Fricas [F]

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = \int \sin(dx+c)^n\sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="fricas")
```

output
```
integral(sin(d*x + c)^n*sin(b*x + a), x)
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)**n,x)
```

output
```
Timed out
```

3.191.7 Maxima [F]

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = \int \sin\left(dx+c\right)^n \sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^n*sin(b*x + a), x)
```

3.191.8 Giac [F]

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = \int \sin\left(dx+c\right)^n \sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^n*sin(b*x + a), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = \int \sin\left(a+b\,x\right)\,\sin\left(c+d\,x\right)^n\,dx$$

input
```
int(sin(a + b*x)*sin(c + d*x)^n,x)
```

output
```
int(sin(a + b*x)*sin(c + d*x)^n, x)
```

3.192 $\int \sin(a+bx)\sin^3(c+dx)\,dx$

3.192.1 Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = -\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)} -\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
-1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)-3/8*sin(a+c+(
b+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.192.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = \frac{1}{8}\left(-\frac{\sin(a-3c+bx-3dx)}{b-3d}+\frac{3\sin(a-c+bx-dx)}{b-d} +\frac{\sin(a+3c+bx+3dx)}{b+3d}-\frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

output
```
(-(Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sin[a - c + b*x - d*x])/(b -
 d) + Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sin[a + c + (b + d)*x])/(b
 + d))/8
```

3.192.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^3(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{8}\cos(a+x(b-3d)-3c)+\frac{3}{8}\cos(a+x(b-d)-c)-\frac{3}{8}\cos(a+x(b+d)+c)+\frac{1}{8}\cos(a+x(b+3d)+3c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

output
```
-1/8*Sin[a - 3*c + (b - 3*d)*x]/(b - 3*d) + (3*Sin[a - c + (b - d)*x])/(8*
(b - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d
)*x]/(8*(b + 3*d))
```

3.192.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.192.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8b+24d}$
risch	$-\frac{\sin(xb-3dx+a-3c)}{8(b-3d)}+\frac{3\sin(xb-dx+a-c)}{8(b-d)}-\frac{3\sin(xb+dx+a+c)}{8(b+d)}+\frac{\sin(xb+3dx+a+3c)}{8b+24d}$
parallelrisch	$\frac{12d^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+12b\,d^2\left(1-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+12(-2b^2d+3d^3)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+8b(\text{t}}{(b-d)(b+3d)}$

input
```
int(sin(b*x+a)*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)-3/8*sin(a+c+(
b+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.34

$$\int \sin(a+bx)\sin^3(c+dx)\,dx =$$
$$-\frac{3\left((b^2d-d^3)\cos(dx+c)^3-(b^2d-3d^3)\cos(dx+c)\right)\sin(bx+a)-\left((b^3-bd^2)\cos(bx+a)\cos(dx+\right.}{b^4-10b^2d^2+9d^4}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
-(3*((b^2*d - d^3)*cos(d*x + c)^3 - (b^2*d - 3*d^3)*cos(d*x + c))*sin(b*x
+ a) - ((b^3 - b*d^2)*cos(b*x + a)*cos(d*x + c)^2 - (b^3 - 7*b*d^2)*cos(b*
x + a))*sin(d*x + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

3.192.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(76) = 152$.

Time = 1.85 (sec) , antiderivative size = 921, normalized size of antiderivative = 10.12

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*sin(a)*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin
(c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*s
in(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*
x)**3/8 + sin(a - 3*d*x)*cos(c + d*x)**3/(8*d) + 7*sin(c + d*x)**3*cos(a -
 3*d*x)/(24*d) + sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b,
-3*d)), (3*x*sin(a - d*x)*sin(c + d*x)**3/8 + 3*x*sin(a - d*x)*sin(c + d*x
)*cos(c + d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 - 3*
x*cos(a - d*x)*cos(c + d*x)**3/8 + 3*sin(a - d*x)*cos(c + d*x)**3/(8*d) +
5*sin(c + d*x)**3*cos(a - d*x)/(8*d) + 3*sin(c + d*x)*cos(a - d*x)*cos(c +
 d*x)**2/(4*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(
a + d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)
*cos(c + d*x)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 + 3*sin(a + d*x)*cos(
c + d*x)**3/(8*d) - 5*sin(c + d*x)**3*cos(a + d*x)/(8*d) - 3*sin(c + d*x)*
cos(a + d*x)*cos(c + d*x)**2/(4*d), Eq(b, d)), (x*sin(a + 3*d*x)*sin(c + d
*x)**3/8 - 3*x*sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c +
 d*x)**2*cos(a + 3*d*x)*cos(c + d*x)/8 - x*cos(a + 3*d*x)*cos(c + d*x)**3/
8 + sin(a + 3*d*x)*cos(c + d*x)**3/(8*d) - 7*sin(c + d*x)**3*cos(a + 3*d*x
)/(24*d) - sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, 3*d)),
 (-b**3*sin(c + d*x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 3*b*
*2*d*sin(a + b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9...
```

3.192.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(83) = 166$.

Time = 0.29 (sec) , antiderivative size = 916, normalized size of antiderivative = 10.07

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))
*cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin
(3*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) - 3*(b^3*sin(3*c) - b^2*d*sin
(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) + 3*(b
^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b +
 d)*x + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9
*d^3*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c
) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*si
n(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d
)*x - a + 6*c) - (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3
*sin(3*c))*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^
2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) -
3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) +
 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin
((b + d)*x + a + 4*c) + 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c
) + 9*d^3*cos(3*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos
(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(
b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b
 - d)*x - a - 2*c) - (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3
*d^3*cos(3*c))*sin(-(b - 3*d)*x - a + 6*c) - (b^3*cos(3*c) + 3*b^2*d*co...
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = \frac{\sin(bx+3\,dx+a+3\,c)}{8\,(b+3\,d)} - \frac{3\,\sin(bx+dx+a+c)}{8\,(b+d)} + \frac{3\,\sin(bx-dx+a-c)}{8\,(b-d)} - \frac{\sin(bx-3\,dx+a-3\,c)}{8\,(b-3\,d)}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*sin(b*x + d*x + a + c)/(b +
 d) + 3/8*sin(b*x - d*x + a - c)/(b - d) - 1/8*sin(b*x - 3*d*x + a - 3*c)/
(b - 3*d)
```

3.192.9 Mupad [B] (verification not implemented)

Time = 21.33 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.42

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = -\mathrm{e}^{a\,1\mathrm{i}-c\,3\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{b+3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b-3\,d)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right) + \mathrm{e}^{a\,1\mathrm{i}+c\,3\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,3\mathrm{i}}\left(\frac{b-3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b+3\,d)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right) + \mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{3\,b+3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b-3\,d)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right) - \mathrm{e}^{a\,1\mathrm{i}+c\,1\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{3\,b-3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b+3\,d)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right)$$

input
```
int(sin(a + b*x)*sin(c + d*x)^3,x)
```

output
```
exp(a*1i + c*3i + b*x*1i + d*x*3i)*((b - 3*d)/(b^2*16i - d^2*144i) + (exp(
- a*2i - b*x*2i)*(b + 3*d))/(b^2*16i - d^2*144i)) - exp(a*1i - c*3i + b*x*
1i - d*x*3i)*((b + 3*d)/(b^2*16i - d^2*144i) + (exp(- a*2i - b*x*2i)*(b -
3*d))/(b^2*16i - d^2*144i)) + exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3
*d)/(b^2*16i - d^2*16i) + (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(b^2*16i - d^
2*16i)) - exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(b^2*16i - d^2*1
6i) + (exp(- a*2i - b*x*2i)*(3*b + 3*d))/(b^2*16i - d^2*16i))
```

3.193 $\int \sin(a+bx)\sin^2(c+dx)\,dx$

3.193.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \sin(a+bx)\sin^2(c+dx)\,dx = -\frac{\cos(a+bx)}{2b} + \frac{\cos(a-2c+(b-2d)x)}{4(b-2d)} + \frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
-1/2*cos(b*x+a)/b+1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cos(a+2*c+(b+2*d)*x
)/(b+2*d)
```

3.193.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \sin(a+bx)\sin^2(c+dx)\,dx = \frac{1}{4}\left(-\frac{2\cos(a)\cos(bx)}{b} + \frac{\cos(a-2c+bx-2dx)}{b-2d} + \frac{\cos(a+2c+bx+2dx)}{b+2d} + \frac{2\sin(a)\sin(bx)}{b}\right)$$

input
```
Integrate[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

output
```
((-2*Cos[a]*Cos[b*x])/b + Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cos[a + 2
*c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

3.193.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^2(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{4}\sin(a+x(b-2d)-2c)-\frac{1}{4}\sin(a+x(b+2d)+2c)+\frac{1}{2}\sin(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

output
```
-1/2*Cos[a + b*x]/b + Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cos[a + 2
*c + (b + 2*d)*x]/(4*(b + 2*d))
```

3.193.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.193.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cos(xb+a)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4b-8d}+\frac{\cos(a+2c+(b+2d)x)}{4b+8d}$
risch	$-\frac{\cos(xb+a)}{2b}+\frac{\cos(xb-2dx+a-2c)}{4b-8d}+\frac{\cos(xb+2dx+a+2c)}{4b+8d}$
parallelrisch	$\frac{b(b+2d)\cos(a-2c+(b-2d)x)+b(b-2d)\cos(a+2c+(b+2d)x)+(-2b^2+8d^2)\cos(xb+a)-8d^2}{4b^3-16b\,d^2}$
norman	$\frac{\frac{4d^2}{b(b^2-4d^2)}+\frac{4d^2\tan(\frac{dx}{2}+\frac{c}{2})^4}{b(b^2-4d^2)}+\frac{8d\tan(\frac{a}{2}+\frac{xb}{2})\tan(\frac{dx}{2}+\frac{c}{2})}{b^2-4d^2}-\frac{8d\tan(\frac{a}{2}+\frac{xb}{2})\tan(\frac{dx}{2}+\frac{c}{2})^3}{b^2-4d^2}+\frac{4b\tan(\frac{a}{2}+\frac{xb}{2})^2\tan(\frac{dx}{2}+\frac{c}{2})^2}{b^2-4d^2}+\frac{2(-2b^2}{}}{(1+\tan(\frac{a}{2}+\frac{xb}{2})^2)(1+\tan(\frac{dx}{2}+\frac{c}{2})^2)^2}$

input
```
int(sin(b*x+a)*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(b*x+a)/b+1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cos(a+2*c+(b+2*d)*x
)/(b+2*d)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \sin(a+bx)\sin^2(c+dx)\,dx$$
$$=\frac{b^2\cos(bx+a)\cos(dx+c)^2+2\,bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-(b^2-2\,d^2)\cos(bx+a)}{b^3-4\,bd^2}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
(b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*x
 + c) - (b^2 - 2*d^2)*cos(b*x + a))/(b^3 - 4*b*d^2)
```

3.193.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(49) = 98$.

Time = 0.67 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.58

$$\int \sin(a+bx)\sin^2(c+dx)\,dx$$
$$= \begin{cases} x\sin(a)\sin^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) \\ \frac{x\sin(a-2dx)\sin^2(c+dx)}{4}-\frac{x\sin(a-2dx)\cos^2(c+dx)}{4}-\frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2}-\frac{3\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{4d}+ \\ \frac{x\sin(a+2dx)\sin^2(c+dx)}{4}-\frac{x\sin(a+2dx)\cos^2(c+dx)}{4}+\frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2}-\frac{3\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{4d}- \\ -\frac{b^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2}+\frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*sin(a)*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)
), (x*sin(a - 2*d*x)*sin(c + d*x)**2/4 - x*sin(a - 2*d*x)*cos(c + d*x)**2/
4 - x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 - 3*sin(a - 2*d*x)*sin(c
+ d*x)*cos(c + d*x)/(4*d) + cos(a - 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, -2
*d)), (x*sin(a + 2*d*x)*sin(c + d*x)**2/4 - x*sin(a + 2*d*x)*cos(c + d*x)*
*2/4 + x*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 - 3*sin(a + 2*d*x)*sin
(c + d*x)*cos(c + d*x)/(4*d) - cos(a + 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b,
 2*d)), (-b**2*sin(c + d*x)**2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*b*d*sin(
a + b*x)*sin(c + d*x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)
**2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(
b**3 - 4*b*d**2), True))
```

3.193.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(56) = 112$.

Time = 0.25 (sec) , antiderivative size = 414, normalized size of antiderivative = 6.68

$$\int \sin(a+bx)\sin^2(c+dx)\,dx$$
$$= \frac{(b^2\cos(2c)-2bd\cos(2c))\cos((b+2d)x+a+4c)+(b^2\cos(2c)-2bd\cos(2c))\cos((b+2d)x+a)+}{}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*cos
(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2
*c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(b
 - 2*d)*x - a) - 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) - 2*
(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*d
*sin(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*si
n((b + 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a
+ 4*c) - (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*si
n(2*c) - 4*d^2*sin(2*c))*sin(b*x + a + 2*c) + 2*(b^2*sin(2*c) - 4*d^2*sin(
2*c))*sin(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)
^2 + b*sin(2*c)^2)*d^2)
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \sin(a+bx)\sin^2(c+dx)\,dx = \frac{\cos(bx+2\,dx+a+2\,c)}{4\,(b+2\,d)} + \frac{\cos(bx-2\,dx+a-2\,c)}{4\,(b-2\,d)} - \frac{\cos(bx+a)}{2\,b}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*cos(b*x - 2*d*x + a - 2*c)/
(b - 2*d) - 1/2*cos(b*x + a)/b
```

3.193.9 Mupad [B] (verification not implemented)

Time = 20.80 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \sin(a+bx)\sin^2(c+dx)\,dx = -\frac{d\,(2\,b\,\cos(a-2\,c+b\,x-2\,d\,x)-2\,b\,\cos(a+2\,c+b\,x+2\,d\,x))+b^2\,\cos(a-2\,c+b\,x-2\,d\,x)+b^2\,\mathrm{co}}{16\,b\,d^2-4\,b^3} - \frac{\cos(a+b\,x)}{2\,b}$$

input
```
int(sin(a + b*x)*sin(c + d*x)^2,x)
```

output
```
- (d*(2*b*cos(a - 2*c + b*x - 2*d*x) - 2*b*cos(a + 2*c + b*x + 2*d*x)) + b
^2*cos(a - 2*c + b*x - 2*d*x) + b^2*cos(a + 2*c + b*x + 2*d*x))/(16*b*d^2
- 4*b^3) - cos(a + b*x)/(2*b)
```

3.193. $\int \sin(a+bx)\sin^2(c+dx)\,dx$

3.194 $\int \sin(a+bx)\sin(c+dx)\,dx$

3.194.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \sin(a+bx)\sin(c+dx)\,dx = \frac{\sin(a-c+(b-d)x)}{2(b-d)} - \frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

output
```
1/2*sin(a-c+(b-d)*x)/(b-d)-1/2*sin(a+c+(b+d)*x)/(b+d)
```

3.194.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\sin(c+dx)\,dx = \frac{\sin(a-c+(b-d)x)}{2(b-d)} - \frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + d*x],x]
```

output
```
Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

3.194.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{1}{2}\cos(a+x(b-d)-c)-\frac{1}{2}\cos(a+x(b+d)+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

input
```
Int[Sin[a + b*x]*Sin[c + d*x],x]
```

output
```
Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

3.194.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.194.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\sin(a-c+(b-d)x)}{2b-2d}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}$	40
risch	$\frac{\sin(xb-dx+a-c)}{2b-2d}-\frac{\sin(xb+dx+a+c)}{2(b+d)}$	41
parallelrisch	$\frac{(b+d)\sin(a-c+(b-d)x)-\sin(a+c+(b+d)x)(b-d)}{2b^2-2d^2}$	49
norman	$\frac{-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}+\frac{2d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b^2-d^2}+\frac{2b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}-\frac{2d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^2-d^2}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$	147

input
```
int(sin(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(a-c+(b-d)*x)/(b-d)-1/2*sin(a+c+(b+d)*x)/(b+d)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sin(a+bx)\sin(c+dx)\,dx = \frac{d\cos(dx+c)\sin(bx+a)-b\cos(bx+a)\sin(dx+c)}{b^2-d^2}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
(d*cos(d*x + c)*sin(b*x + a) - b*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

3.194.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32) = 64$.

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \sin(a+bx)\sin(c+dx)\,dx = \begin{cases} x\sin(a)\sin(c) & \text{for } b=0 \wedge d=0 \\ \frac{x\sin(a-dx)\sin(c+dx)}{2} - \frac{x\cos(a-dx)\cos(c+dx)}{2} - \frac{\sin(a-dx)\cos(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sin(a+dx)\sin(c+dx)}{2} + \frac{x\cos(a+dx)\cos(c+dx)}{2} - \frac{\sin(a+dx)\cos(c+dx)}{2d} & \text{for } b=d \\ -\frac{b\sin(c+dx)\cos(a+bx)}{b^2-d^2} + \frac{d\sin(a+bx)\cos(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c),x)
```

output
```
Piecewise((x*sin(a)*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*sin(c +
d*x)/2 - x*cos(a - d*x)*cos(c + d*x)/2 - sin(a - d*x)*cos(c + d*x)/(2*d),
Eq(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/2
 - sin(a + d*x)*cos(c + d*x)/(2*d), Eq(b, d)), (-b*sin(c + d*x)*cos(a + b*
x)/(b**2 - d**2) + d*sin(a + b*x)*cos(c + d*x)/(b**2 - d**2), True))
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \sin(a+bx)\sin(c+dx)\,dx = -\frac{\sin(bx+dx+a+c)}{2(b+d)} - \frac{\sin(-bx+dx-a+c)}{2(b-d)}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \sin(a+bx)\sin(c+dx)\,dx = -\frac{\sin(bx+dx+a+c)}{2(b+d)} + \frac{\sin(bx-dx+a-c)}{2(b-d)}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \sin(a+bx)\sin(c+dx)\,dx = \frac{d\left(\frac{\sin(a+c+b\,x+d\,x)}{2}+\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2} - \frac{b\left(\frac{\sin(a+c+b\,x+d\,x)}{2}-\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2}$$

input
```
int(sin(a + b*x)*sin(c + d*x),x)
```

output
```
(d*(sin(a + c + b*x + d*x)/2 + sin(a - c + b*x - d*x)/2))/(b^2 - d^2) - (b
*(sin(a + c + b*x + d*x)/2 - sin(a - c + b*x - d*x)/2))/(b^2 - d^2)
```

3.195 $\int \csc(c+bx)\sin(a+bx)\,dx$

3.195.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}$$

output
```
x*cos(a-c)+ln(sin(b*x+c))*sin(a-c)/b
```

3.195.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}$$

input
```
Integrate[Csc[c + b*x]*Sin[a + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

3.195.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5093, 24, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \ 5093$$

$$\sin(a-c)\int \cot(c+bx)dx+\cos(a-c)\int 1dx$$

$$\Big\downarrow \ 24$$

$$\sin(a-c)\int \cot(c+bx)dx+x\cos(a-c)$$

$$\Big\downarrow \ 3042$$

$$\sin(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx+x\cos(a-c)$$

$$\Big\downarrow \ 25$$

$$x\cos(a-c)-\sin(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx$$

$$\Big\downarrow \ 3956$$

$$\frac{\sin(a-c)\log(-\sin(bx+c))}{b}+x\cos(a-c)$$

input
```
Int[Csc[c + b*x]*Sin[a + b*x],x]
```

output
```
x*Cos[a - c] + (Log[-Sin[c + b*x]]*Sin[a - c])/b
```

3.195.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.195.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

method	result
risch	$-2i\sin\left(a-c\right)x-\frac{2i\sin(a-c)a}{b}+x\,\mathrm{e}^{i(a-c)}+\frac{\ln\left(\mathrm{e}^{2i(xb+a)}-\mathrm{e}^{2i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{\frac{\frac{(\cos(a)\sin(c)-\sin(a)\cos(c))\ln\left(1+\tan(xb+a)^2\right)}{2}+(\cos(a)\cos(c)+\sin(a)\sin(c))\arctan(\tan(xb+a))}{\left(\cos(c)^2+\sin(c)^2\right)\left(\cos(a)^2+\sin(a)^2\right)}+\frac{(\sin(a)\cos(c)-\cos(a)\sin(c))\ln(\tan(xb+a)\cos(a)}{\cos(a)^2\cos(c)^2+\cos(c)^2\sin}}{b}$

input
```
int(csc(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I*sin(a-c)*x-2*I/b*sin(a-c)*a+x*exp(I*(a-c))+ln(exp(2*I*(b*x+a))-exp(2*
I*(a-c)))/b*sin(a-c)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \csc(c+bx)\sin(a+bx)\,dx = \frac{bx\cos(-a+c) - \log\left(\frac{1}{2}\sin(bx+c)\right)\sin(-a+c)}{b}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*cos(-a + c) - log(1/2*sin(b*x + c))*sin(-a + c))/b
```

3.195.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(20) = 40$.

Time = 4.45 (sec) , antiderivative size = 333, normalized size of antiderivative = 12.81

$$\int \csc(c+bx)\sin(a+bx)\,dx$$

$$= \left(\begin{cases} 0 \\ x \\ 0 \\ -\frac{bx\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{bx}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{2\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)}{b\tan^2\left(\frac{c}{2}\right)+b} \end{cases}\right) + \left(\begin{cases} \tilde{\infty}x \\ \frac{\log(\sin(bx))}{b} \\ \frac{x}{\sin(c)} \\ \frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan \right.}{b\,\mathrm{t}} \end{cases}\right)$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x, Eq(c, 0)), (0, Eq(b, 0)), (-b*x*ta
n(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) +
 tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))
*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c
/2)**2 + b), True))*cos(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (log(
sin(b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)*
*2 + b) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log
(tan(c/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))
*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2
)**2 + b) + log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(t
an(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b), True))*sin(a)
```

3.195.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(26) = 52$.

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.15

$$\int \csc(c+bx)\sin(a+bx)\,dx = \frac{2\,bx\cos\left(-a+c\right)-\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^2\right)s}{}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(-a + c) - log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + s
in(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2
*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*s
in(-a + c))/b
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(26) = 52$.

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 9.08

$$\int \csc(c+bx)\sin(a+bx)\,dx = \frac{\frac{\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,a\right)^2+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right)^2+1\right)(bx+c)}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)^2+\tan\left(\frac{1}{2}\,c\right)^2+1}-\frac{2\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\right.}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)^2+\tan\left(\frac{1}{2}\right.}}{b}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1
/2*c)^2 + 1) - 2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(
1/2*a) - tan(1/2*c))*log(tan(1/2*b*x + 1/2*c)^2 + 1)/(tan(1/2*a)^2*tan(1/2
*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^2*tan(1/2*c) - ta
n(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2
*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

3.195.9 Mupad [B] (verification not implemented)

Time = 20.65 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\left(\frac{e^{-a\,1i+c\,1i}}{2}-\frac{e^{a\,1i-c\,1i}}{2}\right)+x\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right) + \frac{\ln\left(-e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)}{b}$$

input
```
int(sin(a + b*x)/sin(c + b*x),x)
```

output
```
x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(
a*1i - c*1i)/2) + (log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*((exp(c*1i -
 a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

3.196 $\int \csc^2(c+bx)\sin(a+bx)\,dx$

3.196.1 Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)/b-csc(b*x+c)*sin(a-c)/b
```

3.196.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int \csc^2(c+bx)\sin(a+bx)\,dx$$
$$= -\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}$$

input
```
Integrate[Csc[c + b*x]^2*Sin[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Csc
[c + b*x]*Sin[a - c])/b
```

3.196.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5093, 3042, 25, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)dx-\frac{\sin(a-c)\int 1d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \csc(c+bx)dx-\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cos(a-c)\text{arctanh}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}$$

input
```
Int[Csc[c + b*x]^2*Sin[a + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) - (Csc[c + b*x]*Sin[a - c])/b
```

3.196.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.196.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.19

method	result
risch	$\frac{e^{i(xb+3a)}-e^{i(xb+a+2c)}}{b\left(-e^{2i(xb+a+c)}+e^{2ia}\right)}-\frac{\ln\left(e^{i(xb+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}+\frac{\ln\left(e^{i(xb+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{4(-2\cos(a)\cos(c)-2\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+8\sin(a)\cos(c)-8\cos(a)\sin(c)}{\left(-4\cos(c)^2\sin(a)^2-4\cos(a)^2\cos(c)^2-4\sin(a)^2\sin(c)^2-4\cos(a)^2\sin(c)^2\right)\left(\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\right.}$

input
```
int(csc(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))
-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*cos(a-c)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \csc^2(c+bx)\sin(a+bx)\,dx =$$
$$-\frac{\cos\left(-a+c\right)\log\left(\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)-\cos\left(-a+c\right)\log\left(-\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)\cdots}{2\,b\sin\left(bx+c\right)}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cos(-a + c)*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - cos(-a + c)*l
og(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 2*sin(-a + c))/(b*sin(b*x + c))
```

3.196.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(29) = 58$.

Time = 54.94 (sec) , antiderivative size = 3264, normalized size of antiderivative = 90.67

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq(
b, 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4
*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan
(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c
/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)*
*2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2))
 + log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*ta
n(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan
(c/2) - b*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/2
)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
+ b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) + t
an(b*x/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**
3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x
/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)*
*2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(b
*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*
tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/t
an(c/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*
tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -...
```

3.196.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 454, normalized size of antiderivative = 12.61

$$\int \csc^2(c+bx)\sin(a+bx)\,dx =$$

$$-\frac{2\left(\cos\left(bx+2\,a\right)-\cos\left(bx+2\,c\right)\right)\cos\left(2\,bx+a+2\,c\right)-2\,\cos\left(bx+2\,a\right)\cos\left(a\right)+2\,\cos\left(bx+2\,c\right)\cos\left(a\right)}{}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*(cos(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 2*cos(b*x
 + 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*cos(-a
+ c) - 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x +
 a + 2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(
a)^2)*cos(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x
)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + a + 2*c)^2*cos(-a + c)
- 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a +
2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)
*cos(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 +
 2*sin(b*x)*sin(c) + sin(c)^2) + 2*(sin(b*x + 2*a) - sin(b*x + 2*c))*sin(2
*b*x + a + 2*c) - 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*co
s(2*b*x + a + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a +
 2*c)^2 - 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(36) = 72$.

Time = 0.32 (sec) , antiderivative size = 349, normalized size of antiderivative = 9.69

$$\int \csc^2(c+bx)\sin(a+bx)\,dx$$
$$= \frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}-\frac{\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}bx+\right.}{\tan\left(\frac{1}{2}a\right)}}{}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan
(1/2*c) - tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2
*c)*tan(1/2*a) - tan(1/2*b*x + 1/2*c)*tan(1/2*c))/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2
*tan(1/2*c)^2 - tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2 + 4*tan(1/2*b*x + 1/2*c)
*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*c)*ta
n(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*c) + tan(1/2*a) -
 tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1
)*tan(1/2*b*x + 1/2*c)))/b
```

3.196.9 Mupad [B] (verification not implemented)

Time = 25.17 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.00

$$\int \csc^2(c+bx)\sin(a+bx)\,dx$$
$$= -\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}\,1i+1i)-\frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}+1)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)(e^{a\,2i-c\,2i}+1)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$
$$+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}\,1i+1i)+\frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}+1)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)(e^{a\,2i-c\,2i}+1)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$
$$+\frac{e^{a\,1i+b\,x\,1i}\,(e^{a\,2i-c\,2i}-1)}{b\,(e^{a\,2i-c\,2i}-e^{a\,2i+b\,x\,2i})}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^2,x)
```

output
```
(log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-
c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(
a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log(- exp(a*1i)*exp(b*x
*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp
(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1))/(2*
b*exp(a*2i - c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b
*(exp(a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

3.197 $\int \csc^3(c+bx)\sin(a+bx)\,dx$

3.197.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \csc^3(c+bx)\sin(a+bx)\,dx=-\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\csc^2(c+bx)\sin(a-c)}{2b}$$

output
```
-cos(a-c)*cot(b*x+c)/b-1/2*csc(b*x+c)^2*sin(a-c)/b
```

3.197.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \csc^3(c+bx)\sin(a+bx)\,dx=\frac{(\cos(a)-\cos(a-c)\cos(c+2bx))\csc(c)\csc^2(c+bx)}{2b}$$

input
```
Integrate[Csc[c + b*x]^3*Sin[a + b*x],x]
```

output
```
((Cos[a] - Cos[a - c]*Cos[c + 2*b*x])*Csc[c]*Csc[c + b*x]^2)/(2*b)
```

3.197.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5093, 3042, 25, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 5093}$$

$$\cos(a-c)\int \csc^2(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^2(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a-c)\int \csc(c+bx)^2dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^2\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\frac{\sin(a-c)\int \csc(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\cos(a-c)\int 1d\cot(c+bx)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

input
```
Int[Csc[c + b*x]^3*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Cot[c + b*x])/b) - (Csc[c + b*x]^2*Sin[a - c])/(2*b)
```

3.197.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.197.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
risch	$\frac{i\left(-2\,{\mathrm e}^{i(2xb+5a+c)}+{\mathrm e}^{i(5a-c)}+{\mathrm e}^{i(3a+c)}\right)}{\left(-{\mathrm e}^{2i(xb+a+c)}+{\mathrm e}^{2ia}\right)^{2} b}$
parallelrisch	$-\frac{\csc\left(\frac{xb}{2}+\frac{c}{2}\right)\left(\sin(xb+a)\left(-\frac{\sec\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}}{2}+1\right)\csc\left(\frac{xb}{2}+\frac{c}{2}\right)+\sec\left(\frac{xb}{2}+\frac{c}{2}\right)\cos(xb+a)\right)}{4b}$
default	$\frac{-\frac{\sin(a)\cos(c)-\cos(a)\sin(c)}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^{2}(\tan(xb+a)\cos(a)\cos(c)+\tan(xb+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}-\frac{}{(\cos(a)\cos(c)+\sin(a)\sin(c)}}{b}$

input
```
int(csc(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
I/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(-2*exp(I*(2*b*x+5*a+c))+exp(I*(5*a
-c))+exp(I*(3*a+c)))
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \frac{2\,\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)-\sin\left(-a+c\right)}{2\left(b\cos\left(bx+c\right)^{2}-b\right)}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - sin(-a + c))/(b*cos(b*x + c
)^2 - b)
```

3.197.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**3*sin(b*x+a),x)
```

output
```
Timed out
```

3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(37) = 74$.

Time = 0.22 (sec) , antiderivative size = 399, normalized size of antiderivative = 10.23

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \frac{(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2c))\cos(4bx+a+5c)-2(2\sin(2bx+2a+2c)-\sin(2a)-}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2b}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
((2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(4*b*x + a + 5*c) - 2
*(2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - (
sin(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) - cos(2*a) - c
os(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) + 2*(2
*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(2*b*x + a + 3*c) + (cos
(2*a) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*
b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2
*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(37) = 74.

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.72

$$\int \csc^3(c+bx)\sin(a+bx)\,dx =$$

$$-\frac{\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)^{2}+4\,\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}\right.}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-(tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)*tan(1/2*a)^2 + 4*t
an(b*x + c)*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(b*x + c)
*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + c) + tan(1/2*a) - tan(
1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*t
an(b*x + c)^2)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.198 $\int \csc^4(c+bx)\sin(a+bx)\,dx$

3.198.1 Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{\csc^3(c+bx)\sin(a-c)}{3b}$$

output
```
-1/2*arctanh(cos(b*x+c))*cos(a-c)/b-1/2*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1
/3*csc(b*x+c)^3*sin(a-c)/b
```

3.198.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = -\frac{6\operatorname{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+3\cos(a-c)\cot(c+bx)\csc(c+bx)+2\csc^3(c+bx)\sin}{6b}$$

input
```
Integrate[Csc[c + b*x]^4*Sin[a + b*x],x]
```

output
```
-1/6*(6*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 3*Cos[a - c]*Co
t[c + b*x]*Csc[c + b*x] + 2*Csc[c + b*x]^3*Sin[a - c])/b
```

3.198.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5093, 3042, 25, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc^3(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^3(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)^3dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^3\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^3\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\frac{\sin(a-c)\int \csc^2(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\cos(a-c)\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(-\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

input
```
Int[Csc[c + b*x]^4*Sin[a + b*x],x]
```

output
```
Cos[a - c]*(-1/2*ArcTanh[Cos[c + b*x]]/b - (Cot[c + b*x]*Csc[c + b*x])/(2*
b)) - (Csc[c + b*x]^3*Sin[a - c])/(3*b)
```

3.198.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.198.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.82

method	result
risch	$\frac{-3\,\mathrm{e}^{i(5xb+7a+4c)}-3\,\mathrm{e}^{i(5xb+5a+6c)}-8\,\mathrm{e}^{i(3xb+7a+2c)}+8\,\mathrm{e}^{i(3xb+5a+4c)}+3\,\mathrm{e}^{i(xb+7a)}+3\,\mathrm{e}^{i(xb+5a+2c)}}{6b\left(-\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^{3}}-\frac{\ln\left(\mathrm{e}^{i(xb+a)}+\mathrm{e}^{i(a-c)}\right)\cos(a-c}{2b}$
default	Expression too large to display

input
```
int(csc(b*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))-3*exp(
I*(5*b*x+5*a+6*c))-8*exp(I*(3*b*x+7*a+2*c))+8*exp(I*(3*b*x+5*a+4*c))+3*exp
(I*(b*x+7*a))+3*exp(I*(b*x+5*a+2*c)))-1/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/
b*cos(a-c)+1/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(61) = 122.

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int \csc^4(c+bx)\sin(a+bx)\,dx$$
$$= \frac{6\cos(bx+c)\cos(-a+c)\sin(bx+c)-3\left(\cos(bx+c)^2\cos(-a+c)-\cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c)\right.}{12\left(b\cos(b\right.}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(6*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - 3*(cos(b*x + c)^2*cos(-a +
 c) - cos(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) + 3*(cos(b*x +
 c)^2*cos(-a + c) - cos(-a + c))*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c)
 - 4*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x + c))
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**4*sin(b*x+a),x)
```

output
```
Timed out
```

3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(61) = 122$.

Time = 0.30 (sec) , antiderivative size = 1773, normalized size of antiderivative = 26.46

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(2*(3*cos(5*b*x + 2*a + 4*c) + 3*cos(5*b*x + 6*c) + 8*cos(3*b*x + 2*a
 + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(6*
b*x + a + 6*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c) + cos(
a))*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a +
 2*c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) - 8*cos(3*b
*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) + 16
*(3*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*
x + a + 2*c) - cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) + cos(b*x + 2
*c))*cos(2*b*x + a + 2*c) + 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos
(a) - 3*(cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*b*x + a + 4*c)^2*cos
(-a + c) + 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos(2*b*x + a + 2*c)*c
os(a)*cos(-a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 9*cos(-a + c)*sin
(4*b*x + a + 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 6*cos(-a + c)
*sin(2*b*x + a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*cos(-a + c) - 3*c
os(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c)
 - 6*(3*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x +
 a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a + c)*sin(4*b*x
 + a + 4*c) - 3*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin
(6*b*x + a + 6*c) - 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*si
n(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)...
```

3.198.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2221 vs. $2(61) = 122$.

Time = 0.32 (sec) , antiderivative size = 2221, normalized size of antiderivative = 33.15

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
1/24*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/
2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (2*tan(1/2*b*x + 1/2*c)^3*tan(
1/2*a)^6*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^6
 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^6 + 4*tan(1/2*b*x + 1/
2*c)^3*tan(1/2*a)^6*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*t
an(1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^4 + 2*tan(1
/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^5 - 12*tan(1/2*b*x + 1/2*c)^2*ta
n(1/2*a)^5*tan(1/2*c)^5 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^5
 - 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^6 - 3*tan(1/2*b*x + 1/
2*c)^2*tan(1/2*a)^4*tan(1/2*c)^6 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan
(1/2*c)^6 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c) + 2*tan(1/2*b
*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2
*a)^6*tan(1/2*c)^2 + 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^3 -
24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^3 + 12*tan(1/2*b*x + 1/2
*c)*tan(1/2*a)^6*tan(1/2*c)^3 - 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(
1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*
b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*
a)^2*tan(1/2*c)^5 - 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^5 +
6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*...
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

3.199 $\int \csc^5(c+bx)\sin(a+bx)\,dx$

3.199.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\cos(a-c)\cot^3(c+bx)}{3b}-\frac{\csc^4(c+bx)\sin(a-c)}{4b}$$

output
```
-cos(a-c)*cot(b*x+c)/b-1/3*cos(a-c)*cot(b*x+c)^3/b-1/4*csc(b*x+c)^4*sin(a-
c)/b
```

3.199.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \frac{(3\cos(a)+\cos(a-c)(-4\cos(c+2bx)+\cos(3c+4bx)))\csc\left(\frac{c}{2}\right)\csc^4(c+bx)\sec\left(\frac{c}{2}\right)}{24b}$$

input
```
Integrate[Csc[c + b*x]^5*Sin[a + b*x],x]
```

output
```
((3*Cos[a] + Cos[a - c]*(-4*Cos[c + 2*b*x] + Cos[3*c + 4*b*x]))*Csc[c/2]*C
sc[c + b*x]^4*Sec[c/2])/(24*b)
```

3.199.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5093, 3042, 25, 3086, 15, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^5(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc^4(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^4(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)^4dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^4\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)^4dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^4\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)^4dx-\frac{\sin(a-c)\int \csc^3(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc(c+bx)^4dx-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos(a-c)\int \left(\cot^2(c+bx)+1\right)d\cot(c+bx)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(a-c)\left(\frac{1}{3}\cot^3(bx+c)+\cot(bx+c)\right)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

input
```
Int[Csc[c + b*x]^5*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*(Cot[c + b*x] + Cot[c + b*x]^3/3))/b) - (Csc[c + b*x]^4*Sin[
a - c])/(4*b)
```

3.199.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.199.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.75 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result
risch	$\frac{2i\left(6\,\mathrm{e}^{i(4xb+9a+3c)}-4\,\mathrm{e}^{i(2xb+9a+c)}-4\,\mathrm{e}^{i(2xb+7a+3c)}+\mathrm{e}^{i(9a-c)}+\mathrm{e}^{i(7a+c)}\right)}{3\left(-\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^{4}b}$
parallelrisch	$-\frac{\left(\sec\left(\frac{xb}{2}+\frac{c}{2}\right)\left(\sin(2xb+a+c)-\frac{\sin(4xb+a+3c)}{4}+\sin(a-c)-\frac{\sin(-2xb+a-3c)}{4}\right)\csc\left(\frac{xb}{2}+\frac{c}{2}\right)+3\cos(xb+a)-\cos(-xb+a-2c)-\mathrm{co}}{96b}$
default	$-\frac{3\sin(a)\cos(c)-3\cos(a)\sin(c)}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^{4}(\tan(xb+a)\cos(a)\cos(c)+\tan(xb+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}-\frac{}{(\cos(a)\cos(c)+\sin(a)\sin(c}$

input
```
int(csc(b*x+c)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^4/b*(6*exp(I*(4*b*x+9*a+3*c))-4*exp
(I*(2*b*x+9*a+c))-4*exp(I*(2*b*x+7*a+3*c))+exp(I*(9*a-c))+exp(I*(7*a+c)))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

$$\int \csc^5(c+bx)\sin(a+bx)\,dx$$
$$=\frac{4\left(2\cos\left(bx+c\right)^{3}\cos\left(-a+c\right)-3\cos\left(bx+c\right)\cos\left(-a+c\right)\right)\sin\left(bx+c\right)+3\sin\left(-a+c\right)}{12\left(b\cos\left(bx+c\right)^{4}-2\,b\cos\left(bx+c\right)^{2}+b\right)}$$

input
```
integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(4*(2*cos(b*x + c)^3*cos(-a + c) - 3*cos(b*x + c)*cos(-a + c))*sin(b*
x + c) + 3*sin(-a + c))/(b*cos(b*x + c)^4 - 2*b*cos(b*x + c)^2 + b)
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**5*sin(b*x+a),x)
```

output
```
Timed out
```

3.199.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(56) = 112$.

Time = 0.24 (sec) , antiderivative size = 1076, normalized size of antiderivative = 17.93

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

output
```
-2/3*((6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x +
 4*c) + sin(2*a) + sin(2*c))*cos(8*b*x + a + 9*c) - 4*(6*sin(4*b*x + 2*a +
 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c
))*cos(6*b*x + a + 7*c) + 6*(4*sin(2*b*x + a + 3*c) - sin(a + c))*cos(4*b*
x + 2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) -
4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 4*(4*sin(
2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - 4*(4*sin(
2*b*x + a + 3*c) - sin(a + c))*cos(2*b*x + 4*c) + (sin(2*a) + sin(2*c))*co
s(a + c) - (6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*
b*x + 4*c) + cos(2*a) + cos(2*c))*sin(8*b*x + a + 9*c) + 4*(6*cos(4*b*x +
2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) + co
s(2*c))*sin(6*b*x + a + 7*c) - 6*(4*cos(2*b*x + a + 3*c) - cos(a + c))*sin
(4*b*x + 2*a + 4*c) - 6*(6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*
c) - 4*cos(2*b*x + 4*c) + cos(2*a) + cos(2*c))*sin(4*b*x + a + 5*c) - 4*co
s(a + c)*sin(2*b*x + 2*a + 2*c) - 4*(4*cos(2*b*x + 2*a + 2*c) - cos(2*a) -
 cos(2*c))*sin(2*b*x + a + 3*c) + 4*(4*cos(2*b*x + a + 3*c) - cos(a + c))*
sin(2*b*x + 4*c) - (cos(2*a) + cos(2*c))*sin(a + c) + 4*cos(2*b*x + 2*a +
2*c)*sin(a + c))/(b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 +
 36*b*cos(4*b*x + a + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 - 8*b*cos(2*b*x
 + a + 3*c)*cos(a + c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16...
```

3.199.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(56) = 112.

Time = 0.32 (sec) , antiderivative size = 301, normalized size of antiderivative = 5.02

$$\int \csc^5(c+bx)\sin(a+bx)\,dx =$$

$$-\frac{6\tan(bx+c)^3\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-6\tan(bx+c)^3\tan\left(\frac{1}{2}a\right)^2+24\tan(bx+c)^3\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+$$

input
```
integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/6*(6*tan(b*x + c)^3*tan(1/2*a)^2*tan(1/2*c)^2 - 6*tan(b*x + c)^3*tan(1/
2*a)^2 + 24*tan(b*x + c)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(b*x + c)^2*tan(1/
2*a)^2*tan(1/2*c) - 6*tan(b*x + c)^3*tan(1/2*c)^2 - 6*tan(b*x + c)^2*tan(1
/2*a)*tan(1/2*c)^2 + 2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(b*x
+ c)^3 + 6*tan(b*x + c)^2*tan(1/2*a) - 2*tan(b*x + c)*tan(1/2*a)^2 - 6*tan
(b*x + c)^2*tan(1/2*c) + 8*tan(b*x + c)*tan(1/2*a)*tan(1/2*c) + 3*tan(1/2*
a)^2*tan(1/2*c) - 2*tan(b*x + c)*tan(1/2*c)^2 - 3*tan(1/2*a)*tan(1/2*c)^2
+ 2*tan(b*x + c) + 3*tan(1/2*a) - 3*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*tan(b*x + c)^4)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^5,x)
```

output
```
\text{Hanged}
```

3.200 $\int \csc^6(c+bx)\sin(a+bx)\,dx$

3.200.1 Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = -\frac{3\text{arctanh}(\cos(c+bx))\cos(a-c)}{8b} - \frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b} - \frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b} - \frac{\csc^5(c+bx)\sin(a-c)}{5b}$$

output
```
-3/8*arctanh(cos(b*x+c))*cos(a-c)/b-3/8*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1
/4*cos(a-c)*cot(b*x+c)*csc(b*x+c)^3/b-1/5*csc(b*x+c)^5*sin(a-c)/b
```

3.200.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = -\frac{480\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+2\csc^5(c+bx)(64\sin(a-c)+5\cos(a-c)(14\sin(2(c}{640b}$$

input
```
Integrate[Csc[c + b*x]^6*Sin[a + b*x],x]
```

output
```
-1/640*(480*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 2*Csc[c + b
*x]^5*(64*Sin[a - c] + 5*Cos[a - c]*(14*Sin[2*(c + b*x)] - 3*Sin[4*(c + b*
x)])))/b
```

3.200.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5093, 3042, 25, 3086, 15, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^6(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc^5(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^5(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)^5dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^5\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)^5dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^5\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)^5dx-\frac{\sin(a-c)\int \csc^4(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc(c+bx)^5dx-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\cos(a-c)\left(\frac{3}{4}\int \csc^3(c+bx)dx-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{3}{4}\int \csc(c+bx)^3dx-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\cos(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(\frac{3}{4}\left(-\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

input
```
Int[Csc[c + b*x]^6*Sin[a + b*x],x]
```

output
```
Cos[a - c]*(-1/4*(Cot[c + b*x]*Csc[c + b*x]^3)/b + (3*(-1/2*ArcTanh[Cos[c
+ b*x]]/b - (Cot[c + b*x]*Csc[c + b*x])/(2*b)))/4) - (Csc[c + b*x]^5*Sin[a
 - c])/(5*b)
```

3.200.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.200.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.88 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.73

method	result
risch	$\frac{-15\,e^{i(9xb+11a+8c)}-15\,e^{i(9xb+9a+10c)}+70\,e^{i(7xb+11a+6c)}+70\,e^{i(7xb+9a+8c)}+128\,e^{i(5xb+11a+4c)}-128\,e^{i(5xb+9a+6c)}-70\,e^{i(3xb+11a}}{40b\left(-e^{2i(xb+a+c)}+e^{2ia}\right)^{5}}$
default	Expression too large to display

input
```
int(csc(b*x+c)^6*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/40/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^5*(-15*exp(I*(9*b*x+11*a+8*c))-15*
exp(I*(9*b*x+9*a+10*c))+70*exp(I*(7*b*x+11*a+6*c))+70*exp(I*(7*b*x+9*a+8*c
))+128*exp(I*(5*b*x+11*a+4*c))-128*exp(I*(5*b*x+9*a+6*c))-70*exp(I*(3*b*x+
11*a+2*c))-70*exp(I*(3*b*x+9*a+4*c))+15*exp(I*(b*x+11*a))+15*exp(I*(b*x+9*
a+2*c)))+3/8*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)-3/8*ln(exp(I*(b*x+
a))+exp(I*(a-c)))/b*cos(a-c)
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(86) = 172.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.10

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = -\frac{15\left(\cos(bx+c)^4\cos(-a+c)-2\cos(bx+c)^2\cos(-a+c)+\cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin$$

input
```
integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/80*(15*(cos(b*x + c)^4*cos(-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos
(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 15*(cos(b*x + c)^4*co
s(-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(-a + c))*log(-1/2*cos(b*x +
 c) + 1/2)*sin(b*x + c) - 10*(3*cos(b*x + c)^3*cos(-a + c) - 5*cos(b*x + c
)*cos(-a + c))*sin(b*x + c) - 16*sin(-a + c))/((b*cos(b*x + c)^4 - 2*b*cos
(b*x + c)^2 + b)*sin(b*x + c))
```

3.200.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**6*sin(b*x+a),x)
```

output
```
Timed out
```

3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3879 vs. 2(86) = 172.

Time = 0.43 (sec) , antiderivative size = 3879, normalized size of antiderivative = 41.27

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/80*(2*(15*cos(9*b*x + 2*a + 8*c) + 15*cos(9*b*x + 10*c) - 70*cos(7*b*x +
 2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) + 128*cos(5
*b*x + 6*c) + 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x
 + 2*a) - 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) - 30*(5*cos(8*b*x + a
+ 8*c) - 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x +
 a + 2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) -
10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x + a + 2*c)
 + cos(a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) + 70*cos(7*b*
x + 8*c) + 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b*
x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) + 15*cos(b*x + 2*
c))*cos(8*b*x + a + 8*c) - 140*(10*cos(6*b*x + a + 6*c) - 10*cos(4*b*x + a
 + 4*c) + 5*cos(2*b*x + a + 2*c) - cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(1
0*cos(6*b*x + a + 6*c) - 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c)
- cos(a))*cos(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*
x + 6*c) - 70*cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x +
2*a) + 15*cos(b*x + 2*c))*cos(6*b*x + a + 6*c) + 256*(10*cos(4*b*x + a + 4
*c) - 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*co
s(4*b*x + a + 4*c) - 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) - 1
00*(14*cos(3*b*x + 2*a + 2*c) + 14*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3
*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) + 140*(5*cos(2*b*x + a + 2*c) - c...
```

3.200.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8035 vs. $2(86) = 172$.

Time = 0.36 (sec) , antiderivative size = 8035, normalized size of antiderivative = 85.48

$$\int \csc^6(c + bx)\sin(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

output
```
1/320*(120*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(
1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (4*tan(1/2*b*x + 1/2*c)^5*ta
n(1/2*a)^10*tan(1/2*c)^9 - 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c
)^10 - 5*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^10 + 16*tan(1/2*b
*x + 1/2*c)^5*tan(1/2*a)^10*tan(1/2*c)^7 - 12*tan(1/2*b*x + 1/2*c)^5*tan(1
/2*a)^9*tan(1/2*c)^8 - 15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^
8 + 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*tan(1/2*c)^9 - 20*tan(1/2*b*x +
 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c)^9 + 20*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)
^10*tan(1/2*c)^9 - 16*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan(1/2*c)^10 -
15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2*c)^10 - 20*tan(1/2*b*x + 1/
2*c)^3*tan(1/2*a)^9*tan(1/2*c)^10 - 40*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^1
0*tan(1/2*c)^10 + 24*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^10*tan(1/2*c)^5 - 8
*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^6 - 10*tan(1/2*b*x + 1/2*c
)^4*tan(1/2*a)^10*tan(1/2*c)^6 + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*ta
n(1/2*c)^7 - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c)^7 + 80*tan(
1/2*b*x + 1/2*c)^3*tan(1/2*a)^10*tan(1/2*c)^7 - 48*tan(1/2*b*x + 1/2*c)^5*
tan(1/2*a)^7*tan(1/2*c)^8 - 45*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2
*c)^8 - 60*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2*c)^8 - 120*tan(1/2*
b*x + 1/2*c)^2*tan(1/2*a)^10*tan(1/2*c)^8 + 8*tan(1/2*b*x + 1/2*c)^5*ta...
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^6,x)
```

output
```
\text{Hanged}
```

3.201 $\int \sin^2(a+bx)\sin^n(c+dx)\,dx$

3.201.1 Optimal result

Integrand size = 17, antiderivative size = 410

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = -\frac{i2^{-2-n}e^{-i(2a+cn)-i(2b+dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}\right.\right.}{2b+dn}$$
$$+\frac{i2^{-2-n}e^{i(2a-cn)+i(2b-dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{2b}{d}-n\right.\right.}{2b-dn}$$
$$+\frac{i2^{-1-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},e^{2i(c+dx)}\right)}{dn}$$

output
```
-I*2^(-2-n)*exp(-I*(c*n+2*a)-I*(d*n+2*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-
I*exp(I*(d*x+c)))^n*hypergeom([-n, -b/d-1/2*n],[1-b/d-1/2*n],exp(2*I*(d*x+
c)))/((1-exp(2*I*c+2*I*d*x))^n)/(d*n+2*b)+I*2^(-2-n)*exp(I*(-c*n+2*a)+I*(-
d*n+2*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-
n, b/d-1/2*n],[1+b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(
-d*n+2*b)+I*2^(-1-n)*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n,
-1/2*n],[1-1/2*n],exp(2*I*(d*x+c)))/d/((1-exp(2*I*(d*x+c)))^n)/n
```

3.201.2 Mathematica [F]

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = \int \sin^2(a+bx)\sin^n(c+dx)\,dx$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x]^n,x]
```

output
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x]^n, x]
```

3.201.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx$$

$$\Big\downarrow \text{ 5064}$$

$$2^{-n-2}\int \left(-e^{-2ia-2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n-e^{2ia+2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n+2\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2^{-n-2}\left(-\frac{i\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n,\frac{1}{2}\left(-\frac{2b}{d}-n+2\right),e^{2i(c}\right.}{2b+dn}\right.$$

input
```
Int[Sin[a + b*x]^2*Sin[c + d*x]^n,x]
```

output
```
2^(-2 - n)*(((-I)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(
I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/
2, -n, (2 - (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)
*d*x))^n*(2*b + d*n)) + (I*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*(c + d
*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*b)/d
- n)/2, -n, (2 + (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c +
(2*I)*d*x))^n*(2*b - d*n)) + ((2*I)*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))
)^n*Hypergeometric2F1[-n, -1/2*n, 1 - n/2, E^((2*I)*(c + d*x))])/(d*(1 - E
^((2*I)*(c + d*x)))^n*n))
```

3.201.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

3.201.4 Maple [F]

$$\int \sin(xb+a)^2 \sin(dx+c)^n\, dx$$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^n,x)
```

output
```
int(sin(b*x+a)^2*sin(d*x+c)^n,x)
```

3.201.5 Fricas [F]

$$\int \sin^2(a+bx)\sin^n(c+dx)\, dx = \int \sin(dx+c)^n \sin(bx+a)^2\, dx$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^n, x)
```

3.201.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c)**n,x)
```

output
```
Timed out
```

3.201.7 Maxima [F]

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = \int \sin(dx+c)^n \sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^n*sin(b*x + a)^2, x)
```

3.201.8 Giac [F]

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = \int \sin(dx+c)^n \sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^n*sin(b*x + a)^2, x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = \int \sin\left(a+b\,x\right)^2\sin\left(c+d\,x\right)^n\,dx$$

input
```
int(sin(a + b*x)^2*sin(c + d*x)^n,x)
```

output
```
int(sin(a + b*x)^2*sin(c + d*x)^n, x)
```

3.202 $\int \sin^2(a+bx)\sin(c+dx)\,dx$

3.202.1 Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)} - \frac{\cos(c+dx)}{2d} + \frac{\cos(2a+c+(2b+d)x)}{4(2b+d)}$$

output
```
-1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*cos(d*x+c)/d+1/4*cos(2*a+c+(2*b+d)*x
)/(2*b+d)
```

3.202.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{\cos(2a-c+2bx-dx)}{4(2b-d)} + \frac{\cos(2a+c+(2b+d)x)}{4(2b+d)} + \frac{1}{2}\left(-\frac{\cos(c)\cos(dx)}{d} + \frac{\sin(c)\sin(dx)}{d}\right)$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x],x]
```

output
```
-1/4*Cos[2*a - c + 2*b*x - d*x]/(2*b - d) + Cos[2*a + c + (2*b + d)*x]/(4*
(2*b + d)) + (-((Cos[c]*Cos[d*x])/d) + (Sin[c]*Sin[d*x])/d)/2
```

3.202.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \ 5080$$

$$\int \left(\frac{1}{4}\sin(2a+x(2b-d)-c)-\frac{1}{4}\sin(2a+x(2b+d)+c)+\frac{1}{2}\sin(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\cos(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\cos(c+dx)}{2d}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + d*x],x]
```

output
```
-1/4*Cos[2*a - c + (2*b - d)*x]/(2*b - d) - Cos[c + d*x]/(2*d) + Cos[2*a +
 c + (2*b + d)*x]/(4*(2*b + d))
```

3.202.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.202.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\cos(dx+c)}{2d}+\frac{\cos(2a+c+(2b+d)x)}{8b+4d}$
parallelrisch	$\frac{(-2bd-d^2)\cos(2a-c+(2b-d)x)+(2bd-d^2)\cos(2a+c+(2b+d)x)+(-8b^2+2d^2)\cos(dx+c)+8b^2}{16b^2d-4d^3}$
risch	$-\frac{2\cos(dx+c)b^2}{(2b+d)(2b-d)d}+\frac{d\cos(dx+c)}{2(2b+d)(2b-d)}-\frac{\cos(2xb-dx+2a-c)b}{2(2b+d)(2b-d)}-\frac{d\cos(2xb-dx+2a-c)}{4(2b+d)(2b-d)}+\frac{\cos(2xb+dx+2a+c)b}{2(2b+d)(2b-d)}-\frac{d\cos(}{4(}$
norman	$\frac{-\frac{4b^2}{d(4b^2-d^2)}-\frac{4b^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{d(4b^2-d^2)}-\frac{8b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4b^2-d^2}+\frac{8b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4b^2-d^2}-\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4b^2-d^2}+\frac{2(-4}{}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(sin(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*cos(d*x+c)/d+1/4*cos(2*a+c+(2*b+d)*x
)/(2*b+d)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \sin^2(a+bx)\sin(c+dx)\,dx$$
$$= -\frac{2\,bd\cos(bx+a)\sin(bx+a)\sin(dx+c)+\left(d^2\cos(bx+a)^2+2\,b^2-d^2\right)\cos(dx+c)}{4\,b^2d-d^3}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-(2*b*d*cos(b*x + a)*sin(b*x + a)*sin(d*x + c) + (d^2*cos(b*x + a)^2 + 2*b
^2 - d^2)*cos(d*x + c))/(4*b^2*d - d^3)
```

3.202.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(49) = 98$.

Time = 0.70 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.03

$$\int \sin^2(a+bx)\sin(c+dx)\,dx$$
$$= \begin{cases} x\sin^2(a)\sin(c) \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\sin(c+dx)}{4} - \frac{x\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos^2\left(a-\frac{dx}{2}\right)}{4} + \frac{3\sin\left(a-\frac{dx}{2}\right)\sin(c+dx)\cos\left(a-\frac{dx}{2}\right)}{2d} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\sin(c+dx)}{4} + \frac{x\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos^2\left(a+\frac{dx}{2}\right)}{4} - \frac{\sin^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{d} + \frac{\sin\left(a+\right.}{} \\ \left(\frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\sin(c) \\ -\frac{2b^2\sin^2(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{2b^2\cos^2(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{2bd\sin(a+bx)\sin(c+dx)\cos(a+bx)}{4b^2d-d^3} + \frac{d^2\sin^2(a+bx)\cos(c+dx)}{4b^2d-d^3} \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((x*sin(a)**2*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*
sin(c + d*x)/4 - x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)/2 - x*sin(c
+ d*x)*cos(a - d*x/2)**2/4 + 3*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)/
(2*d) - cos(a - d*x/2)**2*cos(c + d*x)/d, Eq(b, -d/2)), (x*sin(a + d*x/2)*
*2*sin(c + d*x)/4 + x*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)/2 - x*sin
(c + d*x)*cos(a + d*x/2)**2/4 - sin(a + d*x/2)**2*cos(c + d*x)/d + sin(a +
 d*x/2)*sin(c + d*x)*cos(a + d*x/2)/(2*d), Eq(b, d/2)), ((x*sin(a + b*x)**
2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*sin(c), Eq(d,
 0)), (-2*b**2*sin(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b**2*cos
(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*sin(c + d
*x)*cos(a + b*x)/(4*b**2*d - d**3) + d**2*sin(a + b*x)**2*cos(c + d*x)/(4*
b**2*d - d**3), True))
```

3.202.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(62) = 124$.

Time = 0.24 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.46

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{(2\,bd\cos(c) - d^2\cos(c))\cos((2\,b+d)x + 2\,a + 2\,c) + (2\,bd\cos(c) - d^2\cos(c))\cos((2\,b+d)x + 2\,a) -}{}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*((2*b*d*cos(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a + 2*c) + (2*b*d*co
s(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a) - (2*b*d*cos(c) + d^2*cos(c))*co
s(-(2*b - d)*x - 2*a + 2*c) - (2*b*d*cos(c) + d^2*cos(c))*cos(-(2*b - d)*x
 - 2*a) - 2*(4*b^2*cos(c) - d^2*cos(c))*cos(d*x + 2*c) - 2*(4*b^2*cos(c) -
 d^2*cos(c))*cos(d*x) + (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a
+ 2*c) - (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a) - (2*b*d*sin(c
) + d^2*sin(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c)
)*sin(-(2*b - d)*x - 2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*sin(d*x + 2*c) +
 2*(4*b^2*sin(c) - d^2*sin(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b
^2*cos(c)^2 + b^2*sin(c)^2)*d)
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = \frac{\cos(2bx+dx+2a+c)}{4(2b+d)} - \frac{\cos(2bx-dx+2a-c)}{4(2b-d)} - \frac{\cos(dx+c)}{2d}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
1/4*cos(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*cos(2*b*x - d*x + 2*a - c)/
(2*b - d) - 1/2*cos(d*x + c)/d
```

3.202.9 Mupad [B] (verification not implemented)

Time = 20.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.54

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{d^2\cos(2a+c+2bx+dx) - b(2d\cos(2a+c+2bx+dx) - 2d\cos(2a-c+2bx-dx)) + d^2 \text{co}}{16b^2d-4d^3} - \frac{\cos(c+dx)}{2d}$$

input
```
int(sin(a + b*x)^2*sin(c + d*x),x)
```

output
```
- (d^2*cos(2*a + c + 2*b*x + d*x) - b*(2*d*cos(2*a + c + 2*b*x + d*x) - 2*
d*cos(2*a - c + 2*b*x - d*x)) + d^2*cos(2*a - c + 2*b*x - d*x))/(16*b^2*d
- 4*d^3) - cos(c + d*x)/(2*d)
```

3.202. $\int \sin^2(a+bx)\sin(c+dx)\,dx$

3.203 $\int \sin^2(a+bx)\sin^2(c+dx)\,dx$

3.203.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = \frac{x}{4} - \frac{\sin(2a+2bx)}{8b} + \frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)} - \frac{\sin(2c+2dx)}{8d} + \frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
1/4*x-1/8*sin(2*b*x+2*a)/b+1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sin(2*d*x
+2*c)/d+1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

3.203.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = \frac{(-2b^2d+2d^3)\sin(2(a+bx))+bd(b+d)\sin(2(a-c+(b-d)x))+b(b-d)(-2(b+d)\sin(2(c+dx))+}{16b(b-d)d(b+d)}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

output
```
((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*
x)] + b*(b - d)*(-2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a +
 c + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

3.203.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx$$

$$\Big\downarrow \; 5080$$

$$\int \left(\frac{1}{8}\cos(2(a-c)+2x(b-d))+\frac{1}{8}\cos(2(a+c)+2x(b+d))-\frac{1}{4}\cos(2a+2bx)-\frac{1}{4}\cos(2c+2dx)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

output
```
x/4 - Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) -
 Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

3.203.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.203.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}-\frac{\sin(2xb+2a)}{8b}-\frac{\sin(2dx+2c)}{8d}+\frac{\sin((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sin((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sin((2b-2d)x+2a-2c)+4\left(\frac{bd\sin((2b+2d)x+2a+2c)}{4}+(b+d)\left(-\frac{d\sin(2xb+2a)}{2}+b\left(dx-\frac{\sin(2dx+2c)}{2}\right)\right)\right)(b-d)}{16b^3d-16b\,d^3}$
risch	$\frac{x}{4}-\frac{\sin(2xb+2a)}{8b}-\frac{\sin(2dx+2c)b^2}{8d(b-d)(b+d)}+\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}+\frac{\sin(2xb-2dx+2a-2c)b}{16(b-d)(b+d)}+\frac{d\sin(2xb-2dx+2a-2c)}{16(b-d)(b+d)}+\frac{\sin(2xb-}{16(}$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x-1/8*sin(2*b*x+2*a)/b-1/8*sin(2*d*x+2*c)/d+1/8/(2*b-2*d)*sin((2*b-2*d
)*x+2*a-2*c)+1/8/(2*b+2*d)*sin((2*b+2*d)*x+2*a+2*c)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.34

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = -\frac{\left(2\,bd^2\cos\left(bx+a\right)^2+b^3-2\,bd^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx-\right.\right.}{4\left(b^3d-bd^3\right)}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) -
 (b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - (2*b^2*d - d^3
)*cos(b*x + a))*sin(b*x + a))/(b^3*d - b*d^3)
```

3.203.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.

Time = 1.53 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*sin(a)**2*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq
(b, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c
+ d*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*
sin(c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8
- 5*sin(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) + sin(a - d*x)*cos(a -
 d*x)*cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8
*d), Eq(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**
2*cos(c + d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x
)/2 + x*sin(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*
x)**2/8 - 5*sin(a + d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) - sin(a + d*x)
*cos(a + d*x)*cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c +
 d*x)/(8*d), Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(
a + b*x)*cos(a + b*x)/(2*b))*sin(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)*
*2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2
/(4*b**3*d - 4*b*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*
d - 4*b*d**3) - b**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d -
 4*b*d**3) - b**3*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*
b*d**3) - 2*b**2*d*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4*b**3*d ...
```

3.203.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.

Time = 0.25 (sec) , antiderivative size = 620, normalized size of antiderivative = 7.05

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx$$
$$= \frac{8\left(\left(b\cos(2c)^2 + b\sin(2c)^2\right)d^3 - \left(b^3\cos(2c)^2 + b^3\sin(2c)^2\right)d\right)x + \left(b^2d\sin(2c) - bd^2\sin(2c)\right)\cos(2(b+\cdots}{}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c
)^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c)
 - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2
*c) + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*
d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*
cos(2*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a
 - 2*c) + 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) - 2*(b^3*sin(2*c) -
 b*d^2*sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(
2*(b + d)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)
*x + 2*a) + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c
) + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*c
os(2*c) - d^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*c
os(2*c))*sin(2*b*x + 2*a - 2*c) + 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*
d*x) + 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2
+ b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \sin^2(a+bx)\sin^2(c+dx)\,dx = {} & \frac{1}{4}\,x + \frac{\sin\left(2\,bx + 2\,dx + 2\,a + 2\,c\right)}{16\,(b+d)} \\ & + \frac{\sin\left(2\,bx - 2\,dx + 2\,a - 2\,c\right)}{16\,(b-d)} \\ & - \frac{\sin\left(2\,bx + 2\,a\right)}{8\,b} - \frac{\sin\left(2\,dx + 2\,c\right)}{8\,d}\end{aligned}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d
*x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b - 1/8*sin(2*d*x + 2*c)/d
```

3.203.9 Mupad [B] (verification not implemented)

Time = 20.80 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.01

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx$$
$$= \frac{2\,d^3\,\sin\left(2\,a+2\,b\,x\right)-2\,b^3\,\sin\left(2\,c+2\,d\,x\right)+b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\,x\right.}{}$$

input
```
int(sin(a + b*x)^2*sin(c + d*x)^2,x)
```

output
```
(2*d^3*sin(2*a + 2*b*x) - 2*b^3*sin(2*c + 2*d*x) + b*d^2*sin(2*a - 2*c + 2
*b*x - 2*d*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c
 + 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) - 2*b^2*d*sin(2*a
 + 2*b*x) + 2*b*d^2*sin(2*c + 2*d*x) - 4*b*d^3*x + 4*b^3*d*x)/(16*b*d*(b^2
 - d^2))
```

3.204 $\int \sin^2(a+bx)\sin^3(c+dx)\,dx$

3.204.1 Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = \frac{\cos(2a-3c+(2b-3d)x)}{16(2b-3d)} - \frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)} - \frac{3\cos(c+dx)}{8d} + \frac{\cos(3c+3dx)}{24d} + \frac{3\cos(2a+c+(2b+d)x)}{16(2b+d)} - \frac{\cos(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

output
```
1/16*cos(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*cos(2*a-c+(2*b-d)*x)/(2*b-d)-
3/8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+3/16*cos(2*a+c+(2*b+d)*x)/(2*b+d)-1
/16*cos(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.204.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = \frac{1}{48}\left(-\frac{18\cos(c)\cos(dx)}{d} + \frac{2\cos(3c)\cos(3dx)}{d} + \frac{3\cos(2a-3c+2bx-3dx)}{2b-3d} - \frac{9\cos(2a-c+2bx-dx)}{2b-d} + \frac{9\cos(2a+c+2bx+dx)}{2b+d} - \frac{3\cos(2a+3c+2bx+3dx)}{2b+3d} + \frac{18\sin(c)\sin(dx)}{d} - \frac{2\sin(3c)\sin(3dx)}{d}\right)$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

output
```
((-18*Cos[c]*Cos[d*x])/d + (2*Cos[3*c]*Cos[3*d*x])/d + (3*Cos[2*a - 3*c +
2*b*x - 3*d*x])/(2*b - 3*d) - (9*Cos[2*a - c + 2*b*x - d*x])/(2*b - d) + (
9*Cos[2*a + c + 2*b*x + d*x])/(2*b + d) - (3*Cos[2*a + 3*c + 2*b*x + 3*d*x
])/(2*b + 3*d) + (18*Sin[c]*Sin[d*x])/d - (2*Sin[3*c]*Sin[3*d*x])/d)/48
```

3.204.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{16}\sin(2a+x(2b-3d)-3c)+\frac{3}{16}\sin(2a+x(2b-d)-c)-\frac{3}{16}\sin(2a+x(2b+d)+c)+\frac{1}{16}\sin(2a+x(2b$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\cos(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\cos(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\cos(2a+x(2b+3d)+3c)}{16(2b+3d)}-\frac{3\cos(c+dx)}{8d}+\frac{\cos(3c+3dx)}{24d}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

output
```
Cos[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Cos[2*a - c + (2*b -
d)*x])/(16*(2*b - d)) - (3*Cos[c + d*x])/(8*d) + Cos[3*c + 3*d*x]/(24*d) +
 (3*Cos[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Cos[2*a + 3*c + (2*b + 3*
d)*x]/(16*(2*b + 3*d))
```

3.204.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.204.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$\frac{\cos(2a-3c+(2b-3d)x)}{32b-48d}-\frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\cos(dx+c)}{8d}+\frac{\cos(3dx+3c)}{24d}+\frac{3\cos(2a+c+(2b+d)x)}{16(2b+d)}-\frac{\cos(2a+3c}{16(2}$
parallelrisch	$\frac{(24b^3d+36b^2d^2-6b\,d^3-9d^4)\cos(2a-3c+(2b-3d)x)+(-72b^3d-36b^2d^2+162b\,d^3+81d^4)\cos(2a-c+(2b-d)x)+(-24b^3d+36b^2}{}$
risch	$-\frac{3\cos(dx+c)b^2}{2(2b+d)(2b-d)d}+\frac{3d\cos(dx+c)}{8(2b+d)(2b-d)}+\frac{\cos(2xb-3dx+2a-3c)b}{8(2b+3d)(2b-3d)}+\frac{3d\cos(2xb-3dx+2a-3c)}{16(2b+3d)(2b-3d)}-\frac{3\cos(2xb-dx+2a-c)b}{8(2b+d)(2b-d)}$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*cos(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*cos(2*a-c+(2*b-d)*x)/(2*b-d)-
3/8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+3/16*cos(2*a+c+(2*b+d)*x)/(2*b+d)-1
/16*cos(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.33

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx$$
$$=\frac{\left(8\,b^4-38\,b^2d^2+9\,d^4+9\,(4\,b^2d^2-d^4)\cos{(bx+a)}^2\right)\cos{(dx+c)}^3+6\,\left((4\,b^3d-bd^3)\cos{(bx+a)}\cos{(dx+}\right.}{}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/3*((8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos
(d*x + c)^3 + 6*((4*b^3*d - b*d^3)*cos(b*x + a)*cos(d*x + c)^2 - (4*b^3*d
- 7*b*d^3)*cos(b*x + a))*sin(b*x + a)*sin(d*x + c) - 3*(8*b^4 - 26*b^2*d^2
 + 9*d^4 + 3*(4*b^2*d^2 - 3*d^4)*cos(b*x + a)^2)*cos(d*x + c))/(16*b^4*d -
 40*b^2*d^3 + 9*d^5)
```

3.204.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2003 vs. $2(116) = 232$.

Time = 5.43 (sec) , antiderivative size = 2003, normalized size of antiderivative = 13.91

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*sin(a)**2*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x/2
)**2*sin(c + d*x)**3/16 - 3*x*sin(a - 3*d*x/2)**2*sin(c + d*x)*cos(c + d*x
)**2/16 - 3*x*sin(a - 3*d*x/2)*sin(c + d*x)**2*cos(a - 3*d*x/2)*cos(c + d*
x)/8 + x*sin(a - 3*d*x/2)*cos(a - 3*d*x/2)*cos(c + d*x)**3/8 - x*sin(c + d
*x)**3*cos(a - 3*d*x/2)**2/16 + 3*x*sin(c + d*x)*cos(a - 3*d*x/2)**2*cos(c
 + d*x)**2/16 - sin(a - 3*d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/d - 5*sin
(a - 3*d*x/2)**2*cos(c + d*x)**3/(48*d) - sin(a - 3*d*x/2)*sin(c + d*x)**3
*cos(a - 3*d*x/2)/(24*d) + 5*sin(a - 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2
)*cos(c + d*x)**2/(4*d) - 9*cos(a - 3*d*x/2)**2*cos(c + d*x)**3/(16*d), Eq
(b, -3*d/2)), (3*x*sin(a - d*x/2)**2*sin(c + d*x)**3/16 + 3*x*sin(a - d*x/
2)**2*sin(c + d*x)*cos(c + d*x)**2/16 - 3*x*sin(a - d*x/2)*sin(c + d*x)**2
*cos(a - d*x/2)*cos(c + d*x)/8 - 3*x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c +
 d*x)**3/8 - 3*x*sin(c + d*x)**3*cos(a - d*x/2)**2/16 - 3*x*sin(c + d*x)*c
os(a - d*x/2)**2*cos(c + d*x)**2/16 + 17*sin(a - d*x/2)**2*cos(c + d*x)**3
/(48*d) + 13*sin(a - d*x/2)*sin(c + d*x)**3*cos(a - d*x/2)/(8*d) + 7*sin(a
 - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(c + d*x)**2/(4*d) - sin(c + d*x)
**2*cos(a - d*x/2)**2*cos(c + d*x)/d - 49*cos(a - d*x/2)**2*cos(c + d*x)**
3/(48*d), Eq(b, -d/2)), (3*x*sin(a + d*x/2)**2*sin(c + d*x)**3/16 + 3*x*si
n(a + d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/16 + 3*x*sin(a + d*x/2)*sin(c
 + d*x)**2*cos(a + d*x/2)*cos(c + d*x)/8 + 3*x*sin(a + d*x/2)*cos(a + d...
```

3.204.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(132) = 264$.

Time = 0.30 (sec) , antiderivative size = 1362, normalized size of antiderivative = 9.46

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/96*(3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^
4*cos(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) + 3*(8*b^3*d*cos(3*c) - 12*b^2*
d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*cos((2*b + 3*d)*x + 2*a)
 - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*co
s(3*c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos
(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*cos((2*b + d)*x + 2*a - 2*c) +
 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(
3*c))*cos(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(
3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) -
 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(
3*c))*cos(-(2*b - 3*d)*x - 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*c
os(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2
*(16*b^4*cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x) - 2*(
16*b^4*cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x + 6*c) +
 18*(16*b^4*cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x + 4*c
) + 18*(16*b^4*cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x -
2*c) + 3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^
4*sin(3*c))*sin((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*
d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*sin((2*b + 3*d)*x + 2*a)
 - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4...
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = -\frac{\cos(2bx+3dx+2a+3c)}{16(2b+3d)} + \frac{3\cos(2bx+dx+2a+c)}{16(2b+d)} - \frac{3\cos(2bx-dx+2a-c)}{16(2b-d)} + \frac{\cos(2bx-3dx+2a-3c)}{16(2b-3d)} + \frac{\cos(3dx+3c)}{24d} - \frac{3\cos(dx+c)}{8d}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/16*cos(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*cos(2*b*x + d*x +
2*a + c)/(2*b + d) - 3/16*cos(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*cos(
2*b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*cos(3*d*x + 3*c)/d - 3/8*cos
(d*x + c)/d
```

3.204.9 Mupad [B] (verification not implemented)

Time = 22.05 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.26

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = e^{a\,2i-c\,3i+b\,x\,2i-d\,x\,3i}\left(\frac{3\,d\,(2\,b+3\,d)}{384\,b^2\,d-864\,d^3} + \frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{384\,b^2\,d-864\,d^3} - \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-3\,d)}{384\,b^2\,d-864\,d^3}\right) + e^{a\,2i+c\,3i+b\,x\,2i+d\,x\,3i}\left(-\frac{3\,d\,(2\,b-3\,d)}{384\,b^2\,d-864\,d^3} + \frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{384\,b^2\,d-864\,d^3} + \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+3\,d)}{384\,b^2\,d-864\,d^3}\right) - e^{a\,2i-c\,1i+b\,x\,2i-d\,x\,1i}\left(\frac{3\,(2\,b+d)}{32\,(4\,b^2-d^2)} - \frac{3\,e^{-a\,4i-b\,x\,4i}\,(2\,b-d)}{32\,(4\,b^2-d^2)} + \frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{32\,d\,(4\,b^2-d^2)}\right) - e^{a\,2i+c\,1i+b\,x\,2i+d\,x\,1i}\left(-\frac{3\,(2\,b-d)}{32\,(4\,b^2-d^2)} + \frac{3\,e^{-a\,4i-b\,x\,4i}\,(2\,b+d)}{32\,(4\,b^2-d^2)} + \frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{32\,d\,(4\,b^2-d^2)}\right)$$

input
```
int(sin(a + b*x)^2*sin(c + d*x)^3,x)
```

output
```
exp(a*2i - c*3i + b*x*2i - d*x*3i)*((3*d*(2*b + 3*d))/(384*b^2*d - 864*d^3
) + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(384*b^2*d - 864*d^3) - (3*d*e
xp(- a*4i - b*x*4i)*(2*b - 3*d))/(384*b^2*d - 864*d^3)) + exp(a*2i + c*3i
+ b*x*2i + d*x*3i)*((exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(384*b^2*d - 8
64*d^3) - (3*d*(2*b - 3*d))/(384*b^2*d - 864*d^3) + (3*d*exp(- a*4i - b*x*
4i)*(2*b + 3*d))/(384*b^2*d - 864*d^3)) - exp(a*2i - c*1i + b*x*2i - d*x*1
i)*((3*(2*b + d))/(32*(4*b^2 - d^2)) - (3*exp(- a*4i - b*x*4i)*(2*b - d))/
(32*(4*b^2 - d^2)) + (exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(32*d*(4*b^2
- d^2))) - exp(a*2i + c*1i + b*x*2i + d*x*1i)*((3*exp(- a*4i - b*x*4i)*(2*
b + d))/(32*(4*b^2 - d^2)) - (3*(2*b - d))/(32*(4*b^2 - d^2)) + (exp(- a*2
i - b*x*2i)*(24*b^2 - 6*d^2))/(32*d*(4*b^2 - d^2)))
```

3.204. $\int \sin^2(a+bx)\sin^3(c+dx)\,dx$

3.205 $\int \sin^3(a+bx)\sin^n(c+dx)\,dx$

3.205.1 Optimal result

Integrand size = 17, antiderivative size = 600

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx$$
$$= \frac{2^{-3-n}e^{i(3a-cn)+i(3b-dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),\right.}{3b-dn}$$
$$-\frac{3\,2^{-3-n}e^{i(a-cn)+i(b-dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,\frac{b-dn}{2d}\right.}{b-dn}$$
$$-\frac{3\,2^{-3-n}e^{-i(a+cn)-i(b+dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{b}{}\right.}{b+dn}$$
$$+\frac{2^{-3-n}e^{-i(3a+cn)-i(3b+dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{3b}{}\right.}{3b+dn}$$

output
```
2^(-3-n)*exp(I*(-c*n+3*a)+I*(-d*n+3*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*
exp(I*(d*x+c)))^n*hypergeom([-n, 3/2*b/d-1/2*n],[1+3/2*b/d-1/2*n],exp(2*I*
(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(-d*n+3*b)-3*2^(-3-n)*exp(I*(-c*n+a)+
I*(-d*n+b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom(
[-n, 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I
*d*x))^n)/(-d*n+b)-3*2^(-3-n)*exp(-I*(c*n+a)-I*(d*n+b)*x+I*n*(d*x+c))*(I/e
xp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n-b)/d],[1+1/2*(-
d*n-b)/d],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(d*n+b)+2^(-3-n)*ex
p(-I*(c*n+3*a)-I*(d*n+3*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c
)))^n*hypergeom([-n, 1/2*(-d*n-3*b)/d],[1-3/2*b/d-1/2*n],exp(2*I*(d*x+c)))
/((1-exp(2*I*c+2*I*d*x))^n)/(d*n+3*b)
```

3.205.2 Mathematica [F]

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = \int \sin^3(a+bx)\sin^n(c+dx)\,dx$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x]^n,x]
```

output
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x]^n, x]
```

3.205.3 Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 580, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx$$

$$\Big\downarrow \text{5064}$$

$$2^{-n-3}\int \left(3ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n-3ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n-ie^{-3ia-3ibx}\left(ie^{-i(c+dx)}-ie^{i(c\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2^{-n-3}\left(\frac{\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n,\frac{1}{2}\left(\frac{3b}{d}-n+2\right),e^{2i(c+dx)}\right)e}{3b-dn}\right.$$

input
```
Int[Sin[a + b*x]^3*Sin[c + d*x]^n,x]
```

output
```
2^(-3 - n)*((E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(
c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2
 + (3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*
(3*b - d*n)) - (3*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I/E^(I*
(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (
2 + b/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(b
- d*n)) - (3*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(I/E^(I*(c
 + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(b + d*n)/d, 1
- (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n)) + (E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x + I*n*(c + d*x))*(I/E^
(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(3*b + d*n
)/d, (2 - (3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d
*x))^n*(3*b + d*n)))
```

3.205.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

3.205.4 Maple [F]

$$\int \sin{(xb+a)}^{3} \sin{(dx+c)}^{n} dx$$

input
```
int(sin(b*x+a)^3*sin(d*x+c)^n,x)
```

output
```
int(sin(b*x+a)^3*sin(d*x+c)^n,x)
```

3.205.5 Fricas [F]

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = \int \sin(dx+c)^n \sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^n*sin(b*x + a), x)
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c)**n,x)
```

output
```
Timed out
```

3.205.7 Maxima [F]

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = \int \sin(dx+c)^n \sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^n*sin(b*x + a)^3, x)
```

3.205.8 Giac [F]

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = \int \sin(dx+c)^n \sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^n*sin(b*x + a)^3, x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = \int \sin(a+b\,x)^3 \sin(c+d\,x)^n\,dx$$

input
```
int(sin(a + b*x)^3*sin(c + d*x)^n,x)
```

output
```
int(sin(a + b*x)^3*sin(c + d*x)^n, x)
```

3.206 $\int \sin^3(a+bx)\sin(c+dx)\,dx$

3.206.1 Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \frac{3\sin(a-c+(b-d)x)}{8(b-d)} - \frac{\sin(3a-c+(3b-d)x)}{8(3b-d)} - \frac{3\sin(a+c+(b+d)x)}{8(b+d)} + \frac{\sin(3a+c+(3b+d)x)}{8(3b+d)}$$

output
```
3/8*sin(a-c+(b-d)*x)/(b-d)-1/8*sin(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(3*a+c+(3*b+d)*x)/(3*b+d)
```

3.206.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \frac{1}{8}\left(\frac{3\sin(a-c+bx-dx)}{b-d} - \frac{\sin(3a-c+3bx-dx)}{3b-d} + \frac{\sin(3a+c+3bx+dx)}{3b+d} - \frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x],x]
```

output
```
((3*Sin[a - c + b*x - d*x])/(b - d) - Sin[3*a - c + 3*b*x - d*x]/(3*b - d)
 + Sin[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Sin[a + c + (b + d)*x])/(b +
d))/8
```

3.206.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \ 5080$$

$$\int \left(\frac{3}{8}\cos(a+x(b-d)-c)-\frac{1}{8}\cos(3a+x(3b-d)-c)-\frac{3}{8}\cos(a+x(b+d)+c)+\frac{1}{8}\cos(3a+x(3b+d)+c)\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{\sin(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(3a+x(3b+d)+c)}{8(3b+d)}$$

input
```
Int[Sin[a + b*x]^3*Sin[c + d*x],x]
```

output
```
(3*Sin[a - c + (b - d)*x])/(8*(b - d)) - Sin[3*a - c + (3*b - d)*x]/(8*(3*
b - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[3*a + c + (3*b + d)
*x]/(8*(3*b + d))
```

3.206.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.206.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{\sin(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(3a+c+(3b+d)x)}{24b+8d}$
parallelrisch	$\frac{-3(b+d)\left(b+\frac{d}{3}\right)(b-d)\sin(3a-c+(3b-d)x)+27\left(b-\frac{d}{3}\right)\left((b+d)\left(b+\frac{d}{3}\right)\sin(a-c+(b-d)x)-\left(\left(-\frac{b}{9}-\frac{d}{9}\right)\sin(3a+c+(3b+d)x)+\sin\right.\right.}{72b^4-80b^2d^2+8d^4}$
risch	$\frac{27\sin(xb-dx+a-c)b^3}{8(-3b+d)(-b+d)(3b+d)(b+d)}+\frac{27\sin(xb-dx+a-c)b^2d}{8(-3b+d)(-b+d)(3b+d)(b+d)}-\frac{3\sin(xb-dx+a-c)b\,d^2}{8(-3b+d)(-b+d)(3b+d)(b+d)}-\frac{3\sin(xb-dx+a}{8(-3b+d)(-b+d)(3}$

input
```
int(sin(b*x+a)^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
3/8*sin(a-c+(b-d)*x)/(b-d)-1/8*sin(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(3*a+c+(3*b+d)*x)/(3*b+d)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \sin^3(a+bx)\sin(c+dx)\,dx$$
$$=\frac{\left(7\,b^2d-d^3-\left(b^2d-d^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)+3\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)^3-\left(3\,b^3-bd^2\right.\right.}{9\,b^4-10\,b^2d^2+d^4}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="fricas")
```

output
```
((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)
+ 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*sin(d*x
+ c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

3.206.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(76) = 152$.

Time = 2.09 (sec) , antiderivative size = 932, normalized size of antiderivative = 9.61

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c),x)
```

output
```
Piecewise((x*sin(a)**3*sin(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*
sin(c + d*x)/8 - 3*x*sin(a - d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3*x*sin
(a - d*x)*sin(c + d*x)*cos(a - d*x)**2/8 - 3*x*cos(a - d*x)**3*cos(c + d*x
)/8 + sin(a - d*x)**3*cos(c + d*x)/(8*d) + 3*sin(a - d*x)**2*sin(c + d*x)*
cos(a - d*x)/(4*d) + 3*sin(c + d*x)*cos(a - d*x)**3/(8*d), Eq(b, -d)), (x*
sin(a - d*x/3)**3*sin(c + d*x)/8 - 3*x*sin(a - d*x/3)**2*cos(a - d*x/3)*co
s(c + d*x)/8 - 3*x*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2/8 + x*cos
(a - d*x/3)**3*cos(c + d*x)/8 - 9*sin(a - d*x/3)**3*cos(c + d*x)/(8*d) - 3
*sin(a - d*x/3)**2*sin(c + d*x)*cos(a - d*x/3)/(4*d) - sin(c + d*x)*cos(a
- d*x/3)**3/(8*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*sin(c + d*x)/8 + 3*x
*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(c + d*x)/8 - 3*x*sin(a + d*x/3)*sin(
c + d*x)*cos(a + d*x/3)**2/8 - x*cos(a + d*x/3)**3*cos(c + d*x)/8 - 9*sin(
a + d*x/3)**3*cos(c + d*x)/(8*d) + 3*sin(a + d*x/3)**2*sin(c + d*x)*cos(a
+ d*x/3)/(4*d) + sin(c + d*x)*cos(a + d*x/3)**3/(8*d), Eq(b, d/3)), (3*x*s
in(a + d*x)**3*sin(c + d*x)/8 + 3*x*sin(a + d*x)**2*cos(a + d*x)*cos(c + d
*x)/8 + 3*x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)
**3*cos(c + d*x)/8 + sin(a + d*x)**3*cos(c + d*x)/(8*d) - 3*sin(a + d*x)**
2*sin(c + d*x)*cos(a + d*x)/(4*d) - 3*sin(c + d*x)*cos(a + d*x)**3/(8*d),
Eq(b, d)), (-9*b**3*sin(a + b*x)**2*sin(c + d*x)*cos(a + b*x)/(9*b**4 - 10
*b**2*d**2 + d**4) - 6*b**3*sin(c + d*x)*cos(a + b*x)**3/(9*b**4 - 10*b...
```

3.206.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(89) = 178$.

Time = 0.33 (sec) , antiderivative size = 789, normalized size of antiderivative = 8.13

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/16*((3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*sin(c))*cos((3*
b + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^
3*sin(c))*cos((3*b + d)*x + 3*a) + (3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*
sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a + 2*c) - (3*b^3*sin(c) + b^2*d
*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*
sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x + a + 2
*c) + 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b
 + d)*x + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^3*sin(c
))*cos(-(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*si
n(c) - d^3*sin(c))*cos(-(b - d)*x - a) - (3*b^3*cos(c) - b^2*d*cos(c) - 3*
b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*cos(c) -
b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a) - (3*b^
3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*x -
3*a + 2*c) - (3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*s
in(-(3*b - d)*x - 3*a) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) +
 d^3*cos(c))*sin((b + d)*x + a + 2*c) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) -
 b*d^2*cos(c) + d^3*cos(c))*sin((b + d)*x + a) + 3*(9*b^3*cos(c) + 9*b^2*d
*cos(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*
cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a))/
(9*b^4*cos(c)^2 + 9*b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*...
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \frac{\sin(3\,bx+dx+3\,a+c)}{8\,(3\,b+d)} - \frac{\sin(3\,bx-dx+3\,a-c)}{8\,(3\,b-d)} - \frac{3\,\sin(bx+dx+a+c)}{8\,(b+d)} + \frac{3\,\sin(bx-dx+a-c)}{8\,(b-d)}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="giac")
```

output
```
1/8*sin(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*sin(3*b*x - d*x + 3*a - c)/
(3*b - d) - 3/8*sin(b*x + d*x + a + c)/(b + d) + 3/8*sin(b*x - d*x + a - c
)/(b - d)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 22.34 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.09

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = e^{a\,3i-c\,1i+b\,x\,3i-d\,x\,1i}\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}+\frac{e^{-a\,6i-b\,x\,6i}\,(-3\,b^3+b^2\,d+3\,b\,d^2-d^3)}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}-\frac{e^{-a\,2i-b\,x\,2i}\,(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3)}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}-\frac{e^{-a\,4i-b\,x\,4i}\,(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3)}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}\right)-e^{a\,3i+c\,1i+b\,x\,3i+d\,x\,1i}\left(\frac{-3\,b^3+b^2\,d+3\,b\,d^2-d^3}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}+\frac{e^{-a\,6i-b\,x\,6i}\,(-3\,b^3-b^2\,d+3\,b\,d^2+d^3)}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}-\frac{e^{-a\,2i-b\,x\,2i}\,(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3)}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}-\frac{e^{-a\,4i-b\,x\,4i}\,(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3)}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}\right)$$

input
```
int(sin(a + b*x)^3*sin(c + d*x),x)
```

output
```
exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(b^4*1
44i + d^4*16i - b^2*d^2*160i) + (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d - 3
*b^3 - d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) - (exp(- a*2i - b*x*2i)*(
3*b*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i)
- (exp(- a*4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144i +
 d^4*16i - b^2*d^2*160i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2 +
 b^2*d - 3*b^3 - d^3)/(b^4*144i + d^4*16i - b^2*d^2*160i) + (exp(- a*6i -
b*x*6i)*(3*b*d^2 - b^2*d - 3*b^3 + d^3))/(b^4*144i + d^4*16i - b^2*d^2*160
i) - (exp(- a*2i - b*x*2i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144
i + d^4*16i - b^2*d^2*160i) - (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d -
27*b^3 + 3*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i))
```

3.207 $\int \sin^3(a+bx)\sin^2(c+dx)\,dx$

3.207.1 Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)}+\frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}-\frac{\cos(3a+2c+(3b+2d)x)}{16(3b+2d)}$$

output
```
-3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)-
1/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)+3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)-
1/16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

3.207.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = \frac{1}{48}\left(-\frac{18\cos(a)\cos(bx)}{b}+\frac{2\cos(3a)\cos(3bx)}{b}+\frac{9\cos(a-2c+bx-2dx)}{b-2d}-\frac{3\cos(3a-2c+3bx-2dx)}{3b-2d}+\frac{9\cos(a+2c+bx+2dx)}{b+2d}-\frac{3\cos(3a+2c+3bx+2dx)}{3b+2d}+\frac{18\sin(a)\sin(bx)}{b}-\frac{2\sin(3a)\sin(3bx)}{b}\right)$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

output
```
((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b + (9*Cos[a - 2*c + b*
x - 2*d*x])/(b - 2*d) - (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) + (
9*Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) - (3*Cos[3*a + 2*c + 3*b*x + 2*d*x
])/(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

3.207.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{3}{16}\sin(a+x(b-2d)-2c)+\frac{1}{16}\sin(3a+x(3b-2d)-2c)-\frac{3}{16}\sin(a+x(b+2d)+2c)+\frac{1}{16}\sin(3a+x(3b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}-\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}+\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}-\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}$$

input
```
Int[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

output
```
(-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) + (3*Cos[a - 2*c + (b -
2*d)*x])/(16*(b - 2*d)) - Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d))
+ (3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) - Cos[3*a + 2*c + (3*b + 2
*d)*x]/(16*(3*b + 2*d))
```

3.207.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.207.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

method	result
default	$-\frac{3\cos(xb+a)}{8b}+\frac{\cos(3xb+3a)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)}+\frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}-\frac{\cos(3a+}{16}$
parallelrisch	$\frac{-9\left(b+\frac{2d}{3}\right)b(b-2d)(b+2d)\cos(3a-2c+(3b-2d)x)-9b(b-2d)(b+2d)\left(b-\frac{2d}{3}\right)\cos(3a+2c+(3b+2d)x)+81\left(b+\frac{2d}{3}\right)b(b+2d)\left(b-\frac{2}{3}\right.}{}$
risch	$-\frac{3\cos(xb+a)}{8b}+\frac{27\cos(xb-2dx+a-2c)b^3}{16(b+2d)(3b+2d)(3b-2d)(b-2d)}+\frac{27\cos(xb-2dx+a-2c)b^2d}{8(b+2d)(3b+2d)(3b-2d)(b-2d)}-\frac{3\cos(xb-2dx+a-2c)b\,d^2}{4(b+2d)(3b+2d)(3b-2d)(b-2d)}$

input
```
int(sin(b*x+a)^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)-
1/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)+3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)-
1/16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.37

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx$$
$$=\frac{(9\,b^4-38\,b^2d^2+8\,d^4)\cos{(bx+a)}^3+6\left(7\,b^3d-4\,bd^3-(b^3d-4\,bd^3)\cos{(bx+a)}^2\right)\cos{(dx+c)}\sin{(bx+}}{3}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*((9*b^4 - 38*b^2*d^2 + 8*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 -
(b^3*d - 4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) -
 9*((b^4 - 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*c
os(d*x + c)^2 - 3*(9*b^4 - 26*b^2*d^2 + 8*d^4)*cos(b*x + a))/(9*b^5 - 40*b
^3*d^2 + 16*b*d^4)
```

3.207.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(116) = 232$.

Time = 6.37 (sec) , antiderivative size = 2030, normalized size of antiderivative = 14.71

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*sin(a)**3*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**3, Eq
(b, 0)), (3*x*sin(a - 2*d*x)**3*sin(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)**3
*cos(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)**2*sin(c + d*x)*cos(a - 2*d*x)*co
s(c + d*x)/8 + 3*x*sin(a - 2*d*x)*sin(c + d*x)**2*cos(a - 2*d*x)**2/16 - 3
*x*sin(a - 2*d*x)*cos(a - 2*d*x)**2*cos(c + d*x)**2/16 - 3*x*sin(c + d*x)*
cos(a - 2*d*x)**3*cos(c + d*x)/8 - 13*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c
 + d*x)/(16*d) + sin(a - 2*d*x)**2*cos(a - 2*d*x)*cos(c + d*x)**2/(2*d) -
7*sin(a - 2*d*x)*sin(c + d*x)*cos(a - 2*d*x)**2*cos(c + d*x)/(8*d) - 17*si
n(c + d*x)**2*cos(a - 2*d*x)**3/(96*d) + 49*cos(a - 2*d*x)**3*cos(c + d*x)
**2/(96*d), Eq(b, -2*d)), (x*sin(a - 2*d*x/3)**3*sin(c + d*x)**2/16 - x*si
n(a - 2*d*x/3)**3*cos(c + d*x)**2/16 - 3*x*sin(a - 2*d*x/3)**2*sin(c + d*x
)*cos(a - 2*d*x/3)*cos(c + d*x)/8 - 3*x*sin(a - 2*d*x/3)*sin(c + d*x)**2*c
os(a - 2*d*x/3)**2/16 + 3*x*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d
*x)**2/16 + x*sin(c + d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/8 - 15*sin(a -
 2*d*x/3)**3*sin(c + d*x)*cos(c + d*x)/(16*d) + 3*sin(a - 2*d*x/3)**2*cos(
a - 2*d*x/3)*cos(c + d*x)**2/(2*d) + 9*sin(a - 2*d*x/3)*sin(c + d*x)*cos(a
 - 2*d*x/3)**2*cos(c + d*x)/(8*d) + 21*sin(c + d*x)**2*cos(a - 2*d*x/3)**3
/(32*d) + 11*cos(a - 2*d*x/3)**3*cos(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (
x*sin(a + 2*d*x/3)**3*sin(c + d*x)**2/16 - x*sin(a + 2*d*x/3)**3*cos(c ...
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(126) = 252$.

Time = 0.31 (sec) , antiderivative size = 1360, normalized size of antiderivative = 9.86

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^
3*cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*c
os(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a)
 + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*co
s(2*c))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(
2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) -
 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(
2*c))*cos((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) -
 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*co
s(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(
b - 2*d)*x - a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*
cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) - 2*(9*b^4*cos(2*c) - 4
0*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) - 2*(9*b^4*co
s(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) + 1
8*(9*b^4*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2
*c) + 18*(9*b^4*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x
+ a - 2*c) + 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) +
8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b
^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x
+ 3*a) + 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8...
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = -\frac{\cos(3\,bx+2\,dx+3\,a+2\,c)}{16\,(3\,b+2\,d)} - \frac{\cos(3\,bx-2\,dx+3\,a-2\,c)}{16\,(3\,b-2\,d)} + \frac{\cos(3\,bx+3\,a)}{24\,b} + \frac{3\,\cos(bx+2\,dx+a+2\,c)}{16\,(b+2\,d)} + \frac{3\,\cos(bx-2\,dx+a-2\,c)}{16\,(b-2\,d)} - \frac{3\,\cos(bx+a)}{8\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) - 1/16*cos(3*b*x - 2*d*x
+ 3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b + 3/16*cos(b*x + 2*d*x
+ a + 2*c)/(b + 2*d) + 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos
(b*x + a)/b
```

3.207.9 Mupad [B] (verification not implemented)

Time = 23.52 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.17

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = \frac{81\,b^4\,\cos(a-2\,c+b\,x-2\,d\,x)+81\,b^4\,\cos(a+2\,c+b\,x+2\,d\,x)-162\,b^4\,\cos(a+b\,x)-288\,d^4\,\cos(a+}{}$$

input
```
int(sin(a + b*x)^3*sin(c + d*x)^2,x)
```

output
```
(81*b^4*cos(a - 2*c + b*x - 2*d*x) + 81*b^4*cos(a + 2*c + b*x + 2*d*x) - 1
62*b^4*cos(a + b*x) - 288*d^4*cos(a + b*x) - 9*b^4*cos(3*a - 2*c + 3*b*x -
 2*d*x) - 9*b^4*cos(3*a + 2*c + 3*b*x + 2*d*x) + 18*b^4*cos(3*a + 3*b*x) +
 32*d^4*cos(3*a + 3*b*x) + 24*b*d^3*cos(3*a - 2*c + 3*b*x - 2*d*x) - 24*b*
d^3*cos(3*a + 2*c + 3*b*x + 2*d*x) - 6*b^3*d*cos(3*a - 2*c + 3*b*x - 2*d*x
) + 6*b^3*d*cos(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*cos(a - 2*c + b*x
- 2*d*x) - 36*b^2*d^2*cos(a + 2*c + b*x + 2*d*x) + 720*b^2*d^2*cos(a + b*x
) + 36*b^2*d^2*cos(3*a - 2*c + 3*b*x - 2*d*x) + 36*b^2*d^2*cos(3*a + 2*c +
 3*b*x + 2*d*x) - 80*b^2*d^2*cos(3*a + 3*b*x) - 72*b*d^3*cos(a - 2*c + b*x
 - 2*d*x) + 72*b*d^3*cos(a + 2*c + b*x + 2*d*x) + 162*b^3*d*cos(a - 2*c +
b*x - 2*d*x) - 162*b^3*d*cos(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^5
 - 40*b^3*d^2))
```

3.207. $\int \sin^3(a+bx)\sin^2(c+dx)\,dx$

3.208 $\int \sin^3(a+bx)\sin^3(c+dx)\,dx$

3.208.1 Optimal result

Integrand size = 17, antiderivative size = 195

$$\begin{aligned}\int \sin^3(a+bx)\sin^3(c+dx)\,dx =& -\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}\\&+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}-\frac{3\sin(3a-c+(3b-d)x)}{32(3b-d)}\\&-\frac{9\sin(a+c+(b+d)x)}{32(b+d)}-\frac{\sin(3(a+c)+3(b+d)x)}{96(b+d)}\\&+\frac{3\sin(3a+c+(3b+d)x)}{32(3b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}\end{aligned}$$

output
```
-3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+1/96*sin(3*
a-3*c+3*(b-d)*x)/(b-d)-3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*sin(a+c+(b+d
)*x)/(b+d)-1/96*sin(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*
b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.208.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \frac{1}{96}\left(-\frac{9\sin(a-3c+bx-3dx)}{b-3d}+\frac{27\sin(a-c+bx-dx)}{b-d}+\frac{\sin(3(a-c+bx-dx))}{b-d}-\frac{9\sin(3a-c+3bx-dx)}{3b-d}+\frac{9\sin(3a+c+3bx+dx)}{3b+d}+\frac{9\sin(a+3c+bx+3dx)}{b+3d}-\frac{27\sin(a+c+(b+d)x)}{b+d}-\frac{\sin(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

output
```
((-9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(
b - d) + Sin[3*(a - c + b*x - d*x)]/(b - d) - (9*Sin[3*a - c + 3*b*x - d*x
])/(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c +
 b*x + 3*d*x])/(b + 3*d) - (27*Sin[a + c + (b + d)*x])/(b + d) - Sin[3*(a
+ c + (b + d)*x)]/(b + d))/96
```

3.208.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{3}{32}\cos(a+x(b-3d)-3c)+\frac{9}{32}\cos(a+x(b-d)-c)+\frac{1}{32}\cos(3(a-c)+3x(b-d))-\frac{3}{32}\cos(3a+x(3b-$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}-\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\sin(a+x(b+d)+c)}{32(b+d)}-\frac{\sin(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\sin(3a+x(3b+d)+c)}{32(3b+d)}+\frac{3\sin(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

output
```
(-3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x]
)/(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) - (3*Sin[3*a -
c + (3*b - d)*x])/(32*(3*b - d)) - (9*Sin[a + c + (b + d)*x])/(32*(b + d))
 - Sin[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*
x])/(32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

3.208.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.208.4 Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$-\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}-\frac{9\sin(a+c+(b+d)x)}{32(b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\sin((3b-3d)x+3a-3c}{96b-96d}$
parallelrisch	$\frac{-\frac{9(b-3d)(b+3d)(b+d)\left(b+\frac{d}{3}\right)(b-d)\sin(3a-c+(3b-d)x)}{32}+\frac{9\left(\frac{(b-3d)(b+3d)(b+d)\left(b+\frac{d}{3}\right)\sin((3b-3d)x+3a-3c)}{3}-\frac{(b-3d)(b+3d)\left(b+\frac{d}{3}\right)(b-d}{3}\right.}{}}{}$
risch	Expression too large to display

input
```
int(sin(b*x+a)^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)-9/32*sin(a+
c+(b+d)*x)/(b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)+1/32/(3*b-3*d)*sin((3*b
-3*d)*x+3*a-3*c)-3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sin(3*a+c+(3*b+d)*
x)/(3*b+d)-1/32/(3*b+3*d)*sin((3*b+3*d)*x+3*a+3*c)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.49

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = -\frac{\left(\left(63\,b^4d-88\,b^2d^3+9\,d^5-\left(9\,b^4d-82\,b^2d^3+9\,d^5\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-3\left(21\,b^4d-70\,b^2d^3+\right.}{}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*(((63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos
(b*x + a)^2)*cos(d*x + c)^3 - 3*(21*b^4*d - 70*b^2*d^3 + 9*d^5 - (3*b^4*d
- 28*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c))*sin(b*x + a) - ((9*b^5
 - 88*b^3*d^2 + 63*b*d^4)*cos(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)
*cos(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^2 + 3*b*d^4)*cos(b*x + a))*cos(d*x +
 c)^2 - 3*(9*b^5 - 70*b^3*d^2 + 21*b*d^4)*cos(b*x + a))*sin(d*x + c))/(9*b
^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

3.208.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3584 vs. 2(172) = 344.

Time = 18.20 (sec) , antiderivative size = 3584, normalized size of antiderivative = 18.38

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*sin(a)**3*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x
)**3*sin(c + d*x)**3/32 - 9*x*sin(a - 3*d*x)**3*sin(c + d*x)*cos(c + d*x)*
*2/32 - 9*x*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/
32 + 3*x*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)**3/32 + 3*x*sin(a -
 3*d*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/32 - 9*x*sin(a - 3*d*x)*sin(c +
d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**2/32 - 9*x*sin(c + d*x)**2*cos(a - 3*
d*x)**3*cos(c + d*x)/32 + 3*x*cos(a - 3*d*x)**3*cos(c + d*x)**3/32 + 13*si
n(a - 3*d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(320*d) + sin(a - 3*d*x)**3*c
os(c + d*x)**3/(12*d) + 101*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*
x)/(320*d) + 3*sin(a - 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)*
*2/(20*d) + 27*sin(a - 3*d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**3/(320*d) +
sin(c + d*x)**3*cos(a - 3*d*x)**3/(5*d) + 51*sin(c + d*x)*cos(a - 3*d*x)**
3*cos(c + d*x)**2/(320*d), Eq(b, -3*d)), (5*x*sin(a - d*x)**3*sin(c + d*x)
**3/16 + 3*x*sin(a - d*x)**3*sin(c + d*x)*cos(c + d*x)**2/16 - 9*x*sin(a -
 d*x)**2*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/16 - 3*x*sin(a - d*x)**
2*cos(a - d*x)*cos(c + d*x)**3/16 + 3*x*sin(a - d*x)*sin(c + d*x)**3*cos(a
 - d*x)**2/16 + 9*x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)
**2/16 - 3*x*sin(c + d*x)**2*cos(a - d*x)**3*cos(c + d*x)/16 - 5*x*cos(a -
 d*x)**3*cos(c + d*x)**3/16 - 3*sin(a - d*x)**3*sin(c + d*x)**2*cos(c + d*
x)/(16*d) + 3*sin(a - d*x)**3*cos(c + d*x)**3/(16*d) + sin(a - d*x)**2*...
```

3.208.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(179) = 358$.

Time = 0.44 (sec) , antiderivative size = 2612, normalized size of antiderivative = 13.39

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a +
 4*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a -
 2*c) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a
+ 4*c) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2
*d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a
 - 2*c) + 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30
*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a +
 6*c) - 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b
^2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) -
(9*b^5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(
3*c) + 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (
9*b^5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3
*c) + 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^
5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c)
+ 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) + 27*(9*b^5*
sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) +
9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5...
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = -\frac{\sin(3bx+3dx+3a+3c)}{96(b+d)} + \frac{3\sin(3bx+dx+3a+c)}{32(3b+d)} - \frac{3\sin(3bx-dx+3a-c)}{32(3b-d)} + \frac{\sin(3bx-3dx+3a-3c)}{96(b-d)} + \frac{3\sin(bx+3dx+a+3c)}{32(b+3d)} - \frac{9\sin(bx+dx+a+c)}{32(b+d)} + \frac{9\sin(bx-dx+a-c)}{32(b-d)} - \frac{3\sin(bx-3dx+a-3c)}{32(b-3d)}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a
+ c)/(3*b + d) - 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*
x - 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d)
 - 9/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b -
d) - 3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

3.208. $\int \sin^3(a+bx)\sin^3(c+dx)\,dx$

3.208.9 Mupad [B] (verification not implemented)

Time = 26.11 (sec) , antiderivative size = 997, normalized size of antiderivative = 5.11

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \mathrm{e}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-81\,b^3-81\,b^2\,d+9\,b\,d^2+9\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-81\,b^3+81\,b^2\,d+9\,b\,d^2-9\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}\right)-\mathrm{e}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-81\,b^3+81\,b^2\,d+9\,b\,d^2-9\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-81\,b^3-81\,b^2\,d+9\,b\,d^2+9\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}\right)-\mathrm{e}^{a\,3\mathrm{i}-c\,3\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{-b^3-b^2\,d+9\,b\,d^2+9\,d^3}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-b^3+b^2\,d+9\,b\,d^2-9\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-9\,b^3-27\,b^2\,d+9\,b\,d^2+27\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-9\,b^3+27\,b^2\,d+9\,b\,d^2-27\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}\right)+\mathrm{e}^{a\,3\mathrm{i}+c\,3\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,3\mathrm{i}}\left(\frac{-b^3+b^2\,d+9\,b\,d^2-9\,d^3}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-b^3-b^2\,d+9\,b\,d^2+9\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-9\,b^3+27\,b^2\,d+9\,b\,d^2-27\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-9\,b^3-27\,b^2\,d+9\,b\,d^2+27\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}\right)$$

input
```
int(sin(a + b*x)^3*sin(c + d*x)^3,x)
```

output
```
exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(b
^4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2
*d - 9*b^3 - 3*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*2i - b
*x*2i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^
2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(b^
4*576i + d^4*64i - b^2*d^2*640i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9
*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(b^4*576i + d^4*64i - b^2*d^2*640i) + (e
xp(- a*6i - b*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(b^4*576i + d^4*6
4i - b^2*d^2*640i) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 -
9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d
^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i)) - ex
p(a*3i - c*3i + b*x*3i - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(b^4*192
i + d^4*1728i - b^2*d^2*1920i) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d -
b^3 - 9*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp(- a*2i - b*x*2
i)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*
1920i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4
*192i + d^4*1728i - b^2*d^2*1920i)) + exp(a*3i + c*3i + b*x*3i + d*x*3i)*(
(9*b*d^2 + b^2*d - b^3 - 9*d^3)/(b^4*192i + d^4*1728i - b^2*d^2*1920i) + (
exp(- a*6i - b*x*6i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(b^4*192i + d^4*1728
i - b^2*d^2*1920i) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^2*d - 9*b^3 ...
```

3.208. $\int \sin^3(a+bx)\sin^3(c+dx)\,dx$

3.209 $\int \cos^n(c+dx)\sin(a+bx)\,dx$

3.209.1 Optimal result

Integrand size = 15, antiderivative size = 277

$$\int \cos^n(c+dx)\sin(a+bx)\,dx =$$

$$-\frac{2^{-1-n}e^{i(a-cn)+i(b-dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,\frac{b-dn}{2d},\frac{1}{2}(\right.}{b-dn}$$

$$-\frac{2^{-1-n}e^{-i(a+cn)-i(b+dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{b+dn}{2d},\right.}{b+dn}$$

output
```
-2^(-1-n)*exp(I*(-c*n+a)+I*(-d*n+b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*
(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],-exp(2*I*(d*x
+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n+b)-2^(-1-n)*exp(-I*(c*n+a)-I*(d*n+b
)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d
*n-b)/d],[1+1/2*(-d*n-b)/d],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/
(d*n+b)
```

3.209.2 Mathematica [A] (warning: unable to verify)

Time = 2.47 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.65

$$\int \cos^n(c+dx)\sin(a+bx)\,dx =$$

$$-\frac{2^{-1-n}e^{-i(a-c+(b-d)x)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+n}\left((b-dn)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{b}{d}+n\right),-\frac{b+d}{\ldots}\right.\right.}{(b-dn)(b\ldots}$$

input
```
Integrate[Cos[c + d*x]^n*Sin[a + b*x],x]
```

output
```
-((2^(-1 - n)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(1 + n)*((b - d*
n)*Hypergeometric2F1[1, (2 - b/d + n)/2, -1/2*(b + d*(-2 + n))/d, -E^((2*I
)*(c + d*x))] + E^((2*I)*(a + b*x))*(b + d*n)*Hypergeometric2F1[1, (b + d*
(2 + n))/(2*d), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))]))/(E^(I*(a - c + (b
 - d)*x))*(b - d*n)*(b + d*n)))
```

3.209.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^n(c+dx)\,dx$$

$$\Big\downarrow \text{5066}$$

$$2^{-n-1}\int \left(ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n-ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2^{-n-1}\left(-\frac{\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(-n,\frac{b-dn}{2d},\frac{1}{2}\left(\frac{b}{d}-n+2\right),-e^{2i(c+dx)}\right)\exp(\right.}{b-dn}$$

input
```
Int[Cos[c + d*x]^n*Sin[a + b*x],x]
```

output
```
2^(-1 - n)*(-((E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c
 + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 +
b/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(b - d
*n))) - (E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(E^((-I)*(c +
d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(b + d*n)/d, 1 - (b
+ d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(b +
 d*n)))
```

3.209.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

3.209.4 Maple [F]

$$\int \cos \left(dx+c\right)^{n} \sin \left(xb+a\right)dx$$

input
```
int(cos(d*x+c)^n*sin(b*x+a),x)
```

output
```
int(cos(d*x+c)^n*sin(b*x+a),x)
```

3.209.5 Fricas [F]

$$\int \cos^n(c+dx)\sin(a+bx)\,dx=\int \cos \left(dx+c\right)^{n} \sin \left(bx+a\right)\,dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^n*sin(b*x + a), x)
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \cos^n(c+dx)\sin(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**n*sin(b*x+a),x)
```

output
```
Timed out
```

3.209.7 Maxima [F]

$$\int \cos^n(c+dx)\sin(a+bx)\,dx = \int \cos(dx+c)^n \sin(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^n*sin(b*x + a), x)
```

3.209.8 Giac [F]

$$\int \cos^n(c+dx)\sin(a+bx)\,dx = \int \cos(dx+c)^n \sin(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^n*sin(b*x + a), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \cos^n(c+dx)\sin(a+bx)\,dx = \int \cos(c+d\,x)^n\, \sin(a+b\,x)\,dx$$

input
```
int(cos(c + d*x)^n*sin(a + b*x),x)
```

output
```
int(cos(c + d*x)^n*sin(a + b*x), x)
```

3.210 $\int \cos^3(c+dx)\sin(a+bx)\,dx$

3.210.1 Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = -\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)} - \frac{3\cos(a-c+(b-d)x)}{8(b-d)} - \frac{3\cos(a+c+(b+d)x)}{8(b+d)} - \frac{\cos(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
-1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8*cos(a+c+(
b+d)*x)/(b+d)-1/8*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.210.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = \frac{1}{8}\left(-\frac{\cos(a-3c+bx-3dx)}{b-3d} - \frac{3\cos(a-c+bx-dx)}{b-d} - \frac{\cos(a+3c+bx+3dx)}{b+3d} - \frac{3\cos(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cos[c + d*x]^3*Sin[a + b*x],x]
```

output
```
(-(Cos[a - 3*c + b*x - 3*d*x]/(b - 3*d)) - (3*Cos[a - c + b*x - d*x])/(b -
 d) - Cos[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Cos[a + c + (b + d)*x])/(b
 + d))/8
```

3.210.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \ 5085$$

$$\int \left(\frac{1}{8}\sin(a+x(b-3d)-3c)+\frac{3}{8}\sin(a+x(b-d)-c)+\frac{3}{8}\sin(a+x(b+d)+c)+\frac{1}{8}\sin(a+x(b+3d)+3c)\right)d$$

$$\Big\downarrow \ 2009$$

$$-\frac{\cos(a+x(b-3d)-3c)}{8(b-3d)}-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}-\frac{\cos(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Cos[c + d*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*Cos[a - 3*c + (b - 3*d)*x]/(b - 3*d) - (3*Cos[a - c + (b - d)*x])/(8*
(b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) - Cos[a + 3*c + (b + 3*d
)*x]/(8*(b + 3*d))
```

3.210.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.210.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)}-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}-\frac{\cos(a+3c+(b+3d)x)}{8(b+3d)}$
risch	$-\frac{\cos(xb-3dx+a-3c)}{8(b-3d)}-\frac{3\cos(xb-dx+a-c)}{8(b-d)}-\frac{3\cos(xb+dx+a+c)}{8(b+d)}-\frac{\cos(xb+3dx+a+3c)}{8(b+3d)}$
parallelrisch	$\frac{-2b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2(b^2-7d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-12d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)(b^2-3d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-6b\left(-4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2d^2+b^2-3d^2\right)\tan(}{(b-d)(b+3d)(b-3d}$

input
```
int(cos(d*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8*cos(a+c+(
b+d)*x)/(b+d)-1/8*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int \cos^3(c+dx)\sin(a+bx)\,dx$$
$$=\frac{6\,bd^2\cos{(bx+a)}\cos{(dx+c)}-(b^3-bd^2)\cos{(bx+a)}\cos{(dx+c)}^3+3\left(2\,d^3-(b^2d-d^3)\cos{(dx+c)}^2\right)}{b^4-10\,b^2d^2+9\,d^4}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
(6*b*d^2*cos(b*x + a)*cos(d*x + c) - (b^3 - b*d^2)*cos(b*x + a)*cos(d*x +
c)^3 + 3*(2*d^3 - (b^2*d - d^3)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x + c))
/(b^4 - 10*b^2*d^2 + 9*d^4)
```

3.210.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(78) = 156$.

Time = 2.12 (sec) , antiderivative size = 918, normalized size of antiderivative = 10.09

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*x)*
sin(c + d*x)**2*cos(c + d*x)/8 + x*sin(a - 3*d*x)*cos(c + d*x)**3/8 - x*si
n(c + d*x)**3*cos(a - 3*d*x)/8 + 3*x*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d
*x)**2/8 - sin(a - 3*d*x)*sin(c + d*x)**3/(24*d) - sin(a - 3*d*x)*sin(c +
d*x)*cos(c + d*x)**2/(4*d) + 3*cos(a - 3*d*x)*cos(c + d*x)**3/(8*d), Eq(b,
 -3*d)), (3*x*sin(a - d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*sin(a - d*
x)*cos(c + d*x)**3/8 + 3*x*sin(c + d*x)**3*cos(a - d*x)/8 + 3*x*sin(c + d*
x)*cos(a - d*x)*cos(c + d*x)**2/8 + 3*sin(a - d*x)*sin(c + d*x)**3/(8*d) +
 3*sin(a - d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) - cos(a - d*x)*cos(c +
d*x)**3/(8*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**2*cos(c + d*x)/
8 + 3*x*sin(a + d*x)*cos(c + d*x)**3/8 - 3*x*sin(c + d*x)**3*cos(a + d*x)/
8 - 3*x*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)**2/8 + 3*sin(a + d*x)*sin(c
 + d*x)**3/(8*d) + 3*sin(a + d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) + cos
(a + d*x)*cos(c + d*x)**3/(8*d), Eq(b, d)), (-3*x*sin(a + 3*d*x)*sin(c + d
*x)**2*cos(c + d*x)/8 + x*sin(a + 3*d*x)*cos(c + d*x)**3/8 + x*sin(c + d*x
)**3*cos(a + 3*d*x)/8 - 3*x*sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/8
- sin(a + 3*d*x)*sin(c + d*x)**3/(24*d) - sin(a + 3*d*x)*sin(c + d*x)*cos(
c + d*x)**2/(4*d) - 3*cos(a + 3*d*x)*cos(c + d*x)**3/(8*d), Eq(b, 3*d)), (
-b**3*cos(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2
*d*sin(a + b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d...
```

3.210.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(83) = 166$.

Time = 0.25 (sec) , antiderivative size = 912, normalized size of antiderivative = 10.02

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/16*((b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))
*cos((b + 3*d)*x + a + 6*c) + (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos
(3*c) + 3*d^3*cos(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*cos(3*c) - b^2*d*cos
(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b + d)*x + a + 4*c) + 3*(b
^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b +
 d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9
*d^3*cos(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c
) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*co
s(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*cos(-(b - 3*d
)*x - a + 6*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3
*cos(3*c))*cos(-(b - 3*d)*x - a) + (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^
2*sin(3*c) + 3*d^3*sin(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) -
3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*sin((b + 3*d)*x + a) +
 3*(b^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*sin
((b + d)*x + a + 4*c) - 3*(b^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c
) + 9*d^3*sin(3*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*sin(3*c) + b^2*d*sin
(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*sin(-(b - d)*x - a + 4*c) - 3*(
b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*sin(-(b
 - d)*x - a - 2*c) + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3
*d^3*sin(3*c))*sin(-(b - 3*d)*x - a + 6*c) - (b^3*sin(3*c) + 3*b^2*d*si...
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = -\frac{\cos(bx+3\,dx+a+3\,c)}{8\,(b+3\,d)} - \frac{3\,\cos(bx+dx+a+c)}{8\,(b+d)} - \frac{3\,\cos(bx-dx+a-c)}{8\,(b-d)} - \frac{\cos(bx-3\,dx+a-3\,c)}{8\,(b-3\,d)}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/8*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*cos(b*x + d*x + a + c)/(b
+ d) - 3/8*cos(b*x - d*x + a - c)/(b - d) - 1/8*cos(b*x - 3*d*x + a - 3*c)
/(b - 3*d)
```

3.210.9 Mupad [B] (verification not implemented)

Time = 23.19 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.26

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = -\mathrm{e}^{a\,1\mathrm{i}-c\,3\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{b+3\,d}{16\,b^2-144\,d^2}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b-3\,d)}{16\,b^2-144\,d^2}\right) - \mathrm{e}^{a\,1\mathrm{i}+c\,3\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,3\mathrm{i}}\left(\frac{b-3\,d}{16\,b^2-144\,d^2}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b+3\,d)}{16\,b^2-144\,d^2}\right) - \mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{3\,b+3\,d}{16\,b^2-16\,d^2}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b-3\,d)}{16\,b^2-16\,d^2}\right) - \mathrm{e}^{a\,1\mathrm{i}+c\,1\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{3\,b-3\,d}{16\,b^2-16\,d^2}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b+3\,d)}{16\,b^2-16\,d^2}\right)$$

input
```
int(cos(c + d*x)^3*sin(a + b*x),x)
```

output
```
- exp(a*1i - c*3i + b*x*1i - d*x*3i)*((b + 3*d)/(16*b^2 - 144*d^2) + (exp(
- a*2i - b*x*2i)*(b - 3*d))/(16*b^2 - 144*d^2)) - exp(a*1i + c*3i + b*x*1i
 + d*x*3i)*((b - 3*d)/(16*b^2 - 144*d^2) + (exp(- a*2i - b*x*2i)*(b + 3*d)
)/(16*b^2 - 144*d^2)) - exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3*d)/(1
6*b^2 - 16*d^2) + (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(16*b^2 - 16*d^2)) -
exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(16*b^2 - 16*d^2) + (exp(-
 a*2i - b*x*2i)*(3*b + 3*d))/(16*b^2 - 16*d^2))
```

3.211 $\int \cos^2(c+dx)\sin(a+bx)\,dx$

3.211.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = -\frac{\cos(a+bx)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)} -\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
-1/2*cos(b*x+a)/b-1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1/4*cos(a+2*c+(b+2*d)*x
)/(b+2*d)
```

3.211.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = \frac{1}{4}\left(-\frac{2\cos(a)\cos(bx)}{b}-\frac{\cos(a-2c+bx-2dx)}{b-2d} -\frac{\cos(a+2c+bx+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}\right)$$

input
```
Integrate[Cos[c + d*x]^2*Sin[a + b*x],x]
```

output
```
((-2*Cos[a]*Cos[b*x])/b - Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) - Cos[a + 2
*c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

3.211.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{ 5085}$$

$$\int \left(\frac{1}{4}\sin(a+x(b-2d)-2c)+\frac{1}{4}\sin(a+x(b+2d)+2c)+\frac{1}{2}\sin(a+bx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}-\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

input
```
Int[Cos[c + d*x]^2*Sin[a + b*x],x]
```

output
```
-1/2*Cos[a + b*x]/b - Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) - Cos[a + 2
*c + (b + 2*d)*x]/(4*(b + 2*d))
```

3.211.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.211.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}-\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$
risch	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(xb-2dx+a-2c)}{4(b-2d)}-\frac{\cos(xb+2dx+a+2c)}{4(b+2d)}$
parallelrisch	$\frac{(-2b^2+4d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+8d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)b+\left(-4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2b^2+8d^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-8d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(}{(b-2d)(b+2d)b\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)}$
norman	$\frac{\frac{-2b^2+4d^2}{b(b^2-4d^2)}+\frac{(-2b^2+4d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{b(b^2-4d^2)}+\frac{8d^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b(b^2-4d^2)}-\frac{8d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-4d^2}+\frac{8d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2-4d^2}-\frac{4b\tan\left(\frac{a}{2}+\right.}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)}$

input
```
int(cos(d*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(b*x+a)/b-1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1/4*cos(a+2*c+(b+2*d)*x
)/(b+2*d)
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = -\frac{b^2\cos(bx+a)\cos(dx+c)^2+2\,bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-2\,d^2\cos(bx+a)}{b^3-4\,bd^2}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*
x + c) - 2*d^2*cos(b*x + a))/(b^3 - 4*b*d^2)
```

3.211.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(51) = 102$.

Time = 0.77 (sec) , antiderivative size = 405, normalized size of antiderivative = 6.53

$$\int \cos^2(c+dx)\sin(a+bx)\,dx$$
$$= \begin{cases} x\sin(a)\cos^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) \\ -\frac{x\sin(a-2dx)\sin^2(c+dx)}{4}+\frac{x\sin(a-2dx)\cos^2(c+dx)}{4}+\frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2}-\frac{\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{4d}+ \\ -\frac{x\sin(a+2dx)\sin^2(c+dx)}{4}+\frac{x\sin(a+2dx)\cos^2(c+dx)}{4}-\frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2}-\frac{\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{4d}- \\ -\frac{b^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2}+\frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)
), (-x*sin(a - 2*d*x)*sin(c + d*x)**2/4 + x*sin(a - 2*d*x)*cos(c + d*x)**2
/4 + x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 - sin(a - 2*d*x)*sin(c +
 d*x)*cos(c + d*x)/(4*d) + cos(a - 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, -2*
d)), (-x*sin(a + 2*d*x)*sin(c + d*x)**2/4 + x*sin(a + 2*d*x)*cos(c + d*x)*
*2/4 - x*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 - sin(a + 2*d*x)*sin(c
 + d*x)*cos(c + d*x)/(4*d) - cos(a + 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, 2
*d)), (-b**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(a
+ b*x)*sin(c + d*x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)**
2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(b*
*3 - 4*b*d**2), True))
```

3.211.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(56) = 112$.

Time = 0.24 (sec) , antiderivative size = 414, normalized size of antiderivative = 6.68

$$\int \cos^2(c+dx)\sin(a+bx)\,dx =$$
$$-\frac{(b^2\cos(2c)-2bd\cos(2c))\cos((b+2d)x+a+4c)+(b^2\cos(2c)-2bd\cos(2c))\cos((b+2d)x+a)}{}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*co
s(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(
2*c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(
b - 2*d)*x - a) + 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) + 2
*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*
d*sin(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*s
in((b + 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a
 + 4*c) - (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) + 2*(b^2*s
in(2*c) - 4*d^2*sin(2*c))*sin(b*x + a + 2*c) - 2*(b^2*sin(2*c) - 4*d^2*sin
(2*c))*sin(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c
)^2 + b*sin(2*c)^2)*d^2)
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = -\frac{\cos(bx+2\,dx+a+2\,c)}{4\,(b+2\,d)} - \frac{\cos(bx-2\,dx+a-2\,c)}{4\,(b-2\,d)} - \frac{\cos(bx+a)}{2\,b}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) - 1/4*cos(b*x - 2*d*x + a - 2*c)
/(b - 2*d) - 1/2*cos(b*x + a)/b
```

3.211.9 Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = \frac{d\,(2\,b\,\cos(a-2\,c+b\,x-2\,d\,x)-2\,b\,\cos(a+2\,c+b\,x+2\,d\,x))+b^2\,\cos(a-2\,c+b\,x-2\,d\,x)+b^2\,\cos(}{16\,b\,d^2-4\,b^3} - \frac{\cos(a+b\,x)}{2\,b}$$

input
```
int(cos(c + d*x)^2*sin(a + b*x),x)
```

output
```
(d*(2*b*cos(a - 2*c + b*x - 2*d*x) - 2*b*cos(a + 2*c + b*x + 2*d*x)) + b^2
*cos(a - 2*c + b*x - 2*d*x) + b^2*cos(a + 2*c + b*x + 2*d*x))/(16*b*d^2 -
4*b^3) - cos(a + b*x)/(2*b)
```

3.211. $\int \cos^2(c+dx)\sin(a+bx)\,dx$

3.212 $\int \cos(c+dx)\sin(a+bx)\,dx$

3.212.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cos(c+dx)\sin(a+bx)\,dx=-\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}$$

output
```
-1/2*cos(a-c+(b-d)*x)/(b-d)-1/2*cos(a+c+(b+d)*x)/(b+d)
```

3.212.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)\sin(a+bx)\,dx=-\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Cos[c + d*x]*Sin[a + b*x],x]
```

output
```
-1/2*Cos[a - c + (b - d)*x]/(b - d) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

3.212.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \ 5085$$

$$\int \left(\frac{1}{2}\sin(a+x(b-d)-c)+\frac{1}{2}\sin(a+x(b+d)+c)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

input
```
Int[Cos[c + d*x]*Sin[a + b*x],x]
```

output
```
-1/2*Cos[a - c + (b - d)*x]/(b - d) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

3.212.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.212.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}$	40
risch	$-\frac{\cos(xb-dx+a-c)}{2(b-d)}-\frac{\cos(xb+dx+a+c)}{2(b+d)}$	41
parallelrisch	$\frac{2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}b-4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}{(b-d)(b+d)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}$	94
norman	$\frac{\frac{2b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{b^{2}-d^{2}}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{b^{2}-d^{2}}-\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}-d^{2}}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}$	115

input
```
int(cos(d*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(a-c+(b-d)*x)/(b-d)-1/2*cos(a+c+(b+d)*x)/(b+d)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)\sin(a+bx)\,dx=-\frac{b\cos\left(bx+a\right)\cos\left(dx+c\right)+d\sin\left(bx+a\right)\sin\left(dx+c\right)}{b^2-d^2}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b*cos(b*x + a)*cos(d*x + c) + d*sin(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

3.212.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(34) = 68$.

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.60

$$\int \cos(c+dx)\sin(a+bx)\,dx = \begin{cases} x\sin(a)\cos(c) & \text{for } b=0\wedge d=0 \\ \frac{x\sin(a-dx)\cos(c+dx)}{2}+\frac{x\sin(c+dx)\cos(a-dx)}{2}+\frac{\sin(a-dx)\sin(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sin(a+dx)\cos(c+dx)}{2}-\frac{x\sin(c+dx)\cos(a+dx)}{2}+\frac{\sin(a+dx)\sin(c+dx)}{2d} & \text{for } b=d \\ -\frac{b\cos(a+bx)\cos(c+dx)}{b^2-d^2}-\frac{d\sin(a+bx)\sin(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*cos(c +
d*x)/2 + x*sin(c + d*x)*cos(a - d*x)/2 + sin(a - d*x)*sin(c + d*x)/(2*d),
Eq(b, -d)), (x*sin(a + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)*cos(a + d*x)/2
 + sin(a + d*x)*sin(c + d*x)/(2*d), Eq(b, d)), (-b*cos(a + b*x)*cos(c + d*
x)/(b**2 - d**2) - d*sin(a + b*x)*sin(c + d*x)/(b**2 - d**2), True))
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)\sin(a+bx)\,dx = -\frac{\cos(bx+dx+a+c)}{2(b+d)} - \frac{\cos(-bx+dx-a+c)}{2(b-d)}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(-b*x + d*x - a + c)/(b - d)
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)\sin(a+bx)\,dx = -\frac{\cos(bx+dx+a+c)}{2(b+d)} - \frac{\cos(bx-dx+a-c)}{2(b-d)}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(b*x - d*x + a - c)/(b - d)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \cos(c+dx)\sin(a+bx)\,dx = -\frac{b\left(\frac{\cos(a-c+b\,x-d\,x)}{2}+\frac{\cos(a+c+b\,x+d\,x)}{2}\right)}{b^2-d^2} - \frac{d\left(\frac{\cos(a-c+b\,x-d\,x)}{2}-\frac{\cos(a+c+b\,x+d\,x)}{2}\right)}{b^2-d^2}$$

input
```
int(cos(c + d*x)*sin(a + b*x),x)
```

output
```
- (b*(cos(a - c + b*x - d*x)/2 + cos(a + c + b*x + d*x)/2))/(b^2 - d^2) -
(d*(cos(a - c + b*x - d*x)/2 - cos(a + c + b*x + d*x)/2))/(b^2 - d^2)
```

3.213 $\int \sec(c+bx)\sin(a+bx)\,dx$

3.213.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\log(\cos(c+bx))}{b} + x\sin(a-c)$$

output
```
-cos(a-c)*ln(cos(b*x+c))/b+x*sin(a-c)
```

3.213.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\log(\cos(c+bx))}{b} + x\sin(a-c)$$

input
```
Integrate[Sec[c + b*x]*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

3.213.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5091, 24, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \; 5091$$

$$\cos(a-c)\int \tan(c+bx)dx+\sin(a-c)\int 1dx$$

$$\Big\downarrow \; 24$$

$$\cos(a-c)\int \tan(c+bx)dx+x\sin(a-c)$$

$$\Big\downarrow \; 3042$$

$$\cos(a-c)\int \tan(c+bx)dx+x\sin(a-c)$$

$$\Big\downarrow \; 3956$$

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

input
```
Int[Sec[c + b*x]*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

3.213.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.213.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

method	result
risch	$2i\cos\left(a-c\right)x-ix\,\mathrm{e}^{i(a-c)}+\frac{2i\cos(a-c)a}{b}-\frac{\ln\left(\mathrm{e}^{2i(xb+a)}+\mathrm{e}^{2i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{\frac{\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln\left(1+\tan(xb+a)^2\right)}{2}+(\sin(a)\cos(c)-\cos(a)\sin(c))\arctan(\tan(xb+a))}{\left(\cos(c)^2+\sin(c)^2\right)\left(\cos(a)^2+\sin(a)^2\right)}-\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln(\tan(xb+a)\sin(a)}{\cos(a)^2\cos(c)^2+\cos(c)^2\sin}}{b}$

input
```
int(sec(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I*cos(a-c)*x-I*x*exp(I*(a-c))+2*I/b*cos(a-c)*a-ln(exp(2*I*(b*x+a))+exp(2
*I*(a-c)))/b*cos(a-c)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \sec(c+bx)\sin(a+bx)\,dx=-\frac{bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(-\cos\left(bx+c\right)\right)}{b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x*sin(-a + c) + cos(-a + c)*log(-cos(b*x + c)))/b
```

3.213.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(20) = 40$.

Time = 4.32 (sec) , antiderivative size = 435, normalized size of antiderivative = 16.11

$$\int \sec(c+bx)\sin(a+bx)\,dx$$
$$= \left(\begin{cases} -x \\ x \\ 0 \\ -\frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log(\cdots}{} \end{cases}\right.$$
$$+ \left(\begin{cases} -\frac{\log(\sin(bx))}{b} \\ \frac{\log(\sin(bx))}{b} \\ \frac{x}{\cos(c)} \\ -\frac{bx\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{bx}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{2\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log(\mathrm{ta}\cdots}{} \end{cases}\right.$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*tan
(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)
**2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - t
an(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b)
 - log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2
)**2 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*t
an(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1)
- 1/(tan(c/2) + 1))/(b*tan(c/2)**2 + b), True))*cos(a) + Piecewise((-log(s
in(b*x))/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b
, 0)), (-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2
*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) -
tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) -
 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(
b*tan(c/2)**2 + b), True))*sin(a)
```

3.213.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.70

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^2+\sin\left(2\,bx\right)^2-2\,\sin}{2\,b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2))/b
```

3.213.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(27) = 54$.

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.85

$$\int \sec(c+bx)\sin(a+bx)\,dx = \frac{\frac{4\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\,c\right)\right)(bx+c)}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)^2+\tan\left(\frac{1}{2}\,c\right)^2+1}+\frac{\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,a\right)^2+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right)^2}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)^2+\tan\left(\frac{1}{2}\,c\right)^2}}{2\,b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c
)^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*log(tan(b*x + c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

3.213.9 Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.15

$$\int \sec(c+bx)\sin(a+bx)\,dx = x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)+x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}+\frac{e^{a\,1i-c\,1i}\,1i}{2}\right) - \frac{\ln\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)}{b}$$

input
```
int(sin(a + b*x)/cos(c + b*x),x)
```

output
```
x*((exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2) + x*((exp(c*1i - a*1
i)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - (log(exp(a*2i - c*2i) + exp(a*2i + b
*x*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i - c*1i)/2))/b
```

3.214 $\int \sec^2(c+bx)\sin(a+bx)\,dx$

3.214.1 Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \sec^2(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{b}$$

output
```
cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

3.214.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.59

$$\int \sec^2(c+bx)\sin(a+bx)\,dx$$
$$=\frac{\cos(a-c)\sec(c+bx)}{b}-\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}$$

input
```
Integrate[Sec[c + b*x]^2*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x
)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*
Sin[c])]*Sin[a - c])/b
```

3.214.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5091, 3042, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 5091}$$

$$\sin(a-c)\int \sec(c+bx)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\cos(a-c)\int 1d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 24}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}$$

input
```
Int[Sec[c + b*x]^2*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

3.214.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.214.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.38

method	result
risch	$\frac{e^{i(xb+3a)}+e^{i(xb+a+2c)}}{b\left(e^{2i(xb+a+c)}+e^{2ia}\right)}-\frac{\ln\left(e^{i(xb+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}+\frac{\ln\left(e^{i(xb+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{4(2\sin(a)\cos(c)-2\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+8\cos(a)\cos(c)+8\sin(a)\sin(c)}{\left(-4\cos(c)^2\sin(a)^2-4\cos(a)^2\cos(c)^2-4\sin(a)^2\sin(c)^2-4\cos(a)^2\sin(c)^2\right)\left(\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2+\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\right.}$

input
```
int(sec(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-
ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)+ln(exp(I*(b*x+a))+I*exp(I*(a-
c)))/b*sin(a-c)
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+c)\log(\sin(bx+c)+1)\sin(-a+c) - \cos(bx+c)\log(-\sin(bx+c)+1)\sin(-a+c) - 2\cos(-a+c)}{2\,b\cos(bx+c)}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cos(b*x + c)*log(sin(b*x + c) + 1)*sin(-a + c) - cos(b*x + c)*log(-s
in(b*x + c) + 1)*sin(-a + c) - 2*cos(-a + c))/(b*cos(b*x + c))
```

3.214.6 Sympy [F(-2)]

Exception generated.

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+c)**2*sin(b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.214.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(34) = 68$.

Time = 0.42 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \sec^2(c + bx) \sin(a + bx)\, dx$$
$$= \frac{2\left(\cos\left(bx + 2a\right) + \cos\left(bx + 2c\right)\right)\cos\left(2bx + a + 2c\right) + 2\cos\left(bx + 2a\right)\cos\left(a\right) + 2\cos\left(bx + 2c\right)\cos\left(a\right) +}{}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 2*cos(b*x
+ 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*sin(-a +
 c) + 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a + 2*c)^2*s
in(-a + c) + 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)^2 + sin(a
)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*
c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c
)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x +
2*c)*sin(c) + sin(c)^2)) + 2*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x +
 a + 2*c) + 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*
x + a + 2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^
2 + 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

3.214.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(34) = 68$.

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 7.29

$$\int \sec^2(c + bx) \sin(a + bx)\, dx$$
$$= \frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx + \frac{1}{2}c\right) + 1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1} - \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2 + \tan\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 + \tan}\right.}{b}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
2*((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1
/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan
(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(
1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(ta
n(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2
*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)
/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b
*x + 1/2*c)^2 - 1)))/b
```

3.214.9 Mupad [B] (verification not implemented)

Time = 26.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 7.47

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)}{b\,\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)} + \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} - \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^2,x)
```

output
```
(exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i) + exp(a*2
i + b*x*2i))) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i)
- (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*
2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log
(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*
2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2
i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

3.215 $\int \sec^3(c+bx)\sin(a+bx)\,dx$

3.215.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sec^3(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-c)\sec^2(c+bx)}{2b}+\frac{\sin(a-c)\tan(c+bx)}{b}$$

output
```
1/2*cos(a-c)*sec(b*x+c)^2/b+sin(a-c)*tan(b*x+c)/b
```

3.215.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \sec^3(c+bx)\sin(a+bx)\,dx=\frac{\sec(c)\sec^2(c+bx)(\cos(a)+\sin(a-c)\sin(c+2bx))}{2b}$$

input
```
Integrate[Sec[c + b*x]^3*Sin[a + b*x],x]
```

output
```
(Sec[c]*Sec[c + b*x]^2*(Cos[a] + Sin[a - c]*Sin[c + 2*b*x]))/(2*b)
```

3.215.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5091, 3042, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 5091}$$

$$\sin(a-c)\int \sec^2(c+bx)dx+\cos(a-c)\int \sec^2(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx+\cos(a-c)\int \sec(c+bx)^2\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\cos(a-c)\int \sec(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\cos(a-c)\sec^2(bx+c)}{2b}-\frac{\sin(a-c)\int 1d(-\tan(c+bx))}{b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

input
```
Int[Sec[c + b*x]^3*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^2)/(2*b) + (Sin[a - c]*Tan[c + b*x])/b
```

3.215.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.215.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result
parallelrisch	$\frac{-2\cos(2xb+a+c)+1+\cos(2xb+2c)}{2b(\cos(2xb+2c)+1)}$
risch	$\frac{2\,e^{i(2xb+5a+c)}+e^{i(5a-c)}-e^{i(3a+c)}}{\left(e^{2i(xb+a+c)}+e^{2ia}\right)^2 b}$
default	$\frac{\frac{\cos(a)\cos(c)+\sin(a)\sin(c)}{2(\sin(a)\cos(c)-\cos(a)\sin(c))^2(\tan(xb+a)\sin(a)\cos(c)-\tan(xb+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^2}-\frac{}{(\sin(a)\cos(c)-\cos(a)\sin(c))^2}}{b}$

input
```
int(sec(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-2*cos(2*b*x+a+c)+1+cos(2*b*x+2*c))/(cos(2*b*x+2*c)+1)
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = -\frac{2\cos(bx+c)\sin(bx+c)\sin(-a+c) - \cos(-a+c)}{2\,b\cos(bx+c)^2}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c))/(b*cos(b*x +
c)^2)
```

3.215.6 Sympy [F(-2)]

Exception generated.

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+c)**3*sin(b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.215.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(36) = 72.

Time = 0.24 (sec) , antiderivative size = 391, normalized size of antiderivative = 10.29

$$\int \sec^3(c+bx)\sin(a+bx)\,dx$$
$$= \frac{(2\cos(2\,bx+2\,a+2\,c)+\cos(2\,a)-\cos(2\,c))\cos(4\,bx+a+5\,c)+2\,(2\cos(2\,bx+2\,a+2\,c)+\cos(2\,a)}{b\cos(4\,bx+a+5\,c)^2+4\,b\cos(2\,bx+a+3\,c)^2+4\,b\cos(2\,b}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
((2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 2
*(2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) + (
cos(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2
*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(4*b*x + a + 5*c) + 2*(2
*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2*b*x + a + 3*c) + (sin
(2*a) - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*
b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2
*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(36) = 72$.

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.58

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \frac{\tan(bx+c)^2\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 - \tan(bx+c)^2\tan\left(\frac{1}{2}a\right)^2 + 4\tan(bx+c)^2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right) + 4\tan(\cdots}{2\cdots}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*(tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)^2*tan(1/2*a)^
2 + 4*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 4*tan(b*x + c)*tan(1/2*a)^2*t
an(1/2*c) - tan(b*x + c)^2*tan(1/2*c)^2 - 4*tan(b*x + c)*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x + c)^2 + 4*tan(b*x + c)*tan(1/2*a) - 4*tan(b*x + c)*tan(1
/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.216 $\int \sec^4(c+bx)\sin(a+bx)\,dx$

3.216.1 Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \frac{\cos(a-c)\sec^3(c+bx)}{3b} + \frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{2b} + \frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}$$

output
```
1/3*cos(a-c)*sec(b*x+c)^3/b+1/2*arctanh(sin(b*x+c))*sin(a-c)/b+1/2*sec(b*x
+c)*sin(a-c)*tan(b*x+c)/b
```

3.216.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \frac{12\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+\sec^3(c+bx)(4\cos(a-c)+3\sin(a-c)\sin(2(c+bx)))}{12b}$$

input
```
Integrate[Sec[c + b*x]^4*Sin[a + b*x],x]
```

output
```
(12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + Sec[c + b*x]^3*(4*C
os[a - c] + 3*Sin[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

3.216.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5091, 3042, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec^3(c+bx)dx+\cos(a-c)\int \sec^3(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\cos(a-c)\int \sec(c+bx)^3\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int \sec^2(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\sin(a-c)\left(\frac{1}{2}\int \sec(c+bx)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\left(\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4257}$$

$$\sin(a-c)\left(\frac{\text{arctanh}(\sin(bx+c))}{2b}+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

input
```
Int[Sec[c + b*x]^4*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^3)/(3*b) + Sin[a - c]*(ArcTanh[Sin[c + b*x]]/(2*b
) + (Sec[c + b*x]*Tan[c + b*x])/(2*b))
```

3.216.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.216.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.45 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.85

method	result
risch	$\frac{-3\,\mathrm{e}^{i(5xb+7a+4c)}+3\,\mathrm{e}^{i(5xb+5a+6c)}+8\,\mathrm{e}^{i(3xb+7a+2c)}+8\,\mathrm{e}^{i(3xb+5a+4c)}+3\,\mathrm{e}^{i(xb+7a)}-3\,\mathrm{e}^{i(xb+5a+2c)}}{6b\left(\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^{3}}+\frac{\ln\left(\mathrm{e}^{i(xb+a)}+i\mathrm{e}^{i(a-c)}\right)\sin(a-}{2b}$
default	Expression too large to display

input
```
int(sec(b*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))+3*exp(I
*(5*b*x+5*a+6*c))+8*exp(I*(3*b*x+7*a+2*c))+8*exp(I*(3*b*x+5*a+4*c))+3*exp(
I*(b*x+7*a))-3*exp(I*(b*x+5*a+2*c)))+1/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))
/b*sin(a-c)-1/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \sec^4(c+bx)\sin(a+bx)\,dx =$$
$$-\frac{3\,\cos\left(bx+c\right)^{3}\log\left(\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-3\,\cos\left(bx+c\right)^{3}\log\left(-\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)}{12\,b\cos\left(bx+c\right)^{3}}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/12*(3*cos(b*x + c)^3*log(sin(b*x + c) + 1)*sin(-a + c) - 3*cos(b*x + c)
^3*log(-sin(b*x + c) + 1)*sin(-a + c) + 6*cos(b*x + c)*sin(b*x + c)*sin(-a
 + c) - 4*cos(-a + c))/(b*cos(b*x + c)^3)
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**4*sin(b*x+a),x)
```

output
```
Timed out
```

3.216.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(61) = 122$.

Time = 0.44 (sec) , antiderivative size = 1424, normalized size of antiderivative = 21.25

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/12*(2*(3*cos(5*b*x + 2*a + 4*c) - 3*cos(5*b*x + 6*c) - 8*cos(3*b*x + 2*
a + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) + 3*cos(b*x + 2*c))*cos(6
*b*x + a + 6*c) + 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*c) + cos
(a))*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a
+ 2*c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) + 8*cos(3*
b*x + 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 1
6*(3*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b
*x + a + 2*c) + cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) - cos(b*x +
2*c))*cos(2*b*x + a + 2*c) - 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*co
s(a) - 3*(cos(6*b*x + a + 6*c)^2*sin(-a + c) + 9*cos(4*b*x + a + 4*c)^2*si
n(-a + c) + 9*cos(2*b*x + a + 2*c)^2*sin(-a + c) + 6*cos(2*b*x + a + 2*c)*
cos(a)*sin(-a + c) + sin(6*b*x + a + 6*c)^2*sin(-a + c) + 9*sin(4*b*x + a
+ 4*c)^2*sin(-a + c) + 9*sin(2*b*x + a + 2*c)^2*sin(-a + c) + 6*sin(2*b*x
+ a + 2*c)*sin(a)*sin(-a + c) + 2*(3*cos(4*b*x + a + 4*c)*sin(-a + c) + 3*
cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(6*b*x + a + 6*c
) + 6*(3*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(4*b*x
+ a + 4*c) + 2*(3*sin(4*b*x + a + 4*c)*sin(-a + c) + 3*sin(2*b*x + a + 2*c
)*sin(-a + c) + sin(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 6*(3*sin(2*b*x
+ a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4*c) + (cos(a
)^2 + sin(a)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c...
```

3.216.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 495, normalized size of antiderivative = 7.39

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
1/3*(3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a
)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1
))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(3*ta
n(1/2*b*x + 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*b*x + 1/2*c)^5*ta
n(1/2*a)*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^2
 + 3*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*
a)^2 - 3*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^4*tan
(1/2*a)*tan(1/2*c) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2 - 3*tan(1/2*b*x
 + 1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*b
*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1/
2*b*x + 1/2*c)*tan(1/2*a) + tan(1/2*a)^2 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*
c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)/((tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*c)^2 - 1)^3))/b
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

3.217 $\int \sec^5(c+bx)\sin(a+bx)\,dx$

3.217.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \frac{\cos(a-c)\sec^4(c+bx)}{4b} + \frac{\sin(a-c)\tan(c+bx)}{b} + \frac{\sin(a-c)\tan^3(c+bx)}{3b}$$

output
```
1/4*cos(a-c)*sec(b*x+c)^4/b+sin(a-c)*tan(b*x+c)/b+1/3*sin(a-c)*tan(b*x+c)^
3/b
```

3.217.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \frac{\sec(c)\sec^4(c+bx)(3\cos(a)+\sin(a-c)(4\sin(c+2bx)+\sin(3c+4bx)))}{12b}$$

input
```
Integrate[Sec[c + b*x]^5*Sin[a + b*x],x]
```

output
```
(Sec[c]*Sec[c + b*x]^4*(3*Cos[a] + Sin[a - c]*(4*Sin[c + 2*b*x] + Sin[3*c
+ 4*b*x])))/(12*b)
```

3.217.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5091, 3042, 3086, 15, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^5(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec^4(c+bx)dx+\cos(a-c)\int \sec^4(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx+\cos(a-c)\int \sec(c+bx)^4\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int \sec^3(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx+\frac{\cos(a-c)\sec^4(bx+c)}{4b}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\cos(a-c)\sec^4(bx+c)}{4b}-\frac{\sin(a-c)\int \left(\tan^2(c+bx)+1\right)d(-\tan(c+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(a-c)\sec^4(bx+c)}{4b}-\frac{\sin(a-c)\left(-\frac{1}{3}\tan^3(bx+c)-\tan(bx+c)\right)}{b}$$

input
```
Int[Sec[c + b*x]^5*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^4)/(4*b) - (Sin[a - c]*(-Tan[c + b*x] - Tan[c + b
*x]^3/3))/b
```

3.217.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.217.4 Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

method	result
parallelrisch	$\frac{-32\cos(2xb+a+c)-15-20\cos(2xb+2c)-5\cos(4xb+4c)-8\cos(4xb+a+3c)}{12b(\cos(4xb+4c)+4\cos(2xb+2c)+3)}$
risch	$\frac{4\,\mathrm{e}^{i(4xb+9a+3c)}+\frac{8\,\mathrm{e}^{i(2xb+9a+c)}}{3}-\frac{8\,\mathrm{e}^{i(2xb+7a+3c)}}{3}+\frac{2\,\mathrm{e}^{i(9a-c)}}{3}-\frac{2\,\mathrm{e}^{i(7a+c)}}{3}}{\left(\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^4 b}$
default	$-\frac{1}{(\sin(a)\cos(c)-\cos(a)\sin(c))^4(\tan(xb+a)\sin(a)\cos(c)-\tan(xb+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}+\frac{}{2(\sin(a)\cos(c)-\cos(a)\sin(c))}$

input
```
int(sec(b*x+c)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-32*cos(2*b*x+a+c)-15-20*cos(2*b*x+2*c)-5*cos(4*b*x+4*c)-8*cos(4*b*x
+a+3*c))/b/(cos(4*b*x+4*c)+4*cos(2*b*x+2*c)+3)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \sec^5(c+bx)\sin(a+bx)\,dx$$
$$= -\frac{4\left(2\cos\left(bx+c\right)^3+\cos\left(bx+c\right)\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-3\cos\left(-a+c\right)}{12\,b\cos\left(bx+c\right)^4}$$

input
```
integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/12*(4*(2*cos(b*x + c)^3 + cos(b*x + c))*sin(b*x + c)*sin(-a + c) - 3*co
s(-a + c))/(b*cos(b*x + c)^4)
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**5*sin(b*x+a),x)
```

output
```
Timed out
```

3.217.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 1074, normalized size of antiderivative = 18.20

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

output
```
2/3*((6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x +
4*c) + cos(2*a) - cos(2*c))*cos(8*b*x + a + 9*c) + 4*(6*cos(4*b*x + 2*a +
4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) - cos(2*c)
)*cos(6*b*x + a + 7*c) + 6*(4*cos(2*b*x + a + 3*c) + cos(a + c))*cos(4*b*x
 + 2*a + 4*c) + 6*(6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4
*cos(2*b*x + 4*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 4*(4*cos(2
*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) - 4*(4*cos(2
*b*x + a + 3*c) + cos(a + c))*cos(2*b*x + 4*c) + (cos(2*a) - cos(2*c))*cos
(a + c) + 4*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (6*sin(4*b*x + 2*a + 4*c)
+ 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin
(8*b*x + a + 9*c) + 4*(6*sin(4*b*x + 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c)
 - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin(6*b*x + a + 7*c) + 6*(4*s
in(2*b*x + a + 3*c) + sin(a + c))*sin(4*b*x + 2*a + 4*c) + 6*(6*sin(4*b*x
+ 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) -
sin(2*c))*sin(4*b*x + a + 5*c) + 4*(4*sin(2*b*x + 2*a + 2*c) + sin(2*a) -
sin(2*c))*sin(2*b*x + a + 3*c) - 4*(4*sin(2*b*x + a + 3*c) + sin(a + c))*s
in(2*b*x + 4*c) + (sin(2*a) - sin(2*c))*sin(a + c) + 4*sin(2*b*x + 2*a + 2
*c)*sin(a + c))/(b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 +
36*b*cos(4*b*x + a + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 + 8*b*cos(2*b*x
+ a + 3*c)*cos(a + c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16*...
```

3.217.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(55) = 110$.

Time = 0.32 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.54

$$\int \sec^5(c+bx)\sin(a+bx)\,dx$$
$$= \frac{3\,\tan\left(bx+c\right)^4\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - 3\,\tan\left(bx+c\right)^4\tan\left(\frac{1}{2}\,a\right)^2 + 12\,\tan\left(bx+c\right)^4\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right) + 8}{}$$

input
```
integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

output
```
1/12*(3*tan(b*x + c)^4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(b*x + c)^4*tan(1/
2*a)^2 + 12*tan(b*x + c)^4*tan(1/2*a)*tan(1/2*c) + 8*tan(b*x + c)^3*tan(1/
2*a)^2*tan(1/2*c) - 3*tan(b*x + c)^4*tan(1/2*c)^2 - 8*tan(b*x + c)^3*tan(1
/2*a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(b*
x + c)^4 + 8*tan(b*x + c)^3*tan(1/2*a) - 6*tan(b*x + c)^2*tan(1/2*a)^2 - 8
*tan(b*x + c)^3*tan(1/2*c) + 24*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 24*
tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c) - 6*tan(b*x + c)^2*tan(1/2*c)^2 - 24*
tan(b*x + c)*tan(1/2*a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2 + 24*tan(b*x + c)*
tan(1/2*a) - 24*tan(b*x + c)*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan
(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^5,x)
```

output
```
\text{Hanged}
```

3.218 $\int \sec^6(c+bx)\sin(a+bx)\,dx$

3.218.1 Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \frac{\cos(a-c)\sec^5(c+bx)}{5b} + \frac{3\text{arctanh}(\sin(c+bx))\sin(a-c)}{8b} + \frac{3\sec(c+bx)\sin(a-c)\tan(c+bx)}{8b} + \frac{\sec^3(c+bx)\sin(a-c)\tan(c+bx)}{4b}$$

output
```
1/5*cos(a-c)*sec(b*x+c)^5/b+3/8*arctanh(sin(b*x+c))*sin(a-c)/b+3/8*sec(b*x
+c)*sin(a-c)*tan(b*x+c)/b+1/4*sec(b*x+c)^3*sin(a-c)*tan(b*x+c)/b
```

3.218.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \frac{480\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+2\sec^5(c+bx)(64\cos(a-c)+5\sin(a-c)(14\sin(2(c+}{640b}$$

input
```
Integrate[Sec[c + b*x]^6*Sin[a + b*x],x]
```

output
```
(480*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + 2*Sec[c + b*x]^5*(
64*Cos[a - c] + 5*Sin[a - c]*(14*Sin[2*(c + b*x)] + 3*Sin[4*(c + b*x)])))/
(640*b)
```

3.218.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5091, 3042, 3086, 15, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^6(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec^5(c+bx)dx+\cos(a-c)\int \sec^5(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx+\cos(a-c)\int \sec(c+bx)^5\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int \sec^4(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\sin(a-c)\left(\frac{3}{4}\int \sec^3(c+bx)dx+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\left(\frac{3}{4}\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\sin(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int\sec(c+bx)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$\downarrow$ 3042

$$\sin(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int\csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$\downarrow$ 4257

$$\sin(a-c)\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(bx+c))}{2b}+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

input
```
Int[Sec[c + b*x]^6*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^5)/(5*b) + Sin[a - c]*((Sec[c + b*x]^3*Tan[c + b*
x])/(4*b) + (3*(ArcTanh[Sin[c + b*x]]/(2*b) + (Sec[c + b*x]*Tan[c + b*x])/
(2*b)))/4)
```

3.218.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.218.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 49.84 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.76

method	result
risch	$\frac{-15\,\mathrm{e}^{i(9xb+11a+8c)}+15\,\mathrm{e}^{i(9xb+9a+10c)}-70\,\mathrm{e}^{i(7xb+11a+6c)}+70\,\mathrm{e}^{i(7xb+9a+8c)}+128\,\mathrm{e}^{i(5xb+11a+4c)}+128\,\mathrm{e}^{i(5xb+9a+6c)}+70\,\mathrm{e}^{i(3xb+11a}}{40b\left(\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^{5}}$
default	Expression too large to display

input
```
int(sec(b*x+c)^6*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/40/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))^5*(-15*exp(I*(9*b*x+11*a+8*c))+15*e
xp(I*(9*b*x+9*a+10*c))-70*exp(I*(7*b*x+11*a+6*c))+70*exp(I*(7*b*x+9*a+8*c)
)+128*exp(I*(5*b*x+11*a+4*c))+128*exp(I*(5*b*x+9*a+6*c))+70*exp(I*(3*b*x+1
1*a+2*c))-70*exp(I*(3*b*x+9*a+4*c))+15*exp(I*(b*x+11*a))-15*exp(I*(b*x+9*a
+2*c)))-3/8*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)+3/8*ln(exp(I*(b*x
+a))+I*exp(I*(a-c)))/b*sin(a-c)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int \sec^6(c+bx)\sin(a+bx)\,dx =$$
$$-\frac{15\cos(bx+c)^5\log(\sin(bx+c)+1)\sin(-a+c)-15\cos(bx+c)^5\log(-\sin(bx+c)+1)\sin(-a+}{80\,b\cos(bx+c)^5}$$

input
```
integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/80*(15*cos(b*x + c)^5*log(sin(b*x + c) + 1)*sin(-a + c) - 15*cos(b*x +
c)^5*log(-sin(b*x + c) + 1)*sin(-a + c) + 10*(3*cos(b*x + c)^3 + 2*cos(b*x
 + c))*sin(b*x + c)*sin(-a + c) - 16*cos(-a + c))/(b*cos(b*x + c)^5)
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**6*sin(b*x+a),x)
```

output
```
Timed out
```

3.218.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3096 vs. 2(86) = 172.

Time = 0.52 (sec) , antiderivative size = 3096, normalized size of antiderivative = 32.94

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/80*(2*(15*cos(9*b*x + 2*a + 8*c) - 15*cos(9*b*x + 10*c) + 70*cos(7*b*x
+ 2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(
5*b*x + 6*c) - 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*
x + 2*a) + 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) + 30*(5*cos(8*b*x + a
 + 8*c) + 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x
+ a + 2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) +
 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c
) + cos(a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) - 70*cos(7*b
*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b
*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x + 2*a) + 15*cos(b*x + 2
*c))*cos(8*b*x + a + 8*c) + 140*(10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x +
a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(
10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c)
 + cos(a))*cos(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) + 128*cos(5*b
*x + 6*c) + 70*cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x +
 2*a) - 15*cos(b*x + 2*c))*cos(6*b*x + a + 6*c) - 256*(10*cos(4*b*x + a +
4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*c
os(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) -
100*(14*cos(3*b*x + 2*a + 2*c) - 14*cos(3*b*x + 4*c) + 3*cos(b*x + 2*a) -
3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 140*(5*cos(2*b*x + a + 2*c) + ...
```

3.218.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(86) = 172$.

Time = 0.32 (sec) , antiderivative size = 756, normalized size of antiderivative = 8.04

$$\int \sec^6(c + bx)\sin(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

output
```
1/20*(15*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
 tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2
 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 15*(tan(1/2*a)^2*tan(1/2*c) - tan(1/
2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c)
- 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(2
5*tan(1/2*b*x + 1/2*c)^9*tan(1/2*a)^2*tan(1/2*c) - 25*tan(1/2*b*x + 1/2*c)
^9*tan(1/2*a)*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^8*tan(1/2*a)^2*tan(1/
2*c)^2 + 25*tan(1/2*b*x + 1/2*c)^9*tan(1/2*a) + 20*tan(1/2*b*x + 1/2*c)^8*
tan(1/2*a)^2 - 25*tan(1/2*b*x + 1/2*c)^9*tan(1/2*c) - 80*tan(1/2*b*x + 1/2
*c)^8*tan(1/2*a)*tan(1/2*c) - 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*a)^2*tan(1
/2*c) + 20*tan(1/2*b*x + 1/2*c)^8*tan(1/2*c)^2 + 10*tan(1/2*b*x + 1/2*c)^7
*tan(1/2*a)*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^8 - 10*tan(1/2*b*x + 1/
2*c)^7*tan(1/2*a) + 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*c) - 40*tan(1/2*b*x
+ 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^2 + 40*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a
)^2 - 160*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)*tan(1/2*c) + 10*tan(1/2*b*x +
1/2*c)^3*tan(1/2*a)^2*tan(1/2*c) + 40*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2
- 10*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^2 - 40*tan(1/2*b*x + 1/2
*c)^4 + 10*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a) - 10*tan(1/2*b*x + 1/2*c)^3*t
an(1/2*c) - 25*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 25*tan(1/2*b
*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^2 - 25*...
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^6,x)
```

output
```
\text{Hanged}
```

3.219 $\int \cos^n(c+dx)\sin^2(a+bx)\,dx$

3.219.1 Optimal result

Integrand size = 17, antiderivative size = 386

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = -\frac{i2^{-2-n}e^{-i(2a+cn)-i(2b+dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-\right.\right.}{2b+dn} + \frac{i2^{-2-n}e^{i(2a-cn)+i(2b-dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{2b}{d}-n\right)\right.}{2b-dn} + \frac{i2^{-1-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},-e^{2i(c+dx)}\right)}{dn}$$

output
```
-I*2^(-2-n)*exp(-I*(c*n+2*a)-I*(d*n+2*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+e
xp(I*(d*x+c)))^n*hypergeom([-n, -b/d-1/2*n],[1-b/d-1/2*n],-exp(2*I*(d*x+c)
))/((1+exp(2*I*c+2*I*d*x))^n)/(d*n+2*b)+I*2^(-2-n)*exp(I*(-c*n+2*a)+I*(-d*
n+2*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, b/
d-1/2*n],[1+b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n
+2*b)+I*2^(-1-n)*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, -1/2*n]
,[1-1/2*n],-exp(2*I*(d*x+c)))/d/((1+exp(2*I*(d*x+c)))^n)/n
```

3.219.2 Mathematica [A] (warning: unable to verify)

Time = 1.46 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.63

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx =$$

$$-\frac{i2^{-2-n}e^{-2i(a+bx)+i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+n}\left(dn(-2b+dn)\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{d}+\frac{n}{2},1\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^n*Sin[a + b*x]^2,x]
```

output
```
((-I)*2^(-2 - n)*E^((-2*I)*(a + b*x) + I*(c + d*x))*((1 + E^((2*I)*(c + d*
x)))/E^(I*(c + d*x)))^(1 + n)*(d*n*(-2*b + d*n)*Hypergeometric2F1[1, 1 - b
/d + n/2, 1 - b/d - n/2, -E^((2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(2*b
+ d*n)*(d*E^((2*I)*(a + b*x))*n*Hypergeometric2F1[1, 1 + b/d + n/2, 1 + b/
d - n/2, -E^((2*I)*(c + d*x))] + 2*(2*b - d*n)*Hypergeometric2F1[1, (2 + n
)/2, 1 - n/2, -E^((2*I)*(c + d*x))])))/(-4*b^2*d*n + d^3*n^3)
```

3.219.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^n(c+dx)\,dx$$

$$\Big\downarrow \text{5066}$$

$$2^{-n-2}\int \left(-e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n-e^{2ia+2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n+2\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2^{-n-2}\left(-\frac{i\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n,\frac{1}{2}\left(-\frac{2b}{d}-n+2\right),-e^{2i(\right.}{2b+dn}\right.$$

input
```
Int[Cos[c + d*x]^n*Sin[a + b*x]^2,x]
```

output
```
2^(-2 - n)*(((-I)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(
E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2
, -n, (2 - (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)
*d*x))^n*(2*b + d*n)) + (I*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*(c + d
*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*b)/d -
 n)/2, -n, (2 + (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c +
(2*I)*d*x))^n*(2*b - d*n)) + ((2*I)*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))
^n*Hypergeometric2F1[-n, -1/2*n, 1 - n/2, -E^((2*I)*(c + d*x))])/(d*(1 + E
^((2*I)*(c + d*x)))^n*n))
```

3.219.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

3.219.4 Maple [F]

$$\int \cos\left(dx+c\right)^{n} \sin\left(xb+a\right)^{2} dx$$

input
```
int(cos(d*x+c)^n*sin(b*x+a)^2,x)
```

output
```
int(cos(d*x+c)^n*sin(b*x+a)^2,x)
```

3.219.5 Fricas [F]

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = \int \cos\left(dx+c\right)^{n} \sin\left(bx+a\right)^{2}\,dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^n, x)
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**n*sin(b*x+a)**2,x)
```

output
```
Timed out
```

3.219.7 Maxima [F]

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = \int \cos(dx+c)^n\sin(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^n*sin(b*x + a)^2, x)
```

3.219.8 Giac [F]

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = \int \cos(dx+c)^n\sin(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^n*sin(b*x + a)^2, x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = \int \cos\left(c+d\,x\right)^n\sin\left(a+b\,x\right)^2\,dx$$

input
```
int(cos(c + d*x)^n*sin(a + b*x)^2,x)
```

output
```
int(cos(c + d*x)^n*sin(a + b*x)^2, x)
```

3.220 $\int \cos(c+dx)\sin^2(a+bx)\,dx$

3.220.1 Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = -\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)} + \frac{\sin(c+dx)}{2d} - \frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}$$

output
```
-1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/2*sin(d*x+c)/d-1/4*sin(2*a+c+(2*b+d)*x
)/(2*b+d)
```

3.220.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = \frac{1}{4}\left(\frac{2\cos(dx)\sin(c)}{d} + \frac{2\cos(c)\sin(dx)}{d} - \frac{\sin(2a-c+2bx-dx)}{2b-d} - \frac{\sin(2a+c+2bx+dx)}{2b+d}\right)$$

input
```
Integrate[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
((2*Cos[d*x]*Sin[c])/d + (2*Cos[c]*Sin[d*x])/d - Sin[2*a - c + 2*b*x - d*x
]/(2*b - d) - Sin[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

3.220.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(-\frac{1}{4}\cos(2a+x(2b-d)-c)-\frac{1}{4}\cos(2a+x(2b+d)+c)+\frac{1}{2}\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2a+x(2b-d)-c)}{4(2b-d)}-\frac{\sin(2a+x(2b+d)+c)}{4(2b+d)}+\frac{\sin(c+dx)}{2d}$$

input
```
Int[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
-1/4*Sin[2*a - c + (2*b - d)*x]/(2*b - d) + Sin[c + d*x]/(2*d) - Sin[2*a +
 c + (2*b + d)*x]/(4*(2*b + d))
```

3.220.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.220.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\sin(dx+c)}{2d}-\frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}$
parallelrisch	$\frac{(-2bd-d^2)\sin(2a-c+(2b-d)x)+(-2bd+d^2)\sin(2a+c+(2b+d)x)+(8b^2-2d^2)\sin(dx+c)}{16b^2d-4d^3}$
risch	$\frac{2\sin(dx+c)b^2}{d(2b-d)(2b+d)}-\frac{d\sin(dx+c)}{2(2b-d)(2b+d)}-\frac{\sin(2xb-dx+2a-c)b}{2(2b-d)(2b+d)}-\frac{d\sin(2xb-dx+2a-c)}{4(2b-d)(2b+d)}-\frac{\sin(2xb+dx+2a+c)b}{2(2b-d)(2b+d)}+\frac{d\sin(2xb}{4(2b-}$
norman	$\frac{-\frac{4b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{4b^2-d^2}+\frac{4b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{4b^2-d^2}+\frac{4b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4b^2-d^2}-\frac{4b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4b^2-d^2}+\frac{4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(4b^2-d^2)d}+\frac{4b^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right.}{(4b^2-}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^2}$

input
```
int(cos(d*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/2*sin(d*x+c)/d-1/4*sin(2*a+c+(2*b+d)*x
)/(2*b+d)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \cos(c+dx)\sin^2(a+bx)\,dx$$
$$= -\frac{2\,bd\cos(bx+a)\cos(dx+c)\sin(bx+a)-\left(d^2\cos(bx+a)^2+2\,b^2-d^2\right)\sin(dx+c)}{4\,b^2d-d^3}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(b*x + a) - (d^2*cos(b*x + a)^2 + 2*b
^2 - d^2)*sin(d*x + c))/(4*b^2*d - d^3)
```

3.220.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(49) = 98$.

Time = 0.72 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.03

$$\int \cos(c+dx)\sin^2(a+bx)\,dx$$

$$= \begin{cases} x\sin^2(a)\cos(c) \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{x\sin\left(a-\frac{dx}{2}\right)\sin(c+dx)\cos\left(a-\frac{dx}{2}\right)}{2} - \frac{x\cos^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{3\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos(c+dx)}{2d} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4} - \frac{x\sin\left(a+\frac{dx}{2}\right)\sin(c+dx)\cos\left(a+\frac{dx}{2}\right)}{2} - \frac{x\cos^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{\sin^2\left(a+\frac{dx}{2}\right)\sin(c+dx)}{d} + \frac{\sin\left(a+\right.}{} \\ \left(\frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\cos(c) \\ \frac{2b^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3} + \frac{2b^2\sin(c+dx)\cos^2(a+bx)}{4b^2d-d^3} - \frac{2bd\sin(a+bx)\cos(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{d^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3} \end{cases}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*
cos(c + d*x)/4 + x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)/2 - x*cos(a
- d*x/2)**2*cos(c + d*x)/4 + 3*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)/
(2*d) + sin(c + d*x)*cos(a - d*x/2)**2/d, Eq(b, -d/2)), (x*sin(a + d*x/2)*
*2*cos(c + d*x)/4 - x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)/2 - x*cos
(a + d*x/2)**2*cos(c + d*x)/4 + sin(a + d*x/2)**2*sin(c + d*x)/d + sin(a +
 d*x/2)*cos(a + d*x/2)*cos(c + d*x)/(2*d), Eq(b, d/2)), ((x*sin(a + b*x)**
2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*cos(c), Eq(d,
 0)), (2*b**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**3) + 2*b**2*sin(
c + d*x)*cos(a + b*x)**2/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*cos(a + b*
x)*cos(c + d*x)/(4*b**2*d - d**3) - d**2*sin(a + b*x)**2*sin(c + d*x)/(4*b
**2*d - d**3), True))
```

3.220.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(62) = 124$.

Time = 0.24 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.46

$$\int \cos(c+dx)\sin^2(a+bx)\,dx =$$

$$-\frac{(2\,bd\sin(c) - d^2\sin(c))\cos((2\,b+d)x + 2\,a + 2\,c) - (2\,bd\sin(c) - d^2\sin(c))\cos((2\,b+d)x + 2\,a) - (}{}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*((2*b*d*sin(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a + 2*c) - (2*b*d*si
n(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^2*sin(c))*co
s(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*cos(-(2*b - d)*x
 - 2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*cos(d*x + 2*c) + 2*(4*b^2*sin(c) -
 d^2*sin(c))*cos(d*x) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a
+ 2*c) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a) + (2*b*d*cos(c
) + d^2*cos(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*cos(c) + d^2*cos(c)
)*sin(-(2*b - d)*x - 2*a) + 2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x + 2*c) +
 2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b
^2*cos(c)^2 + b^2*sin(c)^2)*d)
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = -\frac{\sin(2bx+dx+2a+c)}{4(2b+d)} - \frac{\sin(2bx-dx+2a-c)}{4(2b-d)} + \frac{\sin(dx+c)}{2d}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*sin(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*sin(2*b*x - d*x + 2*a - c)
/(2*b - d) + 1/2*sin(d*x + c)/d
```

3.220.9 Mupad [B] (verification not implemented)

Time = 21.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.54

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = \frac{\sin(c+d\,x)}{2\,d} - \frac{b\,(2\,d\,\sin(2\,a+c+2\,b\,x+d\,x)+2\,d\,\sin(2\,a-c+2\,b\,x-d\,x))-d^2\,\sin(2\,a+c+2\,b\,x+d\,x)+d^2\,\sin}{16\,b^2\,d-4\,d^3}$$

input
```
int(cos(c + d*x)*sin(a + b*x)^2,x)
```

output
```
sin(c + d*x)/(2*d) - (b*(2*d*sin(2*a + c + 2*b*x + d*x) + 2*d*sin(2*a - c
+ 2*b*x - d*x)) - d^2*sin(2*a + c + 2*b*x + d*x) + d^2*sin(2*a - c + 2*b*x
 - d*x))/(16*b^2*d - 4*d^3)
```

3.220. $\int \cos(c+dx)\sin^2(a+bx)\,dx$

3.221 $\int \cos^2(c+dx)\sin^2(a+bx)\,dx$

3.221.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \frac{x}{4}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)} + \frac{\sin(2c+2dx)}{8d}-\frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
1/4*x-1/8*sin(2*b*x+2*a)/b-1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sin(2*d*x
+2*c)/d-1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

3.221.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \frac{(-2b^2d+2d^3)\sin(2(a+bx))-bd(b+d)\sin(2(a-c+(b-d)x))+b(b-d)(4d(b+d)x+2(b+d)\sin(2}{16b(b-d)d(b+d)}$$

input
```
Integrate[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

output
```
((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] - b*d*(b + d)*Sin[2*(a - c + (b - d)*
x)] + b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sin[2*(c + d*x)] - d*Sin[2*(a +
 c + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

3.221.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(-\frac{1}{8}\cos(2(a-c)+2x(b-d))-\frac{1}{8}\cos(2(a+c)+2x(b+d))-\frac{1}{4}\cos(2a+2bx)+\frac{1}{4}\cos(2c+2dx)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}-\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

input
```
Int[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

output
```
x/4 - Sin[2*a + 2*b*x]/(8*b) - Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
 Sin[2*c + 2*d*x]/(8*d) - Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

3.221.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.221.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}-\frac{\sin(2xb+2a)}{8b}+\frac{\sin(2dx+2c)}{8d}-\frac{\sin((2b-2d)x+2a-2c)}{8(2b-2d)}-\frac{\sin((2b+2d)x+2a+2c)}{8(2b+2d)}$
parallelrisch	$\frac{-bd(b+d)\sin((2b-2d)x+2a-2c)+4\left(-\frac{bd\sin((2b+2d)x+2a+2c)}{4}+(b+d)\left(-\frac{d\sin(2xb+2a)}{2}+b\left(dx+\frac{\sin(2dx+2c)}{2}\right)\right)\right)(b-d)}{16b^3d-16b\,d^3}$
risch	$\frac{x}{4}-\frac{\sin(2xb+2a)}{8b}+\frac{\sin(2dx+2c)b^2}{8(b-d)d(b+d)}-\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}-\frac{\sin(2xb-2dx+2a-2c)b}{16(b-d)(b+d)}-\frac{d\sin(2xb-2dx+2a-2c)}{16(b-d)(b+d)}-\frac{\sin(2xb-}{16(}$

input
```
int(cos(d*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x-1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d-1/8/(2*b-2*d)*sin((2*b-2*d
)*x+2*a-2*c)-1/8/(2*b+2*d)*sin((2*b+2*d)*x+2*a+2*c)
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx$$
$$=\frac{\left(2\,bd^2\cos\left(bx+a\right)^2+b^3-2\,bd^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx+c\right.\right.}{4\left(b^3d-bd^3\right)}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) +
(b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a
))*sin(b*x + a))/(b^3*d - b*d^3)
```

3.221.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.

Time = 1.63 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq
(b, 0)), (x*sin(a - d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a - d*x)**2*cos(c
+ d*x)**2/8 + x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + 3*
x*sin(c + d*x)**2*cos(a - d*x)**2/8 + x*cos(a - d*x)**2*cos(c + d*x)**2/8
+ sin(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) + sin(a - d*x)*cos(a - d
*x)*cos(c + d*x)**2/(2*d) + 3*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8
*d), Eq(b, -d)), (x*sin(a + d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a + d*x)**
2*cos(c + d*x)**2/8 - x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x
)/2 + 3*x*sin(c + d*x)**2*cos(a + d*x)**2/8 + x*cos(a + d*x)**2*cos(c + d*
x)**2/8 + sin(a + d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) - sin(a + d*x)*c
os(a + d*x)*cos(c + d*x)**2/(2*d) + 3*sin(c + d*x)*cos(a + d*x)**2*cos(c +
 d*x)/(8*d), Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(
a + b*x)*cos(a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)*
*2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2
/(4*b**3*d - 4*b*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*
d - 4*b*d**3) + b**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d -
 4*b*d**3) + b**3*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*
b*d**3) - 2*b**2*d*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d ...
```

3.221.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.

Time = 0.31 (sec) , antiderivative size = 620, normalized size of antiderivative = 7.05

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{8\left(\left(b\cos(2c)^2 + b\sin(2c)^2\right)d^3 - \left(b^3\cos(2c)^2 + b^3\sin(2c)^2\right)d\right)x - \left(b^2d\sin(2c) - bd^2\sin(2c)\right)\cos(2(b+\ldots}{\ldots}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c
)^2)*d)*x - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c)
 + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) + (b^2*d*sin(2
*c) + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) - (b^2*d*sin(2*c) + b*
d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*
cos(2*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a
 - 2*c) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) -
 b*d^2*sin(2*c))*cos(2*d*x + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(
2*(b + d)*x + 2*a + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)
*x + 2*a) - (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c
) - (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*c
os(2*c) - d^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*c
os(2*c))*sin(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*
d*x) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2
+ b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \frac{1}{4}\,x - \frac{\sin\left(2\,bx+2\,dx+2\,a+2\,c\right)}{16\,(b+d)} - \frac{\sin\left(2\,bx-2\,dx+2\,a-2\,c\right)}{16\,(b-d)} - \frac{\sin\left(2\,bx+2\,a\right)}{8\,b} + \frac{\sin\left(2\,dx+2\,c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*x - 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) - 1/16*sin(2*b*x - 2*d
*x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

3.221.9 Mupad [B] (verification not implemented)

Time = 22.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.01

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = -\frac{b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-2\,b^3\,\sin\left(2\,c+2\,d\,x\right)-2\,d^3\,\sin\left(2\,a+2\,b\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\right.}{}$$

input
```
int(cos(c + d*x)^2*sin(a + b*x)^2,x)
```

output
```
-(b*d^2*sin(2*a - 2*c + 2*b*x - 2*d*x) - 2*b^3*sin(2*c + 2*d*x) - 2*d^3*si
n(2*a + 2*b*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*
c + 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) + 2*b^2*d*sin(2*
a + 2*b*x) + 2*b*d^2*sin(2*c + 2*d*x) + 4*b*d^3*x - 4*b^3*d*x)/(16*b*d*(b^
2 - d^2))
```

3.222 $\int \cos^3(c+dx)\sin^2(a+bx)\,dx$

3.222.1 Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = -\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)} - \frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\sin(c+dx)}{8d} + \frac{\sin(3c+3dx)}{24d} - \frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)} - \frac{\sin(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

output
```
-1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)
+3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)-
1/16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.222.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = \frac{1}{48}\left(\frac{18\cos(dx)\sin(c)}{d} + \frac{2\cos(3dx)\sin(3c)}{d} + \frac{18\cos(c)\sin(dx)}{d} + \frac{2\cos(3c)\sin(3dx)}{d} - \frac{3\sin(2a-3c+2bx-3dx)}{2b-3d} - \frac{9\sin(2a-c+2bx-dx)}{2b-d} - \frac{9\sin(2a+c+2bx+dx)}{2b+d} - \frac{3\sin(2a+3c+2bx+3dx)}{2b+3d}\right)$$

input
```
Integrate[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

output
```
((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])
/d + (2*Cos[3*c]*Sin[3*d*x])/d - (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b -
 3*d) - (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) - (9*Sin[2*a + c + 2*b*x
+ d*x])/(2*b + d) - (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

3.222.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(-\frac{1}{16}\cos(2a+x(2b-3d)-3c)-\frac{3}{16}\cos(2a+x(2b-d)-c)-\frac{3}{16}\cos(2a+x(2b+d)+c)-\frac{1}{16}\cos(2a+x(2\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}$$

input
```
Int[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/16*Sin[2*a - 3*c + (2*b - 3*d)*x]/(2*b - 3*d) - (3*Sin[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d)
- (3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Sin[2*a + 3*c + (2*b + 3
*d)*x]/(16*(2*b + 3*d))
```

3.222.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.222.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(dx+c)}{8d}+\frac{\sin(3dx+3c)}{24d}-\frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)}-\frac{\sin(2a+3}{16(}$
parallelrisch	$\frac{(-24b^3d-36b^2d^2+6b\,d^3+9d^4)\sin(2a-3c+(2b-3d)x)-72\left(b-\frac{3d}{2}\right)\left(\left(b+\frac{d}{2}\right)\left(b+\frac{3d}{2}\right)d\sin(2a-c+(2b-d)x)+\left(b-\frac{d}{2}\right)\left(\frac{\left(b+\frac{d}{2}\right)d\,s}{}\right.\right.}{768b^4d-1920b^2d^3+432d^5}$
risch	$\frac{3\sin(dx+c)b^2}{2d(2b-d)(2b+d)}-\frac{3d\sin(dx+c)}{8(2b-d)(2b+d)}-\frac{\sin(2xb-3dx+2a-3c)b}{8(2b-3d)(2b+3d)}-\frac{3d\sin(2xb-3dx+2a-3c)}{16(2b-3d)(2b+3d)}-\frac{3\sin(2xb-dx+2a-c)b}{8(2b-d)(2b+d)}-$

input
```
int(cos(d*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)
+3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)-
1/16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.21

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx$$
$$=\frac{6\left(6\,bd^3\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-\left(18\,d^4\cos\left(bx+\right.\right.}{3\left(16\,b^4d-\right.}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a)
*cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 - 16*b^4 + 40*b^2*d
^2 - 18*d^4 - (8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x +
a)^2)*cos(d*x + c)^2)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

3.222.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2003 vs. $2(116) = 232$.

Time = 5.69 (sec) , antiderivative size = 2003, normalized size of antiderivative = 13.91

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*
x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + x*sin(a - 3*d*x/2)**2*cos(c + d*
x)**3/16 - x*sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a - 3*d*x/2)/8 + 3*x*sin
(a - 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c + d*x)**2/8 + 3*x*sin(c
+ d*x)**2*cos(a - 3*d*x/2)**2*cos(c + d*x)/16 - x*cos(a - 3*d*x/2)**2*cos(
c + d*x)**3/16 + 5*sin(a - 3*d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a - 3*
d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d + 5*sin(a - 3*d*x/2)*sin(c + d*x)
**2*cos(a - 3*d*x/2)*cos(c + d*x)/(4*d) - sin(a - 3*d*x/2)*cos(a - 3*d*x/2
)*cos(c + d*x)**3/(24*d) + 9*sin(c + d*x)**3*cos(a - 3*d*x/2)**2/(16*d), E
q(b, -3*d/2)), (3*x*sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + 3*
x*sin(a - d*x/2)**2*cos(c + d*x)**3/16 + 3*x*sin(a - d*x/2)*sin(c + d*x)**
3*cos(a - d*x/2)/8 + 3*x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(c
+ d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a - d*x/2)**2*cos(c + d*x)/16 - 3*x*
cos(a - d*x/2)**2*cos(c + d*x)**3/16 - 17*sin(a - d*x/2)**2*sin(c + d*x)**
3/(48*d) + 7*sin(a - d*x/2)*sin(c + d*x)**2*cos(a - d*x/2)*cos(c + d*x)/(4
*d) + 13*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)**3/(8*d) + 49*sin(c +
d*x)**3*cos(a - d*x/2)**2/(48*d) + sin(c + d*x)*cos(a - d*x/2)**2*cos(c +
d*x)**2/d, Eq(b, -d/2)), (3*x*sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*
x)/16 + 3*x*sin(a + d*x/2)**2*cos(c + d*x)**3/16 - 3*x*sin(a + d*x/2)*sin(
c + d*x)**3*cos(a + d*x/2)/8 - 3*x*sin(a + d*x/2)*sin(c + d*x)*cos(a + ...
```

3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(132) = 264$.

Time = 0.33 (sec) , antiderivative size = 1362, normalized size of antiderivative = 9.46

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4
*sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d
^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a)
+ 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin
(3*c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(
3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) -
9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3
*c))*cos(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3
*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) -
3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3
*c))*cos(-(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*si
n(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) + 2*
(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) - 2*(1
6*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) -
18*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c)
 + 18*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x - 2
*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4
*cos(3*c))*sin((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d
^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a)
- 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*...
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = -\frac{\sin(2bx+3dx+2a+3c)}{16(2b+3d)} - \frac{3\sin(2bx+dx+2a+c)}{16(2b+d)} - \frac{3\sin(2bx-dx+2a-c)}{16(2b-d)} - \frac{\sin(2bx-3dx+2a-3c)}{16(2b-3d)} + \frac{\sin(3dx+3c)}{24d} + \frac{3\sin(dx+c)}{8d}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/16*sin(2*b*x + d*x +
2*a + c)/(2*b + d) - 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/16*sin(
2*b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin
(d*x + c)/d
```

3.222.9 Mupad [B] (verification not implemented)

Time = 22.45 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.44

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = -e^{a\,2i-c\,1i+b\,x\,2i-d\,x\,1i}\left(\frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{b^2\,d\,128i-d^3\,32i} + \frac{3\,d\,(2\,b+d)}{b^2\,d\,128i-d^3\,32i} - \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-d)}{b^2\,d\,128i-d^3\,32i}\right) + e^{a\,2i+c\,1i+b\,x\,2i+d\,x\,1i}\left(-\frac{3\,d\,(2\,b-d)}{b^2\,d\,128i-d^3\,32i} + \frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{b^2\,d\,128i-d^3\,32i} + \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+d)}{b^2\,d\,128i-d^3\,32i}\right) - e^{a\,2i-c\,3i+b\,x\,2i-d\,x\,3i}\left(\frac{3\,d\,(2\,b+3\,d)}{b^2\,d\,384i-d^3\,864i} + \frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{b^2\,d\,384i-d^3\,864i} - \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-3\,d)}{b^2\,d\,384i-d^3\,864i}\right) + e^{a\,2i+c\,3i+b\,x\,2i+d\,x\,3i}\left(-\frac{3\,d\,(2\,b-3\,d)}{b^2\,d\,384i-d^3\,864i} + \frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{b^2\,d\,384i-d^3\,864i} + \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+3\,d)}{b^2\,d\,384i-d^3\,864i}\right)$$

input
```
int(cos(c + d*x)^3*sin(a + b*x)^2,x)
```

output
```
exp(a*2i + c*1i + b*x*2i + d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2)
)/(b^2*d*128i - d^3*32i) - (3*d*(2*b - d))/(b^2*d*128i - d^3*32i) + (3*d*e
xp(- a*4i - b*x*4i)*(2*b + d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*1i +
 b*x*2i - d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d
^3*32i) + (3*d*(2*b + d))/(b^2*d*128i - d^3*32i) - (3*d*exp(- a*4i - b*x*4
i)*(2*b - d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*3i + b*x*2i - d*x*3i)
*((3*d*(2*b + 3*d))/(b^2*d*384i - d^3*864i) + (exp(- a*2i - b*x*2i)*(8*b^2
 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*exp(- a*4i - b*x*4i)*(2*b - 3*d
))/(b^2*d*384i - d^3*864i)) + exp(a*2i + c*3i + b*x*2i + d*x*3i)*((exp(- a
*2i - b*x*2i)*(8*b^2 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*(2*b - 3*d)
)/(b^2*d*384i - d^3*864i) + (3*d*exp(- a*4i - b*x*4i)*(2*b + 3*d))/(b^2*d*
384i - d^3*864i))
```

3.222. $\int \cos^3(c+dx)\sin^2(a+bx)\,dx$

3.223 $\int \cos^n(c+dx)\sin^3(a+bx)\,dx$

3.223.1 Optimal result

Integrand size = 17, antiderivative size = 568

$$\int \cos^n(c+dx)\sin^3(a+bx)\,dx$$
$$= \frac{2^{-3-n}e^{i(3a-cn)+i(3b-dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-\right.}{3b-dn}$$
$$-\frac{3\,2^{-3-n}e^{i(a-cn)+i(b-dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,\frac{b-dn}{2d},\right.}{b-dn}$$
$$-\frac{3\,2^{-3-n}e^{-i(a+cn)-i(b+dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{b+d}{2d}\right.}{b+dn}$$
$$+\frac{2^{-3-n}e^{-i(3a+cn)-i(3b+dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{3b+}{2d}\right.}{3b+dn}$$

output
```
2^(-3-n)*exp(I*(-c*n+3*a)+I*(-d*n+3*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp
(I*(d*x+c)))^n*hypergeom([-n, 3/2*b/d-1/2*n],[1+3/2*b/d-1/2*n],-exp(2*I*(d
*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n+3*b)-3*2^(-3-n)*exp(I*(-c*n+a)+I*
(-d*n+b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n,
1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x
))^n)/(-d*n+b)-3*2^(-3-n)*exp(-I*(c*n+a)-I*(d*n+b)*x+I*n*(d*x+c))*(exp(-I*
(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n-b)/d],[1+1/2*(-d*n-b)/
d],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(d*n+b)+2^(-3-n)*exp(-I*(
c*n+3*a)-I*(d*n+3*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hyp
ergeom([-n, 1/2*(-d*n-3*b)/d],[1-3/2*b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp
(2*I*c+2*I*d*x))^n)/(d*n+3*b)
```

3.223.2 Mathematica [A] (warning: unable to verify)

Time = 2.60 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.58

$$\int \cos^n(c+dx)\sin^3(a+bx)\,dx = 2^{-3-n}e^{i(-3a+c+d(1+n)x)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+n}\left(\frac{e^{-i(3b+dn)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{3b}{d}+n\right),1-\frac{3b}{2d}-\frac{n}{2},-e^{2i(c+dx)}\right)}{3b+dn} - \frac{3e^{2ia-i(b+dn)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{b}{d}+n\right),-\frac{b+d(-2+n)}{2d},-e^{2i(c+dx)}\right)}{b+dn} + e^{i(4a+bx-dnx)}\left(\frac{e^{2i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2+\frac{3b}{d}+n\right),1+\frac{3b}{2d}-\frac{n}{2},-e^{2i(c+dx)}\right)}{3b-dn} - \frac{3\,\text{Hypergeometric2F1}\left(1,\frac{b+d(2+n)}{2d},\frac{1}{2}\left(2+\frac{b}{d}-n\right),-e^{2i(c+dx)}\right)}{b-dn}\right)\right)$$

input
```
Integrate[Cos[c + d*x]^n*Sin[a + b*x]^3,x]
```

output
```
2^(-3 - n)*E^(I*(-3*a + c + d*(1 + n)*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*
(c + d*x)))^(1 + n)*(Hypergeometric2F1[1, (2 - (3*b)/d + n)/2, 1 - (3*b)/(
2*d) - n/2, -E^((2*I)*(c + d*x))]/(E^(I*(3*b + d*n)*x)*(3*b + d*n)) - (3*E
^((2*I)*a - I*(b + d*n)*x)*Hypergeometric2F1[1, (2 - b/d + n)/2, -1/2*(b +
 d*(-2 + n))/d, -E^((2*I)*(c + d*x))])/(b + d*n) + E^(I*(4*a + b*x - d*n*x
))*((E^((2*I)*(a + b*x))*Hypergeometric2F1[1, (2 + (3*b)/d + n)/2, 1 + (3*
b)/(2*d) - n/2, -E^((2*I)*(c + d*x))])/(3*b - d*n) - (3*Hypergeometric2F1[
1, (b + d*(2 + n))/(2*d), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))])/(b - d*n
)))
```

3.223.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^n(c+dx)\,dx$$

$$\Big\downarrow \text{5066}$$

$$2^{-n-3}\int \Big(3ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n-3ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n-ie^{-3ia-3ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2^{-n-3}\Bigg(\frac{\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n,\frac{1}{2}\left(\frac{3b}{d}-n+2\right),-e^{2i(c+dx)}\right)}{3b-dn}$$

input
```
Int[Cos[c + d*x]^n*Sin[a + b*x]^3,x]
```

output
```
2^(-3 - n)*((E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(E^((-I)*
(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2
+ (3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*
(3*b - d*n)) - (3*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E^((-I)
*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2
 + b/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(b
- d*n)) - (3*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(E^((-I)*(
c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(b + d*n)/d, 1 -
 (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n)) + (E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x + I*n*(c + d*x))*(E^((
-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(3*b + d*n)
/d, (2 - (3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d
*x))^n*(3*b + d*n)))
```

3.223.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

3.223.4 Maple [F]

$$\int \cos (dx+c)^n \sin (xb+a)^3 \, dx$$

input
```
int(cos(d*x+c)^n*sin(b*x+a)^3,x)
```

output
```
int(cos(d*x+c)^n*sin(b*x+a)^3,x)
```

3.223.5 Fricas [F]

$$\int \cos^n(c+dx) \sin^3(a+bx) \, dx = \int \cos (dx+c)^n \sin (bx+a)^3 \, dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^n*sin(b*x + a), x)
```

3.223.6 Sympy [F(-1)]

Timed out.

$$\int \cos^n(c+dx) \sin^3(a+bx) \, dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**n*sin(b*x+a)**3,x)
```

output
```
Timed out
```

3.223.7 Maxima [F]

$$\int \cos^n(c+dx)\sin^3(a+bx)\,dx = \int \cos(dx+c)^n \sin(bx+a)^3\,dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^n*sin(b*x + a)^3, x)
```

3.223.8 Giac [F]

$$\int \cos^n(c+dx)\sin^3(a+bx)\,dx = \int \cos(dx+c)^n \sin(bx+a)^3\,dx$$

input
```
integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^n*sin(b*x + a)^3, x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \cos^n(c+dx)\sin^3(a+bx)\,dx = \int \cos(c+d\,x)^n \sin(a+b\,x)^3\,dx$$

input
```
int(cos(c + d*x)^n*sin(a + b*x)^3,x)
```

output
```
int(cos(c + d*x)^n*sin(a + b*x)^3, x)
```

3.224 $\int \cos(c+dx)\sin^3(a+bx)\,dx$

3.224.1 Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \cos(c+dx)\sin^3(a+bx)\,dx = -\frac{3\cos(a-c+(b-d)x)}{8(b-d)}+\frac{\cos(3a-c+(3b-d)x)}{8(3b-d)} -\frac{3\cos(a+c+(b+d)x)}{8(b+d)}+\frac{\cos(3a+c+(3b+d)x)}{8(3b+d)}$$

output
```
-3/8*cos(a-c+(b-d)*x)/(b-d)+1/8*cos(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cos(a+c+(
b+d)*x)/(b+d)+1/8*cos(3*a+c+(3*b+d)*x)/(3*b+d)
```

3.224.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)\sin^3(a+bx)\,dx = \frac{1}{8}\left(-\frac{3\cos(a-c+bx-dx)}{b-d}+\frac{\cos(3a-c+3bx-dx)}{3b-d} +\frac{\cos(3a+c+3bx+dx)}{3b+d}-\frac{3\cos(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
((-3*Cos[a - c + b*x - d*x])/(b - d) + Cos[3*a - c + 3*b*x - d*x]/(3*b - d
) + Cos[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cos[a + c + (b + d)*x])/(b +
 d))/8
```

3.224.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{3}{8}\sin(a+x(b-d)-c)-\frac{1}{8}\sin(3a+x(3b-d)-c)+\frac{3}{8}\sin(a+x(b+d)+c)-\frac{1}{8}\sin(3a+x(3b+d)+c)\right)d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}+\frac{\cos(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}+\frac{\cos(3a+x(3b+d)+c)}{8(3b+d)}$$

input
```
Int[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
(-3*Cos[a - c + (b - d)*x])/(8*(b - d)) + Cos[3*a - c + (3*b - d)*x]/(8*(3
*b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) + Cos[3*a + c + (3*b + d
)*x]/(8*(3*b + d))
```

3.224.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.224.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}+\frac{\cos(3a-c+(3b-d)x)}{24b-8d}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}+\frac{\cos(3a+c+(3b+d)x)}{24b+8d}$
parallelrisch	$\frac{-12\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{6}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{3}-24b^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{5}+12\left(\left(-3b^{3}+b\,d^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-b\,d^{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}+}{(b+d)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{3}(3}$
risch	$-\frac{27\cos(xb-dx+a-c)b^{3}}{8(b+d)(3b+d)(-b+d)(-3b+d)}-\frac{27\cos(xb-dx+a-c)b^{2}d}{8(b+d)(3b+d)(-b+d)(-3b+d)}+\frac{3\cos(xb-dx+a-c)b\,d^{2}}{8(b+d)(3b+d)(-b+d)(-3b+d)}+\frac{3\cos(xb-dx}{8(b+d)(3b+d)(-}$

input
```
int(cos(d*x+c)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(a-c+(b-d)*x)/(b-d)+1/8*cos(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cos(a+c+(
b+d)*x)/(b+d)+1/8*cos(3*a+c+(3*b+d)*x)/(3*b+d)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int \cos(c+dx)\sin^3(a+bx)\,dx =$$
$$-\frac{\left(7\,b^2d-d^3-\left(b^2d-d^3\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)\sin\left(dx+c\right)-3\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)^3-\left(3\,b^3-b\right.\right.}{9\,b^4-10\,b^2d^2+d^4}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*sin(b*x + a)*sin(d*x + c)
 - 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*cos(d*x
 + c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

3.224.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(76) = 152$.

Time = 2.04 (sec) , antiderivative size = 932, normalized size of antiderivative = 9.61

$$\int \cos(c + dx) \sin^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)**3,x)
```

output
```
Piecewise((x*sin(a)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*
cos(c + d*x)/8 + 3*x*sin(a - d*x)**2*sin(c + d*x)*cos(a - d*x)/8 + 3*x*sin
(a - d*x)*cos(a - d*x)**2*cos(c + d*x)/8 + 3*x*sin(c + d*x)*cos(a - d*x)**
3/8 - sin(a - d*x)**3*sin(c + d*x)/(8*d) + 3*sin(a - d*x)**2*cos(a - d*x)*
cos(c + d*x)/(4*d) + 3*cos(a - d*x)**3*cos(c + d*x)/(8*d), Eq(b, -d)), (x*
sin(a - d*x/3)**3*cos(c + d*x)/8 + 3*x*sin(a - d*x/3)**2*sin(c + d*x)*cos(
a - d*x/3)/8 - 3*x*sin(a - d*x/3)*cos(a - d*x/3)**2*cos(c + d*x)/8 - x*sin
(c + d*x)*cos(a - d*x/3)**3/8 + 9*sin(a - d*x/3)**3*sin(c + d*x)/(8*d) - 3
*sin(a - d*x/3)**2*cos(a - d*x/3)*cos(c + d*x)/(4*d) - cos(a - d*x/3)**3*c
os(c + d*x)/(8*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*cos(c + d*x)/8 - 3*x
*sin(a + d*x/3)**2*sin(c + d*x)*cos(a + d*x/3)/8 - 3*x*sin(a + d*x/3)*cos(
a + d*x/3)**2*cos(c + d*x)/8 + x*sin(c + d*x)*cos(a + d*x/3)**3/8 + 9*sin(
a + d*x/3)**3*sin(c + d*x)/(8*d) + 3*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(
c + d*x)/(4*d) + cos(a + d*x/3)**3*cos(c + d*x)/(8*d), Eq(b, d/3)), (3*x*s
in(a + d*x)**3*cos(c + d*x)/8 - 3*x*sin(a + d*x)**2*sin(c + d*x)*cos(a + d
*x)/8 + 3*x*sin(a + d*x)*cos(a + d*x)**2*cos(c + d*x)/8 - 3*x*sin(c + d*x)
*cos(a + d*x)**3/8 - sin(a + d*x)**3*sin(c + d*x)/(8*d) - 3*sin(a + d*x)**
2*cos(a + d*x)*cos(c + d*x)/(4*d) - 3*cos(a + d*x)**3*cos(c + d*x)/(8*d),
Eq(b, d)), (-9*b**3*sin(a + b*x)**2*cos(a + b*x)*cos(c + d*x)/(9*b**4 - 10
*b**2*d**2 + d**4) - 6*b**3*cos(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b...
```

3.224.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(89) = 178$.

Time = 0.28 (sec) , antiderivative size = 785, normalized size of antiderivative = 8.09

$$\int \cos(c + dx) \sin^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/16*((3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*cos(c))*cos((3*b
 + d)*x + 3*a + 2*c) + (3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3
*cos(c))*cos((3*b + d)*x + 3*a) + (3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*c
os(c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a + 2*c) + (3*b^3*cos(c) + b^2*d*
cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*c
os(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b + d)*x + a + 2*
c) - 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b
+ d)*x + a) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3*cos(c)
)*cos(-(b - d)*x - a + 2*c) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos
(c) - d^3*cos(c))*cos(-(b - d)*x - a) + (3*b^3*sin(c) - b^2*d*sin(c) - 3*b
*d^2*sin(c) + d^3*sin(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b
^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*sin(c))*sin((3*b + d)*x + 3*a) + (3*b^3
*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*sin(-(3*b - d)*x - 3
*a + 2*c) - (3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*si
n(-(3*b - d)*x - 3*a) - 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) +
d^3*sin(c))*sin((b + d)*x + a + 2*c) + 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) -
b*d^2*sin(c) + d^3*sin(c))*sin((b + d)*x + a) - 3*(9*b^3*sin(c) + 9*b^2*d*
sin(c) - b*d^2*sin(c) - d^3*sin(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*s
in(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^3*sin(c))*sin(-(b - d)*x - a))/(
9*b^4*cos(c)^2 + 9*b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*c...
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \cos(c+dx)\sin^3(a+bx)\,dx = \frac{\cos(3\,bx+dx+3\,a+c)}{8\,(3\,b+d)} + \frac{\cos(3\,bx-dx+3\,a-c)}{8\,(3\,b-d)} - \frac{3\,\cos(bx+dx+a+c)}{8\,(b+d)} - \frac{3\,\cos(bx-dx+a-c)}{8\,(b-d)}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*cos(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*cos(3*b*x - d*x + 3*a - c)/
(3*b - d) - 3/8*cos(b*x + d*x + a + c)/(b + d) - 3/8*cos(b*x - d*x + a - c
)/(b - d)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 21.98 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.86

$$\int \cos(c+dx)\sin^3(a+bx)\,dx = -\mathrm{e}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{144\,b^4-160\,b^2\,d^2+16\,d^4}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-3\,b^3+b^2\,d+3\,b\,d^2-d^3)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\right)-\mathrm{e}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{-3\,b^3+b^2\,d+3\,b\,d^2-d^3}{144\,b^4-160\,b^2\,d^2+16\,d^4}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-3\,b^3-b^2\,d+3\,b\,d^2+d^3)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\right)$$

input
```
int(cos(c + d*x)*sin(a + b*x)^3,x)
```

output
```
- exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(144
*b^4 + 16*d^4 - 160*b^2*d^2) + (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d - 3*
b^3 - d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*2i - b*x*2i)*(3*b*
d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(
- a*4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(144*b^4 + 16*d^4
- 160*b^2*d^2)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2 + b^2*d - 3
*b^3 - d^3)/(144*b^4 + 16*d^4 - 160*b^2*d^2) + (exp(- a*6i - b*x*6i)*(3*b*
d^2 - b^2*d - 3*b^3 + d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*2i
 - b*x*2i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(144*b^4 + 16*d^4 - 160*
b^2*d^2) - (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(1
44*b^4 + 16*d^4 - 160*b^2*d^2))
```

3.225 $\int \cos^2(c+dx)\sin^3(a+bx)\,dx$

3.225.1 Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b} - \frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)} - \frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}+\frac{\cos(3a+2c+(3b+2d)x)}{16(3b+2d)}$$

output
```
-3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)+
1/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)+
1/16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

3.225.2 Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = \frac{1}{48}\left(-\frac{18\cos(a)\cos(bx)}{b}+\frac{2\cos(3a)\cos(3bx)}{b} - \frac{9\cos(a-2c+bx-2dx)}{b-2d}+\frac{3\cos(3a-2c+3bx-2dx)}{3b-2d} - \frac{9\cos(a+2c+bx+2dx)}{b+2d}+\frac{3\cos(3a+2c+3bx+2dx)}{3b+2d} + \frac{18\sin(a)\sin(bx)}{b}-\frac{2\sin(3a)\sin(3bx)}{b}\right)$$

input
```
Integrate[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

output
```
((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b - (9*Cos[a - 2*c + b*
x - 2*d*x])/(b - 2*d) + (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) - (
9*Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cos[3*a + 2*c + 3*b*x + 2*d*x
])/(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

3.225.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{3}{16}\sin(a+x(b-2d)-2c)-\frac{1}{16}\sin(3a+x(3b-2d)-2c)+\frac{3}{16}\sin(a+x(b+2d)+2c)-\frac{1}{16}\sin(3a+x(3b+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}$$

input
```
Int[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) - (3*Cos[a - 2*c + (b -
2*d)*x])/(16*(b - 2*d)) + Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d))
- (3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cos[3*a + 2*c + (3*b + 2
*d)*x]/(16*(3*b + 2*d))
```

3.225.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.225.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

method	result
default	$-\frac{3\cos(xb+a)}{8b}+\frac{\cos(3xb+3a)}{24b}-\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cos(3a-2c+(3b-2d)x)}{48b-32d}-\frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}+\frac{\cos(3a+}{4}$
parallelrisch	$\frac{9\left(b+\frac{2d}{3}\right)b(b-2d)(b+2d)\cos(3a-2c+(3b-2d)x)+9b(b-2d)(b+2d)\left(b-\frac{2d}{3}\right)\cos(3a+2c+(3b+2d)x)-81\left(b+\frac{2d}{3}\right)b(b+2d)\left(b-\frac{2d}{3}\right.}{}$
risch	$-\frac{3\cos(xb+a)}{8b}-\frac{27\cos(xb-2dx+a-2c)b^{3}}{16(b+2d)(3b+2d)(3b-2d)(b-2d)}-\frac{27\cos(xb-2dx+a-2c)b^{2}d}{8(b+2d)(3b+2d)(3b-2d)(b-2d)}+\frac{3\cos(xb-2dx+a-2c)b\,d^{2}}{4(b+2d)(3b+2d)(3b-2d)(b-2d)}$

input
```
int(cos(d*x+c)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)+
1/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)+
1/16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.30

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = -\frac{2\left(b^2d^2-4\,d^4\right)\cos\left(bx+a\right)^3+6\left(7\,b^3d-4\,bd^3-\left(b^3d-4\,bd^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)\sin}{3\left(9\,b^5\right.}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/3*(2*(b^2*d^2 - 4*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 - (b^3*d -
 4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) - 9*((b^4
 - 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*cos(d*x +
 c)^2 - 6*(7*b^2*d^2 - 4*d^4)*cos(b*x + a))/(9*b^5 - 40*b^3*d^2 + 16*b*d^4
)
```

3.225.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. 2(116) = 232.

Time = 5.64 (sec) , antiderivative size = 2020, normalized size of antiderivative = 14.64

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((x*sin(a)**3*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**3, Eq
(b, 0)), (-3*x*sin(a - 2*d*x)**3*sin(c + d*x)**2/16 + 3*x*sin(a - 2*d*x)**
3*cos(c + d*x)**2/16 + 3*x*sin(a - 2*d*x)**2*sin(c + d*x)*cos(a - 2*d*x)*c
os(c + d*x)/8 - 3*x*sin(a - 2*d*x)*sin(c + d*x)**2*cos(a - 2*d*x)**2/16 +
3*x*sin(a - 2*d*x)*cos(a - 2*d*x)**2*cos(c + d*x)**2/16 + 3*x*sin(c + d*x)
*cos(a - 2*d*x)**3*cos(c + d*x)/8 - 3*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c
 + d*x)/(16*d) + sin(a - 2*d*x)**2*cos(a - 2*d*x)*cos(c + d*x)**2/(2*d) -
sin(a - 2*d*x)*sin(c + d*x)*cos(a - 2*d*x)**2*cos(c + d*x)/(8*d) + sin(c +
 d*x)**2*cos(a - 2*d*x)**3/(96*d) + 31*cos(a - 2*d*x)**3*cos(c + d*x)**2/(
96*d), Eq(b, -2*d)), (-x*sin(a - 2*d*x/3)**3*sin(c + d*x)**2/16 + x*sin(a
- 2*d*x/3)**3*cos(c + d*x)**2/16 + 3*x*sin(a - 2*d*x/3)**2*sin(c + d*x)*co
s(a - 2*d*x/3)*cos(c + d*x)/8 + 3*x*sin(a - 2*d*x/3)*sin(c + d*x)**2*cos(a
 - 2*d*x/3)**2/16 - 3*x*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d*x)*
*2/16 - x*sin(c + d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/8 - sin(a - 2*d*x/
3)**3*sin(c + d*x)*cos(c + d*x)/(16*d) + 3*sin(a - 2*d*x/3)**2*cos(a - 2*d
*x/3)*cos(c + d*x)**2/(2*d) + 15*sin(a - 2*d*x/3)*sin(c + d*x)*cos(a - 2*d
*x/3)**2*cos(c + d*x)/(8*d) + 27*sin(c + d*x)**2*cos(a - 2*d*x/3)**3/(32*d
) + 5*cos(a - 2*d*x/3)**3*cos(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (-x*sin(
a + 2*d*x/3)**3*sin(c + d*x)**2/16 + x*sin(a + 2*d*x/3)**3*cos(c + d*x)...
```

3.225.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. 2(126) = 252.

Time = 0.31 (sec) , antiderivative size = 1360, normalized size of antiderivative = 9.86

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3
*cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*co
s(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a)
+ 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos
(2*c))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2
*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) -
9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2
*c))*cos((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) -
4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*cos
(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b
 - 2*d)*x - a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*c
os(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) + 2*(9*b^4*cos(2*c) - 40
*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) + 2*(9*b^4*cos
(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) - 18
*(9*b^4*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2*
c) - 18*(9*b^4*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x +
 a - 2*c) + 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8
*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^
3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x +
 3*a) + 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*...
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \cos^2(c+dx)\sin^3(a+bx)\,dx = &\frac{\cos(3\,bx+2\,dx+3\,a+2\,c)}{16\,(3\,b+2\,d)}+\frac{\cos(3\,bx-2\,dx+3\,a-2\,c)}{16\,(3\,b-2\,d)}\\ &+\frac{\cos(3\,bx+3\,a)}{24\,b}-\frac{3\,\cos(bx+2\,dx+a+2\,c)}{16\,(b+2\,d)}\\ &-\frac{3\,\cos(bx-2\,dx+a-2\,c)}{16\,(b-2\,d)}-\frac{3\,\cos(bx+a)}{8\,b}\end{aligned}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/16*cos(3*b*x - 2*d*x +
 3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b - 3/16*cos(b*x + 2*d*x +
 a + 2*c)/(b + 2*d) - 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos(
b*x + a)/b
```

3.225.9 Mupad [B] (verification not implemented)

Time = 22.24 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.17

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = -\frac{81\,b^4\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+81\,b^4\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)+162\,b^4\,\cos\left(a+b\,x\right)+288\,d^4\,\cos\left(a\right.}{}$$

input
```
int(cos(c + d*x)^2*sin(a + b*x)^3,x)
```

output
```
-(81*b^4*cos(a - 2*c + b*x - 2*d*x) + 81*b^4*cos(a + 2*c + b*x + 2*d*x) +
162*b^4*cos(a + b*x) + 288*d^4*cos(a + b*x) - 9*b^4*cos(3*a - 2*c + 3*b*x
- 2*d*x) - 9*b^4*cos(3*a + 2*c + 3*b*x + 2*d*x) - 18*b^4*cos(3*a + 3*b*x)
- 32*d^4*cos(3*a + 3*b*x) + 24*b*d^3*cos(3*a - 2*c + 3*b*x - 2*d*x) - 24*b
*d^3*cos(3*a + 2*c + 3*b*x + 2*d*x) - 6*b^3*d*cos(3*a - 2*c + 3*b*x - 2*d*
x) + 6*b^3*d*cos(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*cos(a - 2*c + b*x
 - 2*d*x) - 36*b^2*d^2*cos(a + 2*c + b*x + 2*d*x) - 720*b^2*d^2*cos(a + b*
x) + 36*b^2*d^2*cos(3*a - 2*c + 3*b*x - 2*d*x) + 36*b^2*d^2*cos(3*a + 2*c
+ 3*b*x + 2*d*x) + 80*b^2*d^2*cos(3*a + 3*b*x) - 72*b*d^3*cos(a - 2*c + b*
x - 2*d*x) + 72*b*d^3*cos(a + 2*c + b*x + 2*d*x) + 162*b^3*d*cos(a - 2*c +
 b*x - 2*d*x) - 162*b^3*d*cos(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^
5 - 40*b^3*d^2))
```

3.226 $\int \cos^3(c+dx)\sin^3(a+bx)\,dx$

3.226.1 Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = -\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\cos(a-c+(b-d)x)}{32(b-d)} + \frac{\cos(3(a-c)+3(b-d)x)}{96(b-d)} + \frac{3\cos(3a-c+(3b-d)x)}{32(3b-d)} - \frac{9\cos(a+c+(b+d)x)}{32(b+d)} + \frac{\cos(3(a+c)+3(b+d)x)}{96(b+d)} + \frac{3\cos(3a+c+(3b+d)x)}{32(3b+d)} - \frac{3\cos(a+3c+(b+3d)x)}{32(b+3d)}$$

output
```
-3/32*cos(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cos(a-c+(b-d)*x)/(b-d)+1/96*cos(3*
a-3*c+3*(b-d)*x)/(b-d)+3/32*cos(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*cos(a+c+(b+d
)*x)/(b+d)+1/96*cos(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*cos(3*a+c+(3*b+d)*x)/(3*
b+d)-3/32*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.226.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = \frac{1}{96}\left(-\frac{9\cos(a-3c+bx-3dx)}{b-3d}-\frac{27\cos(a-c+bx-dx)}{b-d}+\frac{\cos(3(a-c+bx-dx))}{b-d}+\frac{9\cos(3a-c+3bx-dx)}{3b-d}+\frac{9\cos(3a+c+3bx+dx)}{3b+d}-\frac{9\cos(a+3c+bx+3dx)}{b+3d}-\frac{27\cos(a+c+(b+d)x)}{b+d}+\frac{\cos(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

output
```
((-9*Cos[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cos[a - c + b*x - d*x])/(
b - d) + Cos[3*(a - c + b*x - d*x)]/(b - d) + (9*Cos[3*a - c + 3*b*x - d*x
])/(3*b - d) + (9*Cos[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cos[a + 3*c +
 b*x + 3*d*x])/(b + 3*d) - (27*Cos[a + c + (b + d)*x])/(b + d) + Cos[3*(a
+ c + (b + d)*x)]/(b + d))/96
```

3.226.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \ 5085$$

$$\int \left(\frac{3}{32}\sin(a+x(b-3d)-3c)+\frac{9}{32}\sin(a+x(b-d)-c)-\frac{1}{32}\sin(3(a-c)+3x(b-d))-\frac{3}{32}\sin(3a+x(3b-d\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{3\cos(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cos(a+x(b-d)-c)}{32(b-d)}+\frac{\cos(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cos(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cos(a+x(b+d)+c)}{32(b+d)}+\frac{\cos(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\cos(3a+x(3b+d)+c)}{32(3b+d)}-\frac{3\cos(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*Cos[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cos[a - c + (b - d)*x]
)/(32*(b - d)) + Cos[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cos[3*a -
c + (3*b - d)*x])/(32*(3*b - d)) - (9*Cos[a + c + (b + d)*x])/(32*(b + d))
 + Cos[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cos[3*a + c + (3*b + d)*
x])/(32*(3*b + d)) - (3*Cos[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

3.226.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.226.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$-\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cos(a-c+(b-d)x)}{32(b-d)}-\frac{9\cos(a+c+(b+d)x)}{32(b+d)}-\frac{3\cos(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\cos((3b-3d)x+3a-3}{96b-96d}$
parallelrisch	$\frac{-36\left(\left(b^2-\frac{61d^2}{9}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\frac{40\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2d^2}{3}+3b^2-7d^2\right)b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6-216\left(\left(b^2-\frac{7d^2}{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/32*cos(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cos(a-c+(b-d)*x)/(b-d)-9/32*cos(a+
c+(b+d)*x)/(b+d)-3/32*cos(a+3*c+(b+3*d)*x)/(b+3*d)+1/32/(3*b-3*d)*cos((3*b
-3*d)*x+3*a-3*c)+3/32*cos(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*cos(3*a+c+(3*b+d)*
x)/(3*b+d)+1/32/(3*b+3*d)*cos((3*b+3*d)*x+3*a+3*c)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.35

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = \frac{\left((9\,b^5-82\,b^3d^2+9\,bd^4)\cos{(bx+a)}^3-3\,(9\,b^5-28\,b^3d^2+3\,bd^4)\cos{(bx+a)}\right)\cos{(dx+c)}^3+\left(122\,b^2d^3-\right.}{}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^
2 + 3*b*d^4)*cos(b*x + a))*cos(d*x + c)^3 + (122*b^2*d^3 - 18*d^5 - 2*(b^2
*d^3 - 9*d^5)*cos(b*x + a)^2 - (63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d -
 82*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x
+ c) - 6*((b^3*d^2 - 9*b*d^4)*cos(b*x + a)^3 - 3*(7*b^3*d^2 - 3*b*d^4)*cos
(b*x + a))*cos(d*x + c))/(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

3.226.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3577 vs. 2(172) = 344.

Time = 17.96 (sec) , antiderivative size = 3577, normalized size of antiderivative = 18.34

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((x*sin(a)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-9*x*sin(a - 3*d*
x)**3*sin(c + d*x)**2*cos(c + d*x)/32 + 3*x*sin(a - 3*d*x)**3*cos(c + d*x)
**3/32 - 3*x*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*x)/32 + 9*x*sin
(a - 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/32 - 9*x*sin(a
- 3*d*x)*sin(c + d*x)**2*cos(a - 3*d*x)**2*cos(c + d*x)/32 + 3*x*sin(a - 3
*d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**3/32 - 3*x*sin(c + d*x)**3*cos(a - 3
*d*x)**3/32 + 9*x*sin(c + d*x)*cos(a - 3*d*x)**3*cos(c + d*x)**2/32 - sin(
a - 3*d*x)**3*sin(c + d*x)**3/(12*d) - 13*sin(a - 3*d*x)**3*sin(c + d*x)*c
os(c + d*x)**2/(320*d) + 3*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x
)*cos(c + d*x)/(20*d) + 101*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)*
*3/(320*d) - 27*sin(a - 3*d*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/(320*d) +
 51*sin(c + d*x)**2*cos(a - 3*d*x)**3*cos(c + d*x)/(320*d) + cos(a - 3*d*x
)**3*cos(c + d*x)**3/(5*d), Eq(b, -3*d)), (3*x*sin(a - d*x)**3*sin(c + d*x
)**2*cos(c + d*x)/16 + 5*x*sin(a - d*x)**3*cos(c + d*x)**3/16 + 3*x*sin(a
- d*x)**2*sin(c + d*x)**3*cos(a - d*x)/16 + 9*x*sin(a - d*x)**2*sin(c + d*
x)*cos(a - d*x)*cos(c + d*x)**2/16 + 9*x*sin(a - d*x)*sin(c + d*x)**2*cos(
a - d*x)**2*cos(c + d*x)/16 + 3*x*sin(a - d*x)*cos(a - d*x)**2*cos(c + d*x
)**3/16 + 5*x*sin(c + d*x)**3*cos(a - d*x)**3/16 + 3*x*sin(c + d*x)*cos(a
- d*x)**3*cos(c + d*x)**2/16 + sin(a - d*x)**3*sin(c + d*x)**3/(48*d) + si
n(a - d*x)**3*sin(c + d*x)*cos(c + d*x)**2/(2*d) + 3*sin(a - d*x)**2*si...
```

3.226.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(179) = 358$.

Time = 0.40 (sec) , antiderivative size = 2612, normalized size of antiderivative = 13.39

$$\int \cos^3(c + dx)\sin^3(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/192*(9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d
^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a +
4*c) + 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d
^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a -
2*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*d
^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a +
 4*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*
d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a
- 2*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*
b^2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a +
6*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^
2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a) + (
9*b^5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3
*c) + 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (9
*b^5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*
c) + 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^5
*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) +
 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*c
os(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9
*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*...
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \cos^3(c+dx)\sin^3(a+bx)\,dx = {} & \frac{\cos(3\,bx+3\,dx+3\,a+3\,c)}{96\,(b+d)} + \frac{3\,\cos(3\,bx+dx+3\,a+c)}{32\,(3\,b+d)} \\ & + \frac{3\,\cos(3\,bx-dx+3\,a-c)}{32\,(3\,b-d)} + \frac{\cos(3\,bx-3\,dx+3\,a-3\,c)}{96\,(b-d)} \\ & - \frac{3\,\cos(bx+3\,dx+a+3\,c)}{32\,(b+3\,d)} - \frac{9\,\cos(bx+dx+a+c)}{32\,(b+d)} \\ & - \frac{9\,\cos(bx-dx+a-c)}{32\,(b-d)} - \frac{3\,\cos(bx-3\,dx+a-3\,c)}{32\,(b-3\,d)}\end{aligned}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/96*cos(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*cos(3*b*x + d*x + 3*a +
 c)/(3*b + d) + 3/32*cos(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*cos(3*b*x
 - 3*d*x + 3*a - 3*c)/(b - d) - 3/32*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d)
- 9/32*cos(b*x + d*x + a + c)/(b + d) - 9/32*cos(b*x - d*x + a - c)/(b - d
) - 3/32*cos(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

3.226. $\int \cos^3(c+dx)\sin^3(a+bx)\,dx$

3.226.9 Mupad [B] (verification not implemented)

Time = 25.69 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.88

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = -e^{a\,3i-c\,1i+b\,x\,3i-d\,x\,1i}\left(\frac{-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3}{576\,b^4-640\,b^2\,d^2+64\,d^4} + \frac{e^{-a\,6i-b\,x\,6i}\,(-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3)}{576\,b^4-640\,b^2\,d^2+64\,d^4} - \frac{e^{-a\,2i-b\,x\,2i}\,(-81\,b^3-81\,b^2\,d+9\,b\,d^2+9\,d^3)}{576\,b^4-640\,b^2\,d^2+64\,d^4} - \frac{e^{-a\,4i-b\,x\,4i}\,(-81\,b^3+81\,b^2\,d+9\,b\,d^2-9\,d^3)}{576\,b^4-640\,b^2\,d^2+64\,d^4}\right) - e^{a\,3i+c\,1i+b\,x\,3i+d\,x\,1i}\left(\frac{-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3}{576\,b^4-640\,b^2\,d^2+64\,d^4} + \frac{e^{-a\,6i-b\,x\,6i}\,(-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3)}{576\,b^4-640\,b^2\,d^2+64\,d^4} - \frac{e^{-a\,2i-b\,x\,2i}\,(-81\,b^3+81\,b^2\,d+9\,b\,d^2-9\,d^3)}{576\,b^4-640\,b^2\,d^2+64\,d^4} - \frac{e^{-a\,4i-b\,x\,4i}\,(-81\,b^3-81\,b^2\,d+9\,b\,d^2+9\,d^3)}{576\,b^4-640\,b^2\,d^2+64\,d^4}\right) - e^{a\,3i-c\,3i+b\,x\,3i-d\,x\,3i}\left(\frac{-b^3-b^2\,d+9\,b\,d^2+9\,d^3}{192\,b^4-1920\,b^2\,d^2+1728\,d^4} + \frac{e^{-a\,6i-b\,x\,6i}\,(-b^3+b^2\,d+9\,b\,d^2-9\,d^3)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4} - \frac{e^{-a\,2i-b\,x\,2i}\,(-9\,b^3-27\,b^2\,d+9\,b\,d^2+27\,d^3)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4} - \frac{e^{-a\,4i-b\,x\,4i}\,(-9\,b^3+27\,b^2\,d+9\,b\,d^2-27\,d^3)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}\right) - e^{a\,3i+c\,3i+b\,x\,3i+d\,x\,3i}\left(\frac{-b^3+b^2\,d+9\,b\,d^2-9\,d^3}{192\,b^4-1920\,b^2\,d^2+1728\,d^4} + \frac{e^{-a\,6i-b\,x\,6i}\,(-b^3-b^2\,d+9\,b\,d^2+9\,d^3)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4} - \frac{e^{-a\,2i-b\,x\,2i}\,(-9\,b^3+27\,b^2\,d+9\,b\,d^2-27\,d^3)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4} - \frac{e^{-a\,4i-b\,x\,4i}\,(-9\,b^3-27\,b^2\,d+9\,b\,d^2+27\,d^3)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}\right)$$

input
```
int(cos(c + d*x)^3*sin(a + b*x)^3,x)
```

output
```
- exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/
(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2*
d - 9*b^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- a*2i - b*x*2
i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2)
 - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 +
 64*d^4 - 640*b^2*d^2)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9*b*d^2 + 3
*b^2*d - 9*b^3 - 3*d^3)/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- a*6i - b
*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^
2) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4
 + 64*d^4 - 640*b^2*d^2) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 81*b^2*d - 81*
b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2)) - exp(a*3i - c*3i + b*x*3i
 - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2
*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d - b^3 - 9*d^3))/(192*b^4 +
1728*d^4 - 1920*b^2*d^2) - (exp(- a*2i - b*x*2i)*(9*b*d^2 - 27*b^2*d - 9*b
^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*4i - b*x*4i)*
(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)
) - exp(a*3i + c*3i + b*x*3i + d*x*3i)*((9*b*d^2 + b^2*d - b^3 - 9*d^3)/(1
92*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 - b^2*d
 - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*2i - b*x*2
i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^...
```

3.226. $\int \cos^3(c+dx)\sin^3(a+bx)\,dx$

3.227 $\int \cos(a+bx)\csc(c+bx)\,dx$

3.227.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cos(a+bx)\csc(c+bx)\,dx = \frac{\cos(a-c)\log(\sin(c+bx))}{b} - x\sin(a-c)$$

output
```
cos(a-c)*ln(sin(b*x+c))/b-x*sin(a-c)
```

3.227.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(a+bx)\csc(c+bx)\,dx$$
$$= \frac{-2i\arctan(\tan(c+bx))\cos(a-c)+\cos(a-c)\left(2ibx+\log\left(\sin^2(c+bx)\right)\right)-2bx\sin(a-c)}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x],x]
```

output
```
((-2*I)*ArcTan[Tan[c + b*x]]*Cos[a - c] + Cos[a - c]*((2*I)*b*x + Log[Sin[
c + b*x]^2]) - 2*b*x*Sin[a - c])/(2*b)
```

3.227.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5092, 24, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)dx-\sin(a-c)\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \cot(c+bx)dx-x\sin(a-c)$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx-x\sin(a-c)$$

$$\Big\downarrow \text{25}$$

$$-\cos(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx-x\sin(a-c)$$

$$\Big\downarrow \text{3956}$$

$$\frac{\cos(a-c)\log(-\sin(bx+c))}{b}-x\sin(a-c)$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x],x]
```

output
```
(Cos[a - c]*Log[-Sin[c + b*x]])/b - x*Sin[a - c]
```

3.227.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.227.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

method	result
risch	$ix\,\mathrm{e}^{i(a-c)}-2i\cos\left(a-c\right)x-\frac{2i\cos(a-c)a}{b}+\frac{\ln\left(\mathrm{e}^{2i(xb+a)}-\mathrm{e}^{2i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{\frac{\frac{(-\cos(a)\cos(c)-\sin(a)\sin(c))\ln\left(1+\tan(xb+a)^2\right)}{2}+(\cos(a)\sin(c)-\sin(a)\cos(c))\arctan(\tan(xb+a))}{\left(\cos(c)^2+\sin(c)^2\right)\left(\cos(a)^2+\sin(a)^2\right)}+\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln(\tan(xb+a)\cos(}{\cos(a)^2\cos(c)^2+\cos(c)^2}}{b}$

input
```
int(cos(b*x+a)/sin(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
I*x*exp(I*(a-c))-2*I*cos(a-c)*x-2*I/b*cos(a-c)*a+ln(exp(2*I*(b*x+a))-exp(2
*I*(a-c)))/b*cos(a-c)
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)\csc(c+bx)\,dx=\frac{bx\sin(-a+c)+\cos(-a+c)\log\left(\frac{1}{2}\sin(bx+c)\right)}{b}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="fricas")
```

output
```
(b*x*sin(-a + c) + cos(-a + c)*log(1/2*sin(b*x + c)))/b
```

3.227.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(20) = 40$.

Time = 4.54 (sec) , antiderivative size = 333, normalized size of antiderivative = 12.33

$$\int \cos(a+bx)\csc(c+bx)\,dx=$$

$$-\left(\begin{cases}0\\ x\\ 0\\ -\frac{bx\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}+\frac{bx}{b\tan^2\left(\frac{c}{2}\right)+b}-\frac{2\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}-\frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}+\frac{2\log\left(\tan^2\left(\frac{bx}{2}\right)\ldots\right.}{b\tan^2\left(\ldots\right.}\end{cases}\right.$$

$$+\left(\begin{cases}\tilde{\infty}x\\ \frac{\log(\sin(bx))}{b}\\ \frac{x}{\sin(c)}\\ \frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)}{b\tan^2\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan\ldots\right.}{b\,\mathrm{t}\ldots}\end{cases}\right.$$

input
```
integrate(cos(b*x+a)/sin(b*x+c),x)
```

output
```
-Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x, Eq(c, 0)), (0, Eq(b, 0)), (-b*x*t
an(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2)
+ tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2)
)*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(
c/2)**2 + b), True))*sin(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (log
(sin(b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)
**2 + b) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + lo
g(tan(c/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2)
)*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/
2)**2 + b) + log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(
tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b), True))*cos(a)
```

3.227.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(27) = 54$.

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.93

$$\int \cos(a+bx)\csc(c+bx)\,dx$$
$$= \frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin\left(c\right)\right.}{}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) +
cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + cos(-a + c)*log(co
s(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) +
 sin(c)^2))/b
```

3.227.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(27) = 54$.

Time = 0.33 (sec) , antiderivative size = 482, normalized size of antiderivative = 17.85

$$\int \cos(a+bx)\csc(c+bx)\,dx =$$
$$-\frac{\frac{4\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)(bx+a)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}+\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}\right.\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}\right.}}{}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
tan(1/2*c))*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*
c)^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*tan(1/2*c)^4 - 2*t
an(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*ta
n(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*ta
n(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8
*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)*tan(1/2*
a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*
tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan
(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1
/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^4 + 4*tan(
1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2
*a)*tan(1/2*c) + 1))/b
```

3.227.9 Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 115, normalized size of antiderivative = 4.26

$$\int \cos(a+bx)\csc(c+bx)\,dx = -x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right) - x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}+\frac{e^{a\,1i-c\,1i}\,1i}{2}\right) + \frac{\ln\left(-e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)}{b}$$

input
```
int(cos(a + b*x)/sin(c + b*x),x)
```

output
```
(log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i
 - c*1i)/2))/b - x*((exp(c*1i - a*1i)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - x
*((exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2)
```

3.228 $\int \cos(a+bx)\csc^2(c+bx)\,dx$

3.228.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = -\frac{\cos(a-c)\csc(c+bx)}{b}+\frac{\operatorname{arctanh}(\cos(c+bx))\sin(a-c)}{b}$$

output
```
-cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b
```

3.228.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.57

$$\int \cos(a+bx)\csc^2(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\csc(c+bx)}{b}+\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c
]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/
2]*Sin[c])]*Sin[a - c])/b
```

3.228.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5092, 3042, 25, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)\csc(c+bx)dx-\sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx-\sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \text{25}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int 1d\csc(c+bx)}{b}-\sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \text{24}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b
```

3.228.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.228.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.29

method	result
risch	$\frac{i\left(e^{i(xb+3a)}+e^{i(xb+a+2c)}\right)}{b\left(-e^{2i(xb+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(xb+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(xb+a)}-e^{i(a-c)}\right)\sin(a-c)}{b}$
default	$-\frac{2\left(-\frac{\left(\cos(a)^2\cos(c)^2+2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(a)^2\sin(c)^2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\left(\cos(a)^2\cos(c)^2+\cos(c)^2\sin(a)^2+\cos(a)^2\sin(c)^2+\sin(a)^2\sin(c)^2\right)(\sin(a)\cos(c)-\cos(a)\sin(c))}+\frac{\cos(a)\cos(c)+\sin(a)\sin(\cdots}{\cos(a)^2\cos(c)^2+\cos(c)^2\sin(a)^2+\cos(a)^2\sin\cdots}\cdots}{\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\cos(a)\cos(c)+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos\cdots}$

input
```
int(cos(b*x+a)/sin(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))
+ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)-ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*sin(a-c)
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = -\frac{\log\left(\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\log\left(-\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)\sin\left(-a+c\right)+}{2\,b\sin\left(bx+c\right)}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) - log(-1/2*cos(
b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) + 2*cos(-a + c))/(b*sin(b*x + c))
```

3.228.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. 2(27) = 54.

Time = 61.01 (sec) , antiderivative size = 3264, normalized size of antiderivative = 93.26

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c)**2,x)
```

output
```
-Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq
(b, 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**
4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*ta
n(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(
c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)
**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)
) + log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*t
an(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*ta
n(c/2) - b*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/
2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) +
tan(b*x/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)*
*3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
 b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*
x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)
**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(
b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b
*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/
tan(c/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3
*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) ...
```

3.228.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 450, normalized size of antiderivative = 12.86

$$\int \cos(a + bx)\csc^2(c + bx)\,dx$$
$$= \frac{2\left(\sin\left(bx + 2\,a\right) + \sin\left(bx + 2\,c\right)\right)\cos\left(2\,bx + a + 2\,c\right) - \left(\cos\left(2\,bx + a + 2\,c\right)^2\sin\left(-a + c\right) - 2\,\cos\left(2\,bx + \right.\right.}{}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(sin(b*x + 2*a) + sin(b*x + 2*c))*cos(2*b*x + a + 2*c) - (cos(2*b*x
 + a + 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + si
n(2*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a +
c) + (cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c)
 + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(2*b*x + a
+ 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b
*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) +
(cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + co
s(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(cos(b*x + 2*a) +
cos(b*x + 2*c))*sin(2*b*x + a + 2*c) - 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*
sin(b*x + 2*c) + 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos
(2*b*x + a + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a +
2*c)^2 - 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

3.228.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(35) = 70$.

Time = 0.41 (sec) , antiderivative size = 893, normalized size of antiderivative = 25.51

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
tan(1/2*c))*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan
(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2
*a) - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/2*a)*ta
n(1/2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2
*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(
1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*a)*tan(
1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/
2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)
^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*b*x + 1/
2*a)*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b
*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(
1/2*b*x + 1/2*a)*tan(1/2*a)^4 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/
2*c) + 2*tan(1/2*a)^4*tan(1/2*c) + 20*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*ta
n(1/2*c)^2 - 12*tan(1/2*a)^3*tan(1/2*c)^2 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2
*a)*tan(1/2*c)^3 + 12*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a)*tan
(1/2*c)^4 - 2*tan(1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^
2 + 2*tan(1/2*a)^3 + 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - 12*tan
(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 + 12*tan(1/2*a)
*tan(1/2*c)^2 - 2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a) - 2*tan(1/2*a) + ...
```

3.228.9 Mupad [B] (verification not implemented)

Time = 26.29 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.20

$$
\begin{aligned}
&\int \cos(a+bx)\csc^2(c+bx)\,dx \\
&= -\frac{\ln\left(\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)-\frac{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)\,1\mathrm{i}}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}{2\,b\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}} \\
&\quad+\frac{\ln\left(\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)+\frac{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)\,1\mathrm{i}}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}{2\,b\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}} \\
&\quad+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+1\right)\,1\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}
\end{aligned}
$$

input
```
int(cos(a + b*x)/sin(c + b*x)^2,x)
```

output
```
(log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*
2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2
i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i)*exp(b*x*1
i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i
) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-e
xp(a*2i - c*2i))^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1)*1i)/(
b*(exp(a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

3.228. $\int \cos(a+bx)\csc^2(c+bx)\,dx$

3.229 $\int \cos(a+bx)\csc^3(c+bx)\,dx$

3.229.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = -\frac{\cos(a-c)\csc^2(c+bx)}{2b}+\frac{\cot(c+bx)\sin(a-c)}{b}$$

output
```
-1/2*cos(a-c)*csc(b*x+c)^2/b+cot(b*x+c)*sin(a-c)/b
```

3.229.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = -\frac{\csc(c)\csc^2(c+bx)(\sin(a)-\cos(c+2bx)\sin(a-c))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

output
```
-1/2*(Csc[c]*Csc[c + b*x]^2*(Sin[a] - Cos[c + 2*b*x]*Sin[a - c]))/b
```

3.229.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5092, 3042, 25, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)\csc^2(c+bx)dx-\sin(a-c)\int \csc^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^2\tan\left(c+bx-\frac{\pi}{2}\right)dx-\sin(a-c)\int \csc(c+bx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\sin(a-c)\int \csc(c+bx)^2dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int \csc(c+bx)d\csc(c+bx)}{b}-\sin(a-c)\int \csc(c+bx)^2dx$$

$$\Big\downarrow \text{15}$$

$$-\sin(a-c)\int \csc(c+bx)^2dx-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sin(a-c)\int 1d\cot(c+bx)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(a-c)\cot(bx+c)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

output
```
-1/2*(Cos[a - c]*Csc[c + b*x]^2)/b + (Cot[c + b*x]*Sin[a - c])/b
```

3.229.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

3.229.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{\sec\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}\csc\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}\cos(2xb+a+c)}{8b}$	36
default	$-\frac{1}{2b(\cos(a)\cos(c)+\sin(a)\sin(c))(\tan(xb+a)\cos(a)\cos(c)+\tan(xb+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}$	55
risch	$-\frac{-2\,\mathrm{e}^{i(2xb+5a+c)}+\mathrm{e}^{i(5a-c)}-\mathrm{e}^{i(3a+c)}}{\left(-\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}$	64

input
```
int(cos(b*x+a)/sin(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b*sec(1/2*x*b+1/2*c)^2*csc(1/2*x*b+1/2*c)^2*cos(2*b*x+a+c)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \frac{2\cos(bx+c)\sin(bx+c)\sin(-a+c)+\cos(-a+c)}{2\left(b\cos(bx+c)^2-b\right)}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) + cos(-a + c))/(b*cos(b*x + c
)^2 - b)
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c)**3,x)
```

output
```
Timed out
```

3.229.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(36) = 72.

Time = 0.23 (sec) , antiderivative size = 395, normalized size of antiderivative = 10.39

$$\int \cos(a+bx)\csc^3(c+bx)\,dx$$
$$= \frac{(2\cos(2bx+2a+2c)-\cos(2a)+\cos(2c))\cos(4bx+a+5c)-2(2\cos(2bx+2a+2c)-\cos(2a)}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2b}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="maxima")
```

output
```
((2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(4*b*x + a + 5*c) - 2
*(2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(2*b*x + a + 3*c) - (
cos(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2
*sin(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(4*b*x + a + 5*c) - 2*(2
*sin(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(2*b*x + a + 3*c) - (sin
(2*a) - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*
b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2
*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

3.229.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(36) = 72$.

Time = 0.33 (sec) , antiderivative size = 327, normalized size of antiderivative = 8.61

$$\int \cos(a+bx)\csc^3(c+bx)\,dx =$$

$$-\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\tan}{2\left(\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2+4\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}\right.\right.}$$

input
```
integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*
a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c
)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*
c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan
(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2
+ 1)/((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2
+ 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b
*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/
2*a) + 2*tan(1/2*c))^2*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1
/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.230 $\int \sin(a+bx)\tan^3(c+bx)\,dx$

3.230.1 Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = -\frac{3\text{arctanh}(\sin(c+bx))\cos(a-c)}{2b}+\frac{\sec(c+bx)\sin(a-c)}{b}+\frac{\sin(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b}$$

output
```
-3/2*arctanh(sin(b*x+c))*cos(a-c)/b+sec(b*x+c)*sin(a-c)/b+sin(b*x+a)/b+1/2
*cos(a-c)*sec(b*x+c)*tan(b*x+c)/b
```

3.230.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \frac{-12\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+\sec^2(c+bx)(2\sin(a-2c-bx)+5\sin(a+bx)+\sin(a+2c+3bx))}{4b}$$

input
```
Integrate[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

output
```
(-12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cos[a - c] + Sec[c + b*x]^2*(2*
Sin[a - 2*c - b*x] + 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

3.230.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.867, Rules used = {5087, 3042, 3091, 3042, 4257, 5090, 3042, 3086, 24, 5087, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^3(bx+c)\,dx$$

$$\Big\downarrow \text{5087}$$

$$\cos(a-c)\int \sec(c+bx)\tan^2(c+bx)dx-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \sec(c+bx)\tan(c+bx)^2dx-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{3091}$$

$$\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{1}{2}\int \sec(c+bx)dx\right)-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx\right)-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{5090}$$

$$-\int \sin(a+bx)\tan(c+bx)dx+\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \sin(a+bx)\tan(c+bx)dx+\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{3086}$$

$$-\int \sin(a+bx)\tan(c+bx)dx+\frac{\sin(a-c)\int 1d\sec(c+bx)}{b}+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$-\int \sin(a+bx)\tan(c+bx)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{5087}$$

$$-\cos(a-c)\int \sec(c+bx)dx+\int \cos(a+bx)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\int \sin\left(a+bx+\frac{\pi}{2}\right)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cos(a-c)\operatorname{arctanh}(\sin(bx+c))}{b}+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\sin(a+bx)}{b}$$

input

```
Int[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

output

```
-((ArcTanh[Sin[c + b*x]]*Cos[a - c])/b) + (Sec[c + b*x]*Sin[a - c])/b + Si
n[a + b*x]/b + Cos[a - c]*(-1/2*ArcTanh[Sin[c + b*x]]/b + (Sec[c + b*x]*Ta
n[c + b*x])/(2*b))
```

3.230. $\int \sin(a+bx)\tan^3(c+bx)\,dx$

3.230.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.230.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.58

method	result
risch	$-\frac{i\mathrm{e}^{i(xb+a)}}{2b}+\frac{i\mathrm{e}^{-i(xb+a)}}{2b}-\frac{i\left(3\,\mathrm{e}^{i(3xb+5a+2c)}-\mathrm{e}^{i(3xb+3a+4c)}+\mathrm{e}^{i(xb+5a)}-3\,\mathrm{e}^{i(xb+3a+2c)}\right)}{2b\left(\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^{2}}+\frac{3\ln\left(\mathrm{e}^{i(xb+a)}-i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{2b}$

input
```
int(sin(b*x+a)*tan(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*(b*x+a))/b+1/2*I/b*exp(-I*(b*x+a))-1/2*I/b/(exp(2*I*(b*x+a+c)
)+exp(2*I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*
x+5*a))-3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*co
s(a-c)-3/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(68) = 136$.

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.22

$$\int \sin(a+bx)\tan^3(c+bx)\,dx =$$

$$-\frac{\frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-2\left(\cos(-2a+2c)^2+\cos(-2a+2c)\right)\cos(bx+a)^2+\cos(-2a+2c)^2-1\right)\log\left(-\cdots\right.}{\sqrt{\cos(-2a+\cdots}}}{\cdots}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*
a + 2*c) - 2*(cos(-2*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2
*a + 2*c)^2 - 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*s
in(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a
) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2
*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*s
in(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*co
s(b*x + a)^2*cos(-2*a + 2*c) - 3*cos(-2*a + 2*c) + 5)*sin(b*x + a) - 4*(4*
cos(b*x + a)^3 - 5*cos(b*x + a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(
-2*a + 2*c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a +
 2*c) + b)
```

3.230.6 Sympy [F]

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \int \sin{(a+bx)}\tan^3{(bx+c)}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)*tan(b*x + c)**3, x)
```

3.230.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(68) = 136.

Time = 0.42 (sec) , antiderivative size = 1027, normalized size of antiderivative = 14.26

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos
(6*b*x + 2*a + 4*c) - 2*(5*sin(4*b*x + 2*a + 2*c) - 2*sin(4*b*x + 4*c) + 2
*sin(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3
*b*x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*sin(3*b*x +
a + 2*c) + sin(b*x + a))*cos(4*b*x + 4*c) - 4*(2*sin(2*b*x + 2*a) - 5*sin(
2*b*x + 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c)
 + 4*cos(3*b*x + a + 2*c)^2*cos(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x +
 a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a +
 4*c)^2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 4*cos(-a + c)*sin(3*b*x +
 a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 + 2*(2*cos(3*b*x + a +
 2*c)*cos(-a + c) + cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) + 2*(2*
cos(-a + c)*sin(3*b*x + a + 2*c) + cos(-a + c)*sin(b*x + a))*sin(5*b*x + a
 + 4*c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(
b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos
(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(
c) + sin(c)^2)) - 2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b
*x + a))*sin(6*b*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*
x + 4*c) + 2*cos(2*b*x + 2*a) - 5*cos(2*b*x + 2*c) - 1)*sin(5*b*x + a + 4*
c) - 10*(2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + 4
*(2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 4*c) + 4*(2*cos(2*...
```

3.230.8 Giac [F]

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \int \sin\left(bx+a\right)\tan\left(bx+c\right)^3\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + c)^3, x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)*tan(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.231 $\int \sin(a+bx)\tan^2(c+bx)\,dx$

3.231.1 Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \sin(a+bx)\tan^2(c+bx)\,dx=\frac{\cos(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{b}$$

output
```
cos(b*x+a)/b+cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

3.231.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.48

$$\int \sin(a+bx)\tan^2(c+bx)\,dx =\frac{\cos(a)\cos(bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}-\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}-\frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
(Cos[a]*Cos[b*x])/b + (Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[
c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x
)/2] - I*Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

3.231.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5087, 3042, 3086, 24, 5090, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^2(bx+c)\,dx$$

$$\Big\downarrow \text{5087}$$

$$\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx - \int \cos(a+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx - \int \cos(a+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int 1d\sec(c+bx)}{b} - \int \cos(a+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{24}$$

$$\frac{\cos(a-c)\sec(bx+c)}{b} - \int \cos(a+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{5090}$$

$$\sin(a-c)\int \sec(c+bx)dx - \int \sin(a+bx)dx + \frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx - \int \sin(a+bx)dx + \frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx + \frac{\cos(a-c)\sec(bx+c)}{b} + \frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[
a - c])/b
```

3.231.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.231.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.25

method	result	size
risch	$\frac{e^{i(xb+a)}}{2b}+\frac{e^{-i(xb+a)}}{2b}+\frac{e^{i(xb+3a)}+e^{i(xb+a+2c)}}{b\left(e^{2i(xb+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(xb+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(xb+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}$	143

input
```
int(sin(b*x+a)*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/b/(exp(2*I*(b*x+a+c))+exp(2*I
*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))+I*exp(I*(a-c)
))/b*sin(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(44) = 88.

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 7.16

$$\int \sin(a+bx)\tan^2(c+bx)\,dx =$$

$$-\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)^{2}-4\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\cdots\right.}{\cdots}}{4\cdots}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)
*sin(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a +
2*c) + (cos(-2*a + 2*c)^2 - 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2
*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(
-2*a + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*
a + 2*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2
*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(co
s(-2*a + 2*c) + 1) + 4*cos(-2*a + 2*c) + 4)/(b*sin(b*x + a)*sin(-2*a + 2*c
) - (b*cos(-2*a + 2*c) + b)*cos(b*x + a))
```

3.231.6 Sympy [F]

$$\int \sin(a+bx)\tan^2(c+bx)\,dx = \int \sin\left(a+bx\right)\tan^2\left(bx+c\right)dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*tan(b*x + c)**2, x)
```

3.231.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(44) = 88$.

Time = 0.43 (sec) , antiderivative size = 520, normalized size of antiderivative = 11.82

$$\int \sin(a+bx)\tan^2(c+bx)\,dx$$
$$= \frac{\left(\cos\left(3\,bx+a+2\,c\right)+\cos\left(bx+a\right)\right)\cos\left(4\,bx+2\,a+2\,c\right)+\left(3\,\cos\left(2\,bx+2\,a\right)+3\,\cos\left(2\,bx+2\,c\right)+1\right)\cos}{}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + (3*cos
(2*b*x + 2*a) + 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x
 + 2*a)*cos(b*x + a) + 3*cos(2*b*x + 2*c)*cos(b*x + a) + (cos(3*b*x + a +
2*c)^2*sin(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos
(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) + 2*sin(3*b*x
 + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log((co
s(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2
*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)
*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2))
+ (sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + 3*(sin(2*
b*x + 2*a) + sin(2*b*x + 2*c))*sin(3*b*x + a + 2*c) + 3*sin(2*b*x + 2*a)*s
in(b*x + a) + 3*sin(2*b*x + 2*c)*sin(b*x + a) + cos(b*x + a))/(b*cos(3*b*x
 + a + 2*c)^2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 +
 b*sin(3*b*x + a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(
b*x + a)^2)
```

3.231.8 Giac [F]

$$\int \sin(a+bx)\tan^2(c+bx)\,dx = \int \sin(bx+a)\tan(bx+c)^2\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + c)^2, x)
```

3.231.9 Mupad [B] (verification not implemented)

Time = 27.60 (sec) , antiderivative size = 294, normalized size of antiderivative = 6.68

$$\int \sin(a+bx)\tan^2(c+bx)\,dx = \frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}\,1i+e^{a\,2i+b\,x\,2i}\,1i\right)} + \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} - \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(sin(a + b*x)*tan(c + b*x)^2,x)
```

output
```
exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) + (exp(a*1i + b*x*1i
)*(exp(a*2i - c*2i) + 1)*1i)/(b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x*2i)*
1i)) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) - (exp(a*
2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2
))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i
)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*2i)*(exp(
a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i)
 - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

3.232 $\int \sin(a+bx)\tan(c+bx)\,dx$

3.232.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \sin(a+bx)\tan(c+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{b} - \frac{\sin(a+bx)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)/b-sin(b*x+a)/b
```

3.232.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.24

$$\int \sin(a+bx)\tan(c+bx)\,dx$$
$$= -\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} - \frac{\cos(bx)\sin(a)}{b} - \frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Sin[a + b*x]*Tan[c + b*x],x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos
[b*x]*Sin[a])/b - (Cos[a]*Sin[b*x])/b
```

3.232.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5087, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \text{5087}$$

$$\cos(a-c)\int \sec(c+bx)dx-\int \cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(a-c)\text{arctanh}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[c + b*x],x]
```

output
```
(ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - Sin[a + b*x]/b
```

3.232.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

3.232.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.41

method	result	size
risch	$\frac{i\mathrm{e}^{i(xb+a)}}{2b}-\frac{i\mathrm{e}^{-i(xb+a)}}{2b}+\frac{\ln\left(\mathrm{e}^{i(xb+a)}+i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(\mathrm{e}^{i(xb+a)}-i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{b}$	99

input
```
int(sin(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))+I*exp(I*(
a-c)))/b*cos(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)
```

3.232.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 6.48

$$\int \sin(a+bx)\tan(c+bx)\,dx$$
$$=\frac{\sqrt{2}\sqrt{\cos\left(-2\,a+2\,c\right)+1}\log\left(\frac{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\frac{2\,\sqrt{2}((\cos(-2\,a+2\,c)+1)\sin(bx+a)+c}{\sqrt{\cos(-2\,a+2\,c)}}}{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\cos(\cdot}\right.}{4\,b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a +
2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c
) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)) - 4*sin(b*x +
a))/b
```

3.232.6 Sympy [F]

$$\int \sin(a+bx)\tan(c+bx)\,dx = \int \sin{(a+bx)}\tan{(bx+c)}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c),x)
```

output
```
Integral(sin(a + b*x)*tan(b*x + c), x)
```

3.232.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(29) = 58$.

Time = 0.39 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.52

$$\int \sin(a+bx)\tan(c+bx)\,dx = -\frac{\cos{(-a+c)}\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)+2\sin{(bx+a)}}{2b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

output
```
-1/2*(cos(-a + c)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*
c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c
)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x +
2*c)*sin(c) + sin(c)^2)) + 2*sin(b*x + a))/b
```

3.232.8 Giac [F]

$$\int \sin(a+bx)\tan(c+bx)\,dx = \int \sin\left(bx+a\right)\tan\left(bx+c\right)\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + c), x)
```

3.232.9 Mupad [B] (verification not implemented)

Time = 26.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 7.83

$$\int \sin(a+bx)\tan(c+bx)\,dx$$
$$= -\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b} + \frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b} + \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}+1\right) - \frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}+1\right)1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} - \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}+1\right) + \frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}+1\right)1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

input
```
int(sin(a + b*x)*tan(c + b*x),x)
```

output
```
(exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) + (log(- e
xp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-c*2i)*(e
xp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2
i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)*(exp(a*
2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1
i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2
i)^(1/2))
```

3.233 $\int \cot(c+bx)\sin(a+bx)\,dx$

3.233.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \cot(c+bx)\sin(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(c+bx))\sin(a-c)}{b}+\frac{\sin(a+bx)}{b}$$

output
```
-arctanh(cos(b*x+c))*sin(a-c)/b+sin(b*x+a)/b
```

3.233.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int \cot(c+bx)\sin(a+bx)\,dx=\frac{\cos(bx)\sin(a)}{b}-\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}+\frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Cot[c + b*x]*Sin[a + b*x],x]
```

output
```
(Cos[b*x]*Sin[a])/b - ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)
/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]
*Sin[a - c])/b + (Cos[a]*Sin[b*x])/b
```

3.233.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5089, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \text{5089}$$

$$\sin(a-c)\int \csc(c+bx)dx+\int \cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc(c+bx)dx+\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\sin(a-c)\int \csc(c+bx)dx+\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a+bx)}{b}-\frac{\sin(a-c)\text{arctanh}(\cos(bx+c))}{b}$$

input
```
Int[Cot[c + b*x]*Sin[a + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Sin[a - c])/b) + Sin[a + b*x]/b
```

3.233.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.233.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

method	result	size
risch	$-\frac{i\mathrm{e}^{i(xb+a)}}{2b}+\frac{i\mathrm{e}^{-i(xb+a)}}{2b}+\frac{\ln\left(\mathrm{e}^{i(xb+a)}-\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(\mathrm{e}^{i(xb+a)}+\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}$	95

input
```
int(cot(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*(b*x+a))/b+1/2*I/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))-exp(I*(a
-c)))/b*sin(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 6.79

$$\int \cot(c+bx)\sin(a+bx)\,dx = \frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx+a)-\sin(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)+3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)-1}\right)\mathrm{s}}{\sqrt{\cos(-2a+2c)+1}}}{4b}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*
x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - s
in(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) +
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2
*a + 2*c) - cos(-2*a + 2*c) - 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1
) + 4*sin(b*x + a))/b
```

3.233.6 Sympy [F]

$$\int \cot(c+bx)\sin(a+bx)\,dx = \int \sin\left(a+bx\right)\cot\left(bx+c\right)dx$$

input
```
integrate(cot(b*x+c)*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*cot(b*x + c), x)
```

3.233.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(29) = 58.

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.62

$$\int \cot(c+bx)\sin(a+bx)\,dx = \frac{\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^2\right)\sin\left(-a+c\right)-\log\left(c\right.}{}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) + 2*sin(b
*x + a))/b
```

3.233.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(29) = 58.

Time = 0.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 7.79

$$\int \cot(c+bx)\sin(a+bx)\,dx = -\frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}c\right)} - \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\right.\right.}{\tan\left(\right.}\right.}{b}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-2*((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*tan(
1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)^2
*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) - (ta
n(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))
*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*
a)^2 + tan(1/2*c)^2 + 1) + (tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*b*x) - 2*t
an(1/2*a))/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)))/b
```

3.233.9 Mupad [B] (verification not implemented)

Time = 25.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 8.03

$$\begin{aligned}\int \cot(c+bx)\sin(a+bx)\,dx &= \frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b} - \frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b} \\ &\quad - \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right) - \frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} \\ &\quad + \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right) + \frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}\end{aligned}$$

input
```
int(cot(c + b*x)*sin(a + b*x),x)
```

output
```
(exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1i + b*x*1i)*1i)/(2*b) - (log(- e
xp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(e
xp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*
2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) + (log((exp(a*2i)*exp(-c*2i)*(ex
p(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(
b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2
i - c*2i))^(1/2))
```

3.233. $\int \cot(c + bx)\sin(a + bx)\,dx$

3.234 $\int \cot^2(c+bx)\sin(a+bx)\,dx$

3.234.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \cot^2(c+bx)\sin(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{b}+\frac{\cos(a+bx)}{b}-\frac{\csc(c+bx)\sin(a-c)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)/b+cos(b*x+a)/b-csc(b*x+c)*sin(a-c)/b
```

3.234.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int \cot^2(c+bx)\sin(a+bx)\,dx=-\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}+\frac{\cos(a)\cos(bx)}{b}-\frac{\csc(c+bx)\sin(a-c)}{b}-\frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Cot[c + b*x]^2*Sin[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos
[a]*Cos[b*x])/b - (Csc[c + b*x]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

3.234.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5089, 3042, 25, 3086, 24, 5088, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cot^2(bx+c)\,dx$$

$$\Big\downarrow \text{5089}$$

$$\int \cos(a+bx)\cot(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)\cot(c+bx)dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\int \cos(a+bx)\cot(c+bx)dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \cos(a+bx)\cot(c+bx)dx-\frac{\sin(a-c)\int 1d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\int \cos(a+bx)\cot(c+bx)dx-\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{5088}$$

$$\cos(a-c)\int \csc(c+bx)dx-\int \sin(a+bx)dx-\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)dx-\int \sin(a+bx)dx-\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\cos(a-c)\int \csc(c+bx)dx-\frac{\sin(a-c)\csc(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cos(a-c)\text{arctanh}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

input
```
Int[Cot[c + b*x]^2*Sin[a + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b - (Csc[c + b*x]*S
in[a - c])/b
```

3.234.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.234.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.11

method	result	size
risch	$\frac{e^{i(xb+a)}}{2b}+\frac{e^{-i(xb+a)}}{2b}+\frac{e^{i(xb+3a)}-e^{i(xb+a+2c)}}{b\left(-e^{2i(xb+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(xb+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(e^{i(xb+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}$	143

input
```
int(cot(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/b/(-exp(2*I*(b*x+a+c))+exp(2*
I*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*cos(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(46) = 92.

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.87

$$\int \cot^2(c+bx)\sin(a+bx)\,dx$$

$$=\frac{4\left(\cos\left(-2a+2c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(-2a+2c)+\left(\cos(-2a+2c)^2+2\cos(\cdots\right.\right.}{\cdots}}{4\left(b\cos\cdots\right.}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*
a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) + (cos(-2*a + 2*c)^2 + 2*cos(-2
*a + 2*c) + 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*co
s(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)
*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) -
cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*si
n(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) +
1) + 4*(cos(b*x + a)^2 + 1)*sin(-2*a + 2*c))/(b*cos(b*x + a)*sin(-2*a + 2*
c) + (b*cos(-2*a + 2*c) + b)*sin(b*x + a))
```

3.234.6 Sympy [F]

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = \int \sin(a+bx)\cot^2(bx+c)\,dx$$

input
```
integrate(cot(b*x+c)**2*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*cot(b*x + c)**2, x)
```

3.234.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 612, normalized size of antiderivative = 13.30

$$\int \cot^2(c+bx)\sin(a+bx)\,dx$$
$$= \frac{(\cos(3\,bx+a+2\,c)-\cos(bx+a))\cos(4\,bx+2\,a+2\,c)-(3\,\cos(2\,bx+2\,a)-3\,\cos(2\,bx+2\,c)+1)\,\mathrm{co}}{}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*((cos(3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - (3*cos
(2*b*x + 2*a) - 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x
 + 2*a)*cos(b*x + a) - 3*cos(2*b*x + 2*c)*cos(b*x + a) - (cos(3*b*x + a +
2*c)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos
(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 2*cos(-a +
c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log(cos
(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) +
sin(c)^2) + (cos(3*b*x + a + 2*c)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*c
os(b*x + a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b
*x + a + 2*c)^2 - 2*cos(-a + c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a
 + c)*sin(b*x + a)^2)*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + (sin(3*b*x + a + 2*c) - sin(b*x +
 a))*sin(4*b*x + 2*a + 2*c) - 3*(sin(2*b*x + 2*a) - sin(2*b*x + 2*c))*sin(
3*b*x + a + 2*c) + 3*sin(2*b*x + 2*a)*sin(b*x + a) - 3*sin(2*b*x + 2*c)*si
n(b*x + a) + cos(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a +
 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin
(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 577, normalized size of antiderivative = 12.54

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(
1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1)
)/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + ta
n(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^
2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + (tan(1/2*b*x)^3*tan(1/
2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*b*x
)^3*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^2 - t
an(1/2*b*x)^3*tan(1/2*c)^3 + 6*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^3 + 3*
tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c
)^4 - tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^3*tan(1/2*c) - 6*tan(1/2*b*
x)^2*tan(1/2*a)*tan(1/2*c) - 3*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c) - 2*ta
n(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1
/2*b*x)*tan(1/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*a)
 + 3*tan(1/2*b*x)*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c) + 4*tan(1/2*c)^2)/(
(tan(1/2*b*x)^4*tan(1/2*c) + tan(1/2*b*x)^3*tan(1/2*c)^2 - tan(1/2*b*x)^3
+ tan(1/2*b*x)*tan(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))*(tan(1/2*a)^2*tan
(1/2*c) + tan(1/2*c))))/b
```

3.234.9 Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 290, normalized size of antiderivative = 6.30

$$\int \cot^2(c+bx)\sin(a+bx)\,dx$$
$$= \frac{e^{-a\,1i-b\,x\,1i}}{2\,b} + \frac{e^{a\,1i+b\,x\,1i}}{2\,b} + \frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)}$$
$$- \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$
$$+ \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

input
```
int(cot(c + b*x)^2*sin(a + b*x),x)
```

output
```
exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) + (exp(a*1i + b*x*1i
)*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x*2i)*
1i)) - (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(
a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/
2))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) + (log((exp(a*2i)
*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) -
exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(a*2i - c*2i) +
1))/(2*b*exp(a*2i - c*2i)^(1/2))
```

3.234. $\int \cot^2(c+bx)\sin(a+bx)\,dx$

3.235 $\int \cot^3(c+bx)\sin(a+bx)\,dx$

3.235.1 Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{3\text{arctanh}(\cos(c+bx))\sin(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b} - \frac{\sin(a+bx)}{b}$$

output
```
-cos(a-c)*csc(b*x+c)/b+3/2*arctanh(cos(b*x+c))*sin(a-c)/b-1/2*cot(b*x+c)*c
sc(b*x+c)*sin(a-c)/b-sin(b*x+a)/b
```

3.235.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \frac{12\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+\csc^2(c+bx)(2\sin(a-2c-bx)-5\sin(a+bx)+\sin(a-}{4b}$$

input
```
Integrate[Cot[c + b*x]^3*Sin[a + b*x],x]
```

output
```
(12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Sin[a - c] + Csc[c + b*x]^2*(2*S
in[a - 2*c - b*x] - 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

3.235.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {5089, 3042, 3091, 3042, 4257, 5088, 3042, 25, 3086, 24, 5089, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cot^3(bx+c)\,dx$$

$$\Big\downarrow \text{5089}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\int \cot^2(c+bx)\csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\int \sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\left(-\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\left(-\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{5088}$$

$$-\int \cot(c+bx)\sin(a+bx)dx+\cos(a-c)\int \cot(c+bx)\csc(c+bx)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \cot(c+bx)\sin(a+bx)dx+\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+bx)\sin(a+bx)dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx+\sin(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \; 3086$$

$$-\frac{\cos(a-c)\int 1d\csc(c+bx)}{b}-\int \cot(c+bx)\sin(a+bx)dx+\sin(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \; 24$$

$$-\int \cot(c+bx)\sin(a+bx)dx+\sin(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \; 5089$$

$$-\sin(a-c)\int \csc(c+bx)dx-\int \cos(a+bx)dx+\sin(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\sin(a-c)\int \csc(c+bx)dx-\int \sin\left(a+bx+\frac{\pi}{2}\right)dx+\sin(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \; 3117$$

$$-\sin(a-c)\int \csc(c+bx)dx+\sin(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \; 4257$$

$$\frac{\sin(a-c)\operatorname{arctanh}(\cos(bx+c))}{b}+\sin(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Int[Cot[c + b*x]^3*Sin[a + b*x],x]
```

3.235. $\int \cot^3(c+bx)\sin(a+bx)\,dx$

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b + (A
rcTanh[Cos[c + b*x]]/(2*b) - (Cot[c + b*x]*Csc[c + b*x])/(2*b))*Sin[a - c]
 - Sin[a + b*x]/b
```

3.235.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

3.235. $\int \cot^3(c+bx)\sin(a+bx)\,dx$

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.235.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.49

method	result
risch	$\frac{i\,\mathrm{e}^{i(xb+a)}}{2b}-\frac{i\,\mathrm{e}^{-i(xb+a)}}{2b}+\frac{i\left(-3\,\mathrm{e}^{i(3xb+5a+2c)}-\mathrm{e}^{i(3xb+3a+4c)}+\mathrm{e}^{i(xb+5a)}+3\,\mathrm{e}^{i(xb+3a+2c)}\right)}{2b\left(-\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^{2}}+\frac{3\ln\left(\mathrm{e}^{i(xb+a)}+\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{2b}-$

input
```
int(cot(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+1/2*I/b/(-exp(2*I*(b*x+a+c)
)+exp(2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b
*x+5*a))+3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin
(a-c)-3/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*sin(a-c)
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.03

$$\int \cot^3(c+bx)\sin(a+bx)\,dx$$

$$=\frac{\frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)^2-1\right)\cos(bx+a)\sin(bx+a)+\left(2\cos(bx+a)^2\cos(-2a+2c)-\cos(-2a+2c)-1\right)\sin(-2a+2c)\right)\log\left(-\frac{2\cos(bx+a)^2\cos(-2a}{}\right.}{\sqrt{\cos(-2a+2c)+1}}}{}$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*c
os(b*x + a)^2*cos(-2*a + 2*c) - cos(-2*a + 2*c) - 1)*sin(-2*a + 2*c))*log(
-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a
+ 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(
-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x +
 a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(
-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*cos(b*x + a)^2*cos(-2*a
 + 2*c) - 3*cos(-2*a + 2*c) - 5)*sin(b*x + a) - 4*(4*cos(b*x + a)^3 - 5*co
s(b*x + a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos
(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) - b)
```

3.235.6 Sympy [F]

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \int \sin(a+bx)\cot^3(bx+c)\,dx$$

input
```
integrate(cot(b*x+c)**3*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*cot(b*x + c)**3, x)
```

3.235.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(70) = 140$.

Time = 0.32 (sec) , antiderivative size = 1254, normalized size of antiderivative = 16.95

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(2*(sin(5*b*x + a + 4*c) - 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(
6*b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*b*x + 4*c) - 2*
sin(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3*
b*x + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*sin(3*b*x + a
 + 2*c) - sin(b*x + a))*cos(4*b*x + 4*c) + 4*(2*sin(2*b*x + 2*a) + 5*sin(2
*b*x + 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c)
+ 4*cos(3*b*x + a + 2*c)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x +
a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-
a + c) + 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin
(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a +
2*c)*sin(-a + c) - cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*s
in(3*b*x + a + 2*c)*sin(-a + c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a
+ 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin
(b*x)*sin(c) + sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*cos(3
*b*x + a + 2*c)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a
 + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c) +
4*sin(3*b*x + a + 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin(b*x + a)
*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a + 2*c)*sin(
-a + c) - cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*sin(3*b*x
+ a + 2*c)*sin(-a + c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c)...
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(70) = 140$.

Time = 0.36 (sec) , antiderivative size = 870, normalized size of antiderivative = 11.76

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/4*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*
tan(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*
a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) -
 12*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(
1/2*c))*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + t
an(1/2*a)^2 + tan(1/2*c)^2 + 1) + 8*(tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*b
*x) - 2*tan(1/2*a))/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)) - (2*tan(1/2
*b*x)^3*tan(1/2*a)*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^7
 + tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^8 - 4*tan(1/2*b*x)^3*tan(1/2*a)^2*
tan(1/2*c)^4 + 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^5 - 5*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*c)^5 + 2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^6 - 4*
tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*b*x)^2*tan(1/2*c)^7 - 2*t
an(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^7 - 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*
c)^3 + 5*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^3 + 4*tan(1/2*b*x)^3*tan(1
/2*c)^4 - 22*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^4 + 4*tan(1/2*b*x)*tan(1
/2*a)^2*tan(1/2*c)^4 + 5*tan(1/2*b*x)^2*tan(1/2*c)^5 - 14*tan(1/2*b*x)*tan
(1/2*a)*tan(1/2*c)^5 + 2*tan(1/2*a)^2*tan(1/2*c)^5 + 4*tan(1/2*b*x)*tan(1/
2*c)^6 + 2*tan(1/2*a)*tan(1/2*c)^6 - 2*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c
) - tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c) + 2*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*c)^2 - 4*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^2 - 5*tan(1/2*b*x)...
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(cot(c + b*x)^3*sin(a + b*x),x)
```

output
```
\text{Hanged}
```

3.236 $\int \sin(a+bx)\tan(c+dx)\,dx$

3.236.1 Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \sin(a+bx)\tan(c+dx)\,dx = \frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b} - \frac{ie^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b} - \frac{ie^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}$$

output
```
1/2*I/b/exp(I*(b*x+a))+1/2*I*exp(I*(b*x+a))/b-I*hypergeom([1, -1/2*b/d],[1
-1/2*b/d],-exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))-I*exp(I*(b*x+a))*hypergeom([
1, 1/2*b/d],[1+1/2*b/d],-exp(2*I*(d*x+c)))/b
```

3.236.2 Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.81

$$\int \sin(a+bx)\tan(c+dx)\,dx = -\frac{ie^{-i(a+bx)}\left(-1-e^{2i(a+bx)}+2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)+2e^{2i(a+bx)}\,\text{Hypergeomet}\right.}{2b}$$

input
```
Integrate[Sin[a + b*x]*Tan[c + d*x],x]
```

output
```
((-1/2*I)*(-1 - E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
 b/(2*d), -E^((2*I)*(c + d*x))] + 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[
1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))]))/(b*E^(I*(a + b*x)))
```

3.236.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5068, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan(c+dx)\,dx$$

$$\Big\downarrow \ 5068$$

$$\int \left(-\frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{1}{2}e^{-i(a+bx)}-\frac{1}{2}e^{i(a+bx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{ie^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2i(c+dx)}\right)}{b}+\frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b}$$

input
```
Int[Sin[a + b*x]*Tan[c + d*x],x]
```

output
```
(I/2)/(b*E^(I*(a + b*x))) + ((I/2)*E^(I*(a + b*x)))/b - (I*Hypergeometric2
F1[1, -1/2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) -
(I*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c
 + d*x))])/b
```

3.236.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5068
```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

3.236.4 Maple [F]

$$\int \sin(xb+a)\tan(dx+c)\,dx$$

input
```
int(sin(b*x+a)*tan(d*x+c),x)
```

output
```
int(sin(b*x+a)*tan(d*x+c),x)
```

3.236.5 Fricas [F]

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral(sin(b*x + a)*tan(d*x + c), x)
```

3.236.6 Sympy [F]

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin(a+bx)\tan(c+dx)\,dx$$

input
```
integrate(sin(b*x+a)*tan(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*tan(c + d*x), x)
```

3.236.7 Maxima [F]

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

input
```
integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)*tan(d*x + c), x)
```

3.236.8 Giac [F]

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

input
```
integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(d*x + c), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin\left(a+b\,x\right)\,\tan(c+d\,x)\,dx$$

input
```
int(sin(a + b*x)*tan(c + d*x),x)
```

output
```
int(sin(a + b*x)*tan(c + d*x), x)
```

3.237 $\int \cot(c+dx)\sin(a+bx)\,dx$

3.237.1 Optimal result

Integrand size = 13, antiderivative size = 139

$$\int \cot(c+dx)\sin(a+bx)\,dx = -\frac{ie^{-i(a+bx)}}{2b} - \frac{ie^{i(a+bx)}}{2b} + \frac{ie^{-i(a+bx)}\operatorname{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b} + \frac{ie^{i(a+bx)}\operatorname{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2i(c+dx)}\right)}{b}$$

output
```
-1/2*I/b/exp(I*(b*x+a))-1/2*I*exp(I*(b*x+a))/b+I*hypergeom([1, -1/2*b/d],[
1-1/2*b/d],exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))+I*exp(I*(b*x+a))*hypergeom([
1, 1/2*b/d],[1+1/2*b/d],exp(2*I*(d*x+c)))/b
```

3.237.2 Mathematica [A] (verified)

Time = 5.37 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.87

$$\int \cot(c+dx)\sin(a+bx)\,dx = \frac{-\cos(a)\cos(bx)\cot(c) - \frac{ie^{-i(a-2c+bx)}\left(be^{2idx}\operatorname{Hypergeometric2F1}\left(1,1-\frac{b}{2d},2-\frac{b}{2d},e^{2i(c+dx)}\right)-(b-2d)\operatorname{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\right.\right.}{(b-2d)(-1+e^{2ic})}}{}$$

input
```
Integrate[Cot[c + d*x]*Sin[a + b*x],x]
```

output
```
(-(Cos[a]*Cos[b*x]*Cot[c]) - (I*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 -
b/(2*d), 2 - b/(2*d), E^((2*I)*(c + d*x))] - (b - 2*d)*Hypergeometric2F1[1
, -1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))]))/((b - 2*d)*E^(I*(a - 2*c +
 b*x))*(-1 + E^((2*I)*c))) - (I*E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hyp
ergeometric2F1[1, 1 + b/(2*d), 2 + b/(2*d), E^((2*I)*(c + d*x))] - (b + 2*
d)*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))]))/((b +
 2*d)*(-1 + E^((2*I)*c))) + Cot[c]*Sin[a]*Sin[b*x])/b
```

3.237.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5070, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cot(c+dx)\,dx$$

$$\Big\downarrow \text{5070}$$

$$\int \left(\frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{1}{2}e^{-i(a+bx)}+\frac{1}{2}e^{i(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2i(c+dx)}\right)}{b}-\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}$$

input
```
Int[Cot[c + d*x]*Sin[a + b*x],x]
```

output
```
(-1/2*I)/(b*E^(I*(a + b*x))) - ((I/2)*E^(I*(a + b*x)))/b + (I*Hypergeometr
ic2F1[1, -1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x)))
+ (I*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(
c + d*x))])/b
```

3.237.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5070
```
Int[Cot[(c_.) + (d_.)*(x_)]*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-E^((
-I)*(a + b*x))/2 + E^(I*(a + b*x))/2 + 1/(E^(I*(a + b*x))*(1 - E^(2*I*(c +
d*x)))) - E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

3.237.4 Maple [F]

$$\int \cot (dx+c) \sin (xb+a) \, dx$$

input
```
int(cot(d*x+c)*sin(b*x+a),x)
```

output
```
int(cot(d*x+c)*sin(b*x+a),x)
```

3.237.5 Fricas [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \cot (dx+c) \sin (bx+a) \, dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(cot(d*x + c)*sin(b*x + a), x)
```

3.237.6 Sympy [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \sin (a+bx) \cot (c+dx) \, dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*cot(c + d*x), x)
```

3.237.7 Maxima [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \cot(dx+c)\sin(bx+a)\;dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)*sin(b*x + a), x)
```

3.237.8 Giac [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \cot(dx+c)\sin(bx+a)\;dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(cot(d*x + c)*sin(b*x + a), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \cot(c+d\,x)\;\sin(a+b\,x)\;dx$$

input
```
int(cot(c + d*x)*sin(a + b*x),x)
```

output
```
int(cot(c + d*x)*sin(a + b*x), x)
```

3.238 $\int \cos(a+bx)\cos^3(c+dx)\,dx$

3.238.1 Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)} +\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)+3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.238.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \frac{1}{8}\left(\frac{\sin(a-3c+bx-3dx)}{b-3d}+\frac{3\sin(a-c+bx-dx)}{b-d} +\frac{\sin(a+3c+bx+3dx)}{b+3d}+\frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

output
```
(Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sin[a - c + b*x - d*x])/(b - d)
 + Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sin[a + c + (b + d)*x])/(b +
d))/8
```

3.238.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{8}\cos(a+x(b-3d)-3c)+\frac{3}{8}\cos(a+x(b-d)-c)+\frac{3}{8}\cos(a+x(b+d)+c)+\frac{1}{8}\cos(a+x(b+3d)+3c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}+\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

output
```
Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(
b - d)) + (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)
*x]/(8*(b + 3*d))
```

3.238.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.238.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sin(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}+\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{\sin(xb-3dx+a-3c)b}{8(b-3d)(b+3d)}+\frac{3\sin(xb-3dx+a-3c)d}{8(b-3d)(b+3d)}+\frac{3\sin(xb-dx+a-c)b}{8(b-d)(b+d)}+\frac{3\sin(xb-dx+a-c)d}{8(b-d)(b+d)}+\frac{3\sin(xb+dx+a+c)b}{8(b-d)(b+d)}-$
parallelrisch	$-2b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)(b^2-7d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+6d\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1\right)(b^2-3d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+6b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)(b^2$

input
```
int(cos(b*x+a)*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)+3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \cos(a+bx)\cos^3(c+dx)\,dx =$$
$$-\frac{\left(6\,bd^2\cos\left(dx+c\right)-\left(b^3-bd^2\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-3\,\left(2\,d^3\cos\left(bx+a\right)-\left(b^2d-d^3\right)\cos\left(bx+\right.\right.}{b^4-10\,b^2d^2+9\,d^4}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="fricas")
```

output
```
-((6*b*d^2*cos(d*x + c) - (b^3 - b*d^2)*cos(d*x + c)^3)*sin(b*x + a) - 3*(
2*d^3*cos(b*x + a) - (b^2*d - d^3)*cos(b*x + a)*cos(d*x + c)^2)*sin(d*x +
c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

3.238.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(76) = 152$.

Time = 2.02 (sec) , antiderivative size = 918, normalized size of antiderivative = 10.09

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)**3,x)
```

output
```
Piecewise((x*cos(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin
(c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*s
in(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*
x)**3/8 - 3*sin(a - 3*d*x)*cos(c + d*x)**3/(8*d) - sin(c + d*x)**3*cos(a -
 3*d*x)/(24*d) - sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b,
-3*d)), (-3*x*sin(a - d*x)*sin(c + d*x)**3/8 - 3*x*sin(a - d*x)*sin(c + d*
x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3
*x*cos(a - d*x)*cos(c + d*x)**3/8 + sin(a - d*x)*cos(c + d*x)**3/(8*d) + 3
*sin(c + d*x)**3*cos(a - d*x)/(8*d) + 3*sin(c + d*x)*cos(a - d*x)*cos(c +
d*x)**2/(4*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(a
 + d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)*
cos(c + d*x)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 - sin(a + d*x)*cos(c +
 d*x)**3/(8*d) + 3*sin(c + d*x)**3*cos(a + d*x)/(8*d) + 3*sin(c + d*x)*cos
(a + d*x)*cos(c + d*x)**2/(4*d), Eq(b, d)), (-x*sin(a + 3*d*x)*sin(c + d*x
)**3/8 + 3*x*sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin(c + d
*x)**2*cos(a + 3*d*x)*cos(c + d*x)/8 + x*cos(a + 3*d*x)*cos(c + d*x)**3/8
+ sin(a + 3*d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*d) + 7*sin(a + 3*d*x)*cos
(c + d*x)**3/(24*d) - sin(c + d*x)**3*cos(a + 3*d*x)/(8*d), Eq(b, 3*d)), (
b**3*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*
d*sin(c + d*x)*cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d*...
```

3.238.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(83) = 166$.

Time = 0.30 (sec) , antiderivative size = 914, normalized size of antiderivative = 10.04

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))
*cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin
(3*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*sin(3*c) - b^2*d*sin
(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) - 3*(b
^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b +
 d)*x + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9
*d^3*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c
) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) - (b^3*si
n(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d
)*x - a + 6*c) + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3
*sin(3*c))*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^
2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) -
3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) -
 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin
((b + d)*x + a + 4*c) - 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c
) + 9*d^3*cos(3*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos
(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(
b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b
 - d)*x - a - 2*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3
*d^3*cos(3*c))*sin(-(b - 3*d)*x - a + 6*c) + (b^3*cos(3*c) + 3*b^2*d*co...
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(bx+3\,dx+a+3\,c)}{8\,(b+3\,d)} + \frac{3\,\sin(bx+dx+a+c)}{8\,(b+d)} + \frac{3\,\sin(bx-dx+a-c)}{8\,(b-d)} + \frac{\sin(bx-3\,dx+a-3\,c)}{8\,(b-3\,d)}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*sin(b*x + d*x + a + c)/(b +
 d) + 3/8*sin(b*x - d*x + a - c)/(b - d) + 1/8*sin(b*x - 3*d*x + a - 3*c)/
(b - 3*d)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 22.87 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.44

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = e^{a\,1i-c\,3i+b\,x\,1i-d\,x\,3i}\left(\frac{b+3\,d}{b^2\,16i-d^2\,144i}-\frac{e^{-a\,2i-b\,x\,2i}\,(b-3\,d)}{b^2\,16i-d^2\,144i}\right)+e^{a\,1i+c\,3i+b\,x\,1i+d\,x\,3i}\left(\frac{b-3\,d}{b^2\,16i-d^2\,144i}-\frac{e^{-a\,2i-b\,x\,2i}\,(b+3\,d)}{b^2\,16i-d^2\,144i}\right)+e^{a\,1i-c\,1i+b\,x\,1i-d\,x\,1i}\left(\frac{3\,b+3\,d}{b^2\,16i-d^2\,16i}-\frac{e^{-a\,2i-b\,x\,2i}\,(3\,b-3\,d)}{b^2\,16i-d^2\,16i}\right)+e^{a\,1i+c\,1i+b\,x\,1i+d\,x\,1i}\left(\frac{3\,b-3\,d}{b^2\,16i-d^2\,16i}-\frac{e^{-a\,2i-b\,x\,2i}\,(3\,b+3\,d)}{b^2\,16i-d^2\,16i}\right)$$

input
```
int(cos(a + b*x)*cos(c + d*x)^3,x)
```

output
```
exp(a*1i - c*3i + b*x*1i - d*x*3i)*((b + 3*d)/(b^2*16i - d^2*144i) - (exp(
- a*2i - b*x*2i)*(b - 3*d))/(b^2*16i - d^2*144i)) + exp(a*1i + c*3i + b*x*
1i + d*x*3i)*((b - 3*d)/(b^2*16i - d^2*144i) - (exp(- a*2i - b*x*2i)*(b +
3*d))/(b^2*16i - d^2*144i)) + exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3
*d)/(b^2*16i - d^2*16i) - (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(b^2*16i - d^
2*16i)) + exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(b^2*16i - d^2*1
6i) - (exp(- a*2i - b*x*2i)*(3*b + 3*d))/(b^2*16i - d^2*16i))
```

3.239 $\int \cos(a+bx)\cos^2(c+dx)\,dx$

3.239.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{\sin(a+bx)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\sin(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
1/2*sin(b*x+a)/b+1/4*sin(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sin(a+2*c+(b+2*d)*x)
/(b+2*d)
```

3.239.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{1}{4}\left(\frac{2\cos(bx)\sin(a)}{b}+\frac{2\cos(a)\sin(bx)}{b}+\frac{\sin(a-2c+bx-2dx)}{b-2d}+\frac{\sin(a+2c+bx+2dx)}{b+2d}\right)$$

input
```
Integrate[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

output
```
((2*Cos[b*x]*Sin[a])/b + (2*Cos[a]*Sin[b*x])/b + Sin[a - 2*c + b*x - 2*d*x
]/(b - 2*d) + Sin[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

3.239.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{4}\cos(a+x(b-2d)-2c)+\frac{1}{4}\cos(a+x(b+2d)+2c)+\frac{1}{2}\cos(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sin(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sin(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

output
```
Sin[a + b*x]/(2*b) + Sin[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sin[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

3.239.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.239.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sin(xb+a)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4b-8d}+\frac{\sin(a+2c+(b+2d)x)}{4b+8d}$
parallelrisch	$\frac{b(b+2d)\sin(a-2c+(b-2d)x)+2\left(\frac{b\sin(a+2c+(b+2d)x)}{2}+\sin(xb+a)(b+2d)\right)(b-2d)}{4b^3-16b\,d^2}$
risch	$\frac{\sin(xb+a)}{2b}+\frac{\sin(xb-2dx+a-2c)b}{4(b-2d)(b+2d)}+\frac{\sin(xb-2dx+a-2c)d}{2(b-2d)(b+2d)}+\frac{\sin(xb+2dx+a+2c)b}{4(b-2d)(b+2d)}-\frac{\sin(xb+2dx+a+2c)d}{2(b-2d)(b+2d)}$
norman	$\frac{-\frac{4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-4d^2}+\frac{4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2-4d^2}+\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-4d^2}-\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2-4d^2}+\frac{2\left(b^2-2d^2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\left(b^2-4d^2\right)b}+\frac{2\left(b^2-\right.}{}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(cos(b*x+a)*cos(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(b*x+a)/b+1/4*sin(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sin(a+2*c+(b+2*d)*x)
/(b+2*d)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = -\frac{2\,bd\cos(bx+a)\cos(dx+c)\sin(dx+c)-\left(b^2\cos(dx+c)^2-2\,d^2\right)\sin(bx+a)}{b^3-4\,bd^2}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - 2*d
^2)*sin(b*x + a))/(b^3 - 4*b*d^2)
```

3.239.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(49) = 98$.

Time = 0.76 (sec) , antiderivative size = 405, normalized size of antiderivative = 6.53

$$\int \cos(a+bx)\cos^2(c+dx)\,dx$$
$$= \begin{cases} x\cos(a)\cos^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\cos(a) \\ -\frac{x\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{2}-\frac{x\sin^2(c+dx)\cos(a-2dx)}{4}+\frac{x\cos(a-2dx)\cos^2(c+dx)}{4}-\frac{\sin(a-2dx)\cos^2(c+dx)}{2d}-\frac{\sin(c+d}{} \\ \frac{x\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{2}-\frac{x\sin^2(c+dx)\cos(a+2dx)}{4}+\frac{x\cos(a+2dx)\cos^2(c+dx)}{4}+\frac{\sin(a+2dx)\cos^2(c+dx)}{2d}-\frac{\sin(c+dx)}{} \\ \frac{b^2\sin(a+bx)\cos^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sin(c+dx)\cos(a+bx)\cos(c+dx)}{b^3-4bd^2}-\frac{2d^2\sin(a+bx)\sin^2(c+dx)}{b^3-4bd^2}-\frac{2d^2\sin(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)**2,x)
```

output
```
Piecewise((x*cos(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a), Eq(b, 0)
), (-x*sin(a - 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos(
a - 2*d*x)/4 + x*cos(a - 2*d*x)*cos(c + d*x)**2/4 - sin(a - 2*d*x)*cos(c +
 d*x)**2/(2*d) - sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/(4*d), Eq(b, -2*
d)), (x*sin(a + 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos
(a + 2*d*x)/4 + x*cos(a + 2*d*x)*cos(c + d*x)**2/4 + sin(a + 2*d*x)*cos(c
+ d*x)**2/(2*d) - sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/(4*d), Eq(b, 2*
d)), (b**2*sin(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(c +
d*x)*cos(a + b*x)*cos(c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*sin
(c + d*x)**2/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*cos(c + d*x)**2/(b**3
 - 4*b*d**2), True))
```

3.239.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(56) = 112$.

Time = 0.27 (sec) , antiderivative size = 416, normalized size of antiderivative = 6.71

$$\int \cos(a+bx)\cos^2(c+dx)\,dx =$$
$$-\frac{(b^2\sin(2c)-2bd\sin(2c))\cos((b+2d)x+a+4c)-(b^2\sin(2c)-2bd\sin(2c))\cos((b+2d)x+a)}{}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*((b^2*sin(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a + 4*c) - (b^2*si
n(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a) - (b^2*sin(2*c) + 2*b*d*sin(
2*c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*cos(-(
b - 2*d)*x - a) + 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a + 2*c) - 2
*(b^2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a - 2*c) - (b^2*cos(2*c) - 2*b*
d*cos(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*cos(2*c) - 2*b*d*cos(2*c))*s
in((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a
 + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*c
os(2*c) - 4*d^2*cos(2*c))*sin(b*x + a + 2*c) - 2*(b^2*cos(2*c) - 4*d^2*cos
(2*c))*sin(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c
)^2 + b*sin(2*c)^2)*d^2)
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{\sin(bx+2\,dx+a+2\,c)}{4\,(b+2\,d)} + \frac{\sin(bx-2\,dx+a-2\,c)}{4\,(b-2\,d)} + \frac{\sin(bx+a)}{2\,b}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*sin(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*sin(b*x - 2*d*x + a - 2*c)/
(b - 2*d) + 1/2*sin(b*x + a)/b
```

3.239.9 Mupad [B] (verification not implemented)

Time = 22.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{\sin(a+b\,x)}{2\,b} - \frac{d\,(2\,b\,\sin(a-2\,c+b\,x-2\,d\,x)-2\,b\,\sin(a+2\,c+b\,x+2\,d\,x))+b^2\,\sin(a-2\,c+b\,x-2\,d\,x)+b^2\,\sin}{16\,b\,d^2-4\,b^3}$$

input
```
int(cos(a + b*x)*cos(c + d*x)^2,x)
```

output
```
sin(a + b*x)/(2*b) - (d*(2*b*sin(a - 2*c + b*x - 2*d*x) - 2*b*sin(a + 2*c
+ b*x + 2*d*x)) + b^2*sin(a - 2*c + b*x - 2*d*x) + b^2*sin(a + 2*c + b*x +
 2*d*x))/(16*b*d^2 - 4*b^3)
```

3.239. $\int \cos(a+bx)\cos^2(c+dx)\,dx$

3.240 $\int \cos(a+bx)\cos(c+dx)\,dx$

3.240.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{\sin(a-c+(b-d)x)}{2(b-d)} + \frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

output
```
1/2*sin(a-c+(b-d)*x)/(b-d)+1/2*sin(a+c+(b+d)*x)/(b+d)
```

3.240.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{\sin(a-c+(b-d)x)}{2(b-d)} + \frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Cos[a + b*x]*Cos[c + d*x],x]
```

output
```
Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

3.240.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{2}\cos(a+x(b-d)-c)+\frac{1}{2}\cos(a+x(b+d)+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

input
```
Int[Cos[a + b*x]*Cos[c + d*x],x]
```

output
```
Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

3.240.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.240.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\sin(a-c+(b-d)x)}{2b-2d}+\frac{\sin(a+c+(b+d)x)}{2b+2d}$	40
parallelrisch	$\frac{(b+d)\sin(a-c+(b-d)x)+\sin(a+c+(b+d)x)(b-d)}{2b^2-2d^2}$	48
risch	$\frac{\sin(xb-dx+a-c)b}{2(b-d)(b+d)}+\frac{\sin(xb-dx+a-c)d}{2(b-d)(b+d)}+\frac{\sin(xb+dx+a+c)b}{2(b-d)(b+d)}-\frac{\sin(xb+dx+a+c)d}{2(b-d)(b+d)}$	108
norman	$\frac{\frac{2b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b^2-d^2}-\frac{2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}-\frac{2b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^2-d^2}+\frac{2d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$	147

input
```
int(cos(b*x+a)*cos(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(a-c+(b-d)*x)/(b-d)+1/2*sin(a+c+(b+d)*x)/(b+d)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{b\cos(dx+c)\sin(bx+a)-d\cos(bx+a)\sin(dx+c)}{b^2-d^2}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="fricas")
```

output
```
(b*cos(d*x + c)*sin(b*x + a) - d*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

3.240.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(32) = 64.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \cos(a+bx)\cos(c+dx)\,dx = \begin{cases} x\cos(a)\cos(c) & \text{for } b=0\wedge d=0 \\ -\frac{x\sin(a-dx)\sin(c+dx)}{2}+\frac{x\cos(a-dx)\cos(c+dx)}{2}-\frac{\sin(a-dx)\cos(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sin(a+dx)\sin(c+dx)}{2}+\frac{x\cos(a+dx)\cos(c+dx)}{2}+\frac{\sin(a+dx)\cos(c+dx)}{2d} & \text{for } b=d \\ \frac{b\sin(a+bx)\cos(c+dx)}{b^2-d^2}-\frac{d\sin(c+dx)\cos(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c),x)
```

output
```
Piecewise((x*cos(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (-x*sin(a - d*x)*sin(c +
 d*x)/2 + x*cos(a - d*x)*cos(c + d*x)/2 - sin(a - d*x)*cos(c + d*x)/(2*d),
 Eq(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/
2 + sin(a + d*x)*cos(c + d*x)/(2*d), Eq(b, d)), (b*sin(a + b*x)*cos(c + d*
x)/(b**2 - d**2) - d*sin(c + d*x)*cos(a + b*x)/(b**2 - d**2), True))
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{\sin(bx+dx+a+c)}{2(b+d)} - \frac{\sin(-bx+dx-a+c)}{2(b-d)}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="maxima")
```

output
```
1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{\sin(bx+dx+a+c)}{2(b+d)} + \frac{\sin(bx-dx+a-c)}{2(b-d)}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="giac")
```

output
```
1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

3.240.9 Mupad [B] (verification not implemented)

Time = 22.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{b\left(\frac{\sin(a+c+b\,x+d\,x)}{2}+\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2} - \frac{d\left(\frac{\sin(a+c+b\,x+d\,x)}{2}-\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2}$$

input
```
int(cos(a + b*x)*cos(c + d*x),x)
```

output
```
(b*(sin(a + c + b*x + d*x)/2 + sin(a - c + b*x - d*x)/2))/(b^2 - d^2) - (d
*(sin(a + c + b*x + d*x)/2 - sin(a - c + b*x - d*x)/2))/(b^2 - d^2)
```

3.241 $\int \cos(a+bx)\sec(c+bx)\,dx$

3.241.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}$$

output
```
x*cos(a-c)+ln(cos(b*x+c))*sin(a-c)/b
```

3.241.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

3.241.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5094, 24, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{ 5094}$$

$$\cos(a-c)\int 1dx-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{ 24}$$

$$x\cos(a-c)-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$x\cos(a-c)-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

3.241.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

3.241.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

method	result
risch	$-2i\sin\left(a-c\right)x-\frac{2i\sin(a-c)a}{b}+x\,\mathrm{e}^{i(a-c)}+\frac{\ln\left(\mathrm{e}^{2i(xb+a)}+\mathrm{e}^{2i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{\frac{(\sin(a)\cos(c)-\cos(a)\sin(c))\ln(\tan(xb+a)\sin(a)\cos(c)-\tan(xb+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{\cos(a)^{2}\cos(c)^{2}+\cos(c)^{2}\sin(a)^{2}+\cos(a)^{2}\sin(c)^{2}+\sin(a)^{2}\sin(c)^{2}}+\frac{\frac{(\cos(a)\sin(c)-\sin(a)\cos(c))\ln\left(1+\mathrm{ta}\right.}{2}}{\left(\cos(c)\right.}}{b}$

input
```
int(cos(b*x+a)*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*I*sin(a-c)*x-2*I/b*sin(a-c)*a+x*exp(I*(a-c))+ln(exp(2*I*(b*x+a))+exp(2*
I*(a-c)))/b*sin(a-c)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \cos(a+bx)\sec(c+bx)\,dx=\frac{bx\cos\left(-a+c\right)-\log\left(-\cos\left(bx+c\right)\right)\sin\left(-a+c\right)}{b}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

output
```
(b*x*cos(-a + c) - log(-cos(b*x + c))*sin(-a + c))/b
```

3.241.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(20) = 40$.

Time = 3.52 (sec) , antiderivative size = 435, normalized size of antiderivative = 16.73

$$\int \cos(a+bx)\sec(c+bx)\,dx = -\left(\begin{cases} -x \\ x \\ 0 \\ -\frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \cdots \end{cases}\right) + \left(\begin{cases} -\frac{\log(\sin(bx))}{b} \\ \frac{\log(\sin(bx))}{b} \\ \frac{x}{\cos(c)} \\ -\frac{bx\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{bx}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{2\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \cdots \end{cases}\right)$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x)
```

output
```
-Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*ta
n(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2
)**2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) -
tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b
) - log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/
2)**2 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*
tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1)
 - 1/(tan(c/2) + 1))/(b*tan(c/2)**2 + b), True))*sin(a) + Piecewise((-log(
sin(b*x))/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(
b, 0)), (-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) +
2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) -
 tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b)
- 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/
(b*tan(c/2)**2 + b), True))*cos(a)
```

3.241.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.85

$$\int \cos(a+bx)\sec(c+bx)\,dx$$
$$= \frac{2\,bx\cos\left(-a+c\right) - \log\left(\cos\left(2\,bx\right)^2 + 2\,\cos\left(2\,bx\right)\cos\left(2\,c\right) + \cos\left(2\,c\right)^2 + \sin\left(2\,bx\right)^2 - 2\,\sin\left(2\,bx\right)\sin\left(2\,c\right) + \right.}{2\,b}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(-a + c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*
c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(-a + c))/b
```

3.241.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(26) = 52$.

Time = 0.34 (sec) , antiderivative size = 440, normalized size of antiderivative = 16.92

$$\int \cos(a+bx)\sec(c+bx)\,dx$$
$$= \frac{\frac{\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,a\right)^2 + 4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right) - \tan\left(\frac{1}{2}\,c\right)^2 + 1\right)(bx+a)}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,a\right)^2 + \tan\left(\frac{1}{2}\,c\right)^2 + 1} - \frac{\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right) - \tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,a\right) - \tan\left(\frac{1}{2}\,c\right.\right.}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,a\right)^2 + \tan\left(\frac{1}{2}\,c\right)}}{}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1
/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/
2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 + ta
n(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*
a)^3*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)
- 4*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 -
 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(2*tan(b*x + a)*tan(1/2*a)
^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(
1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1
/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^4*tan(1/2
*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)*t
an(1/2*c)^4 + tan(1/2*a)^3 - tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)))/b
```

3.241.9 Mupad [B] (verification not implemented)

Time = 22.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.19

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)+x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right) + \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}{b}$$

input
```
int(cos(a + b*x)/cos(c + b*x),x)
```

output
```
x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(
a*1i - c*1i)/2) + (log(exp(a*2i - c*2i) + exp(a*2i + b*x*2i))*((exp(c*1i -
 a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

3.242 $\int \cos(a+bx)\sec^2(c+bx)\,dx$

3.242.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{b} - \frac{\sec(c+bx)\sin(a-c)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b
```

3.242.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = -\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} - \frac{\sec(c+bx)\sin(a-c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Sec
[c + b*x]*Sin[a - c])/b
```

3.242.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5094, 3042, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int \sec(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a-c)\int 1d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(a-c)\text{arctanh}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

output
```
(ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b
```

3.242.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

3.242.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.40

method	result
risch	$\frac{i\left(\mathrm{e}^{i(xb+3a)}-\mathrm{e}^{i(xb+a+2c)}\right)}{b\left(\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{\ln\left(\mathrm{e}^{i(xb+a)}+i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(\mathrm{e}^{i(xb+a)}-i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{b}$
default	$-\frac{2\left(-\frac{\left(\cos(a)^2\sin(c)^2-2\cos(a)\cos(c)\sin(a)\sin(c)+\cos(c)^2\sin(a)^2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\left(\cos(a)^2\cos(c)^2+\cos(c)^2\sin(a)^2+\cos(a)^2\sin(c)^2+\sin(a)^2\sin(c)^2\right)(\cos(a)\cos(c)+\sin(a)\sin(c))}-\frac{\sin(a)\cos(c)-\cos(a)\sin(}{\cos(a)^2\cos(c)^2+\cos(c)^2\sin(a)^2+\cos(a)^2\sin}\right.}{\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2+\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sin(a)\cos(c)+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\cos(a)\sin(c)-\cos(a)\cos(c)-\sin}$

input
```
int(cos(b*x+a)*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))+
ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-
c)))/b*cos(a-c)
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \cos(a+bx)\sec^2(c+bx)\,dx$$
$$= \frac{\cos(bx+c)\cos(-a+c)\log(\sin(bx+c)+1) - \cos(bx+c)\cos(-a+c)\log(-\sin(bx+c)+1) + 2\sin}{2\,b\cos(bx+c)}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + c)*cos(-a + c)*log(sin(b*x + c) + 1) - cos(b*x + c)*cos(-a
+ c)*log(-sin(b*x + c) + 1) + 2*sin(-a + c))/(b*cos(b*x + c))
```

3.242.6 Sympy [F(-2)]

Exception generated.

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.242.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(35) = 70$.

Time = 0.40 (sec) , antiderivative size = 391, normalized size of antiderivative = 11.17

$$\int \cos(a+bx)\sec^2(c+bx)\,dx =$$

$$-\frac{2\left(\sin\left(bx+2\,a\right)-\sin\left(bx+2\,c\right)\right)\cos\left(2\,bx+a+2\,c\right)+\left(\cos\left(2\,bx+a+2\,c\right)^2\cos\left(-a+c\right)+2\,\cos\left(2\,bx\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(b*x + 2*a) - sin(b*x + 2*c))*cos(2*b*x + a + 2*c) + (cos(2*b*
x + a + 2*c)^2*cos(-a + c) + 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + c
os(-a + c)*sin(2*b*x + a + 2*c)^2 + 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin
(a) + (cos(a)^2 + sin(a)^2)*cos(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2
- 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + s
in(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x
+ 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(cos(b*x + 2*a) - cos(
b*x + 2*c))*sin(2*b*x + a + 2*c) + 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(
b*x + 2*c) - 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b
*x + a + 2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)
^2 + 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

3.242.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. $2(35) = 70$.

Time = 0.38 (sec) , antiderivative size = 1341, normalized size of antiderivative = 38.31

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*ta
n(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1
/2*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*tan(1/2*
a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) -
tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*
tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/
2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a
) - tan(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^
2 + tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan
(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*tan(1/2*
c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1/2*a)*ta
n(1/2*c) + tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - (tan(1/2*a)^3*tan
(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/
2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3
- tan(1/2*a)^3 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*c)^2 + t
an(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + tan(
1/2*a) - tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*
c) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - t
an(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) - 1)
)/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2...
```

3.242.9 Mupad [B] (verification not implemented)

Time = 26.91 (sec) , antiderivative size = 246, normalized size of antiderivative = 7.03

$$\int \cos(a+bx)\sec^2(c+bx)\,dx$$
$$= \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} - \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} + \frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^2,x)
```

output
```
(log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-
c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*
2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)
*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*e
xp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*
2i - c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp
(a*2i - c*2i) + exp(a*2i + b*x*2i)))
```

3.242. $\int \cos(a+bx)\sec^2(c+bx)\,dx$

3.243 $\int \cos(a+bx)\sec^3(c+bx)\,dx$

3.243.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{\sec^2(c+bx)\sin(a-c)}{2b}+\frac{\cos(a-c)\tan(c+bx)}{b}$$

output
```
-1/2*sec(b*x+c)^2*sin(a-c)/b+cos(a-c)*tan(b*x+c)/b
```

3.243.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{\sec(c)\sec^2(c+bx)(\sin(a)-\cos(a-c)\sin(c+2bx))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

output
```
-1/2*(Sec[c]*Sec[c + b*x]^2*(Sin[a] - Cos[a - c]*Sin[c + 2*b*x]))/b
```

3.243.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5094, 3042, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int \sec^2(c+bx)dx - \sin(a-c)\int \sec^2(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2 dx - \sin(a-c)\int \sec(c+bx)^2\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2 dx - \frac{\sin(a-c)\int \sec(c+bx)d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2 dx - \frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos(a-c)\int 1d(-\tan(c+bx))}{b} - \frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\cos(a-c)\tan(bx+c)}{b} - \frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

output
```
-1/2*(Sec[c + b*x]^2*Sin[a - c])/b + (Cos[a - c]*Tan[c + b*x])/b
```

3.243.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

3.243.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\sin(2xb+a+c)}{b(\cos(2xb+2c)+1)}$	26
default	$-\frac{1}{2b(\sin(a)\cos(c)-\cos(a)\sin(c))(\tan(xb+a)\sin(a)\cos(c)-\tan(xb+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^2}$	56
risch	$\frac{i\left(2\,\mathrm{e}^{i(2xb+5a+c)}+\mathrm{e}^{i(5a-c)}+\mathrm{e}^{i(3a+c)}\right)}{\left(\mathrm{e}^{2i(xb+a+c)}+\mathrm{e}^{2ia}\right)^2 b}$	61

input
```
int(cos(b*x+a)*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*sin(2*b*x+a+c)/(cos(2*b*x+2*c)+1)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)+\sin(-a+c)}{2\,b\cos(bx+c)^2}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) + sin(-a + c))/(b*cos(b*x + c
)^2)
```

3.243.6 Sympy [F(-2)]

Exception generated.

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.243.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 10.05

$$\int \cos(a+bx)\sec^3(c+bx)\,dx =$$

$$-\frac{(2\sin(2bx+2a+2c)+\sin(2a)+\sin(2c))\cos(4bx+a+5c)+2(2\sin(2bx+2a+2c)+\sin(2a}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos(}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
-((2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) +
2*(2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(2*b*x + a + 3*c) +
(sin(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) + cos(2*a) +
cos(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) - 2*(
2*cos(2*b*x + 2*a + 2*c) + cos(2*a) + cos(2*c))*sin(2*b*x + a + 3*c) - (co
s(2*a) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*co
s(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*
c)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x
+ a + 3*c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2
*b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(
2*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

3.243.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(36) = 72$.

Time = 0.34 (sec) , antiderivative size = 315, normalized size of antiderivative = 8.29

$$\int \cos(a+bx)\sec^3(c+bx)\,dx =$$

$$-\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\tan}{4\left(2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)\right.}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*
a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c
)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*
c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan
(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2
+ 1)/((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*
tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan
(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*
c)^2 + 1)^2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a
) - tan(1/2*c))*b)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.244 $\int \cos^2(a+bx)\cos^3(c+dx)\,dx$

3.244.1 Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx=\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}+\frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)}+\frac{\sin(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

output
```
1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)+1
/16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.244.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx=\frac{1}{48}\left(\frac{18\cos(dx)\sin(c)}{d}+\frac{2\cos(3dx)\sin(3c)}{d}+\frac{18\cos(c)\sin(dx)}{d}+\frac{2\cos(3c)\sin(3dx)}{d}+\frac{3\sin(2a-3c+2bx-3dx)}{2b-3d}+\frac{9\sin(2a-c+2bx-dx)}{2b-d}+\frac{9\sin(2a+c+2bx+dx)}{2b+d}+\frac{3\sin(2a+3c+2bx+3dx)}{2b+3d}\right)$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

output
```
((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])
/d + (2*Cos[3*c]*Sin[3*d*x])/d + (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b -
 3*d) + (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sin[2*a + c + 2*b*x
+ d*x])/(2*b + d) + (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

3.244.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{16}\cos(2a+x(2b-3d)-3c)+\frac{3}{16}\cos(2a+x(2b-d)-c)+\frac{3}{16}\cos(2a+x(2b+d)+c)+\frac{1}{16}\cos(2a+x(2b$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}$$

input
```
Int[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

output
```
Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sin[2*a - c + (2*b -
d)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) +
 (3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sin[2*a + 3*c + (2*b + 3*
d)*x]/(16*(2*b + 3*d))
```

3.244.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.244.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sin(2a-3c+(2b-3d)x)}{32b-48d}+\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(dx+c)}{8d}+\frac{\sin(3dx+3c)}{24d}+\frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)}+\frac{\sin(2a+3c+}{32b+}$
parallelrisch	$\frac{(24b^3d+36b^2d^2-6b\,d^3-9d^4)\sin(2a-3c+(2b-3d)x)+72\left(\left(b+\frac{d}{2}\right)\left(b+\frac{3d}{2}\right)d\sin(2a-c+(2b-d)x)+\left(\frac{\left(b+\frac{d}{2}\right)d\sin(2a+3c+(2b+3d)}{3}\right.\right.}{768b^4d-1920b^2d^3+432d^5}$
risch	$\frac{3\sin(dx+c)b^2}{2(2b-d)d(2b+d)}-\frac{3d\sin(dx+c)}{8(2b-d)(2b+d)}+\frac{\sin(2xb-3dx+2a-3c)b}{8(2b-3d)(2b+3d)}+\frac{3d\sin(2xb-3dx+2a-3c)}{16(2b-3d)(2b+3d)}+\frac{3\sin(2xb-dx+2a-c)b}{8(2b-d)(2b+d)}+$

input
```
int(cos(b*x+a)^2*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)+1
/16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.13

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx =$$
$$-\frac{6\left(6\,bd^3\cos(bx+a)\cos(dx+c)-(4\,b^3d-bd^3)\cos(bx+a)\cos(dx+c)^3\right)\sin(bx+a)-\left(18\,d^4\cos(bx\right.}{3\,(16\,b^4d-40\,b}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a
)*cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 + 16*b^4 - 40*b^2*
d^2 + (8*b^4 - 2*b^2*d^2 - 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos(d*x + c
)^2)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

3.244.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. $2(116) = 232$.

Time = 5.66 (sec) , antiderivative size = 2006, normalized size of antiderivative = 13.93

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*cos(d*x+c)**3,x)
```

output
```
Piecewise((x*cos(a)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x
/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - x*sin(a - 3*d*x/2)**2*cos(c + d*x
)**3/16 + x*sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a - 3*d*x/2)/8 - 3*x*sin(
a - 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c + d*x)**2/8 - 3*x*sin(c +
 d*x)**2*cos(a - 3*d*x/2)**2*cos(c + d*x)/16 + x*cos(a - 3*d*x/2)**2*cos(c
 + d*x)**3/16 + 11*sin(a - 3*d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a - 3*
d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*sin(a - 3*d*x/2)*sin(c + d*x)
**2*cos(a - 3*d*x/2)*cos(c + d*x)/(4*d) - 5*sin(a - 3*d*x/2)*cos(a - 3*d*x
/2)*cos(c + d*x)**3/(8*d) + 7*sin(c + d*x)**3*cos(a - 3*d*x/2)**2/(16*d),
Eq(b, -3*d/2)), (-3*x*sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 -
3*x*sin(a - d*x/2)**2*cos(c + d*x)**3/16 - 3*x*sin(a - d*x/2)*sin(c + d*x)
**3*cos(a - d*x/2)/8 - 3*x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(
c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a - d*x/2)**2*cos(c + d*x)/16 + 3*
x*cos(a - d*x/2)**2*cos(c + d*x)**3/16 + sin(a - d*x/2)**2*sin(c + d*x)**3
/(48*d) + sin(a - d*x/2)*sin(c + d*x)**2*cos(a - d*x/2)*cos(c + d*x)/(4*d)
 + 3*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)**3/(8*d) + 31*sin(c + d*x)
**3*cos(a - d*x/2)**2/(48*d) + sin(c + d*x)*cos(a - d*x/2)**2*cos(c + d*x)
**2/d, Eq(b, -d/2)), (-3*x*sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/
16 - 3*x*sin(a + d*x/2)**2*cos(c + d*x)**3/16 + 3*x*sin(a + d*x/2)*sin(c +
 d*x)**3*cos(a + d*x/2)/8 + 3*x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x...
```

3.244.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. 2(132) = 264.

Time = 0.31 (sec) , antiderivative size = 1362, normalized size of antiderivative = 9.46

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^
4*sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*
d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a)
 + 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*si
n(3*c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin
(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) -
 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(
3*c))*cos(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(
3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) -
 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(
3*c))*cos(-(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*s
in(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2
*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) + 2*(
16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) +
 18*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c
) - 18*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x -
2*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^
4*cos(3*c))*sin((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*
d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a)
 - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4...
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(2bx+3dx+2a+3c)}{16(2b+3d)} + \frac{3\sin(2bx+dx+2a+c)}{16(2b+d)} + \frac{3\sin(2bx-dx+2a-c)}{16(2b-d)} + \frac{\sin(2bx-3dx+2a-3c)}{16(2b-3d)} + \frac{\sin(3dx+3c)}{24d} + \frac{3\sin(dx+c)}{8d}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*sin(2*b*x + d*x + 2
*a + c)/(2*b + d) + 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*sin(2
*b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin(
d*x + c)/d
```

3.244.9 Mupad [B] (verification not implemented)

Time = 24.01 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.44

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = -e^{a\,2i-c\,1i+b\,x\,2i-d\,x\,1i}\left(\frac{e^{-a\,2i-b\,x\,2i}\,(24b^2-6d^2)}{b^2\,d\,128i-d^3\,32i} - \frac{3d\,(2b+d)}{b^2\,d\,128i-d^3\,32i} + \frac{3d\,e^{-a\,4i-b\,x\,4i}\,(2b-d)}{b^2\,d\,128i-d^3\,32i}\right) + e^{a\,2i+c\,1i+b\,x\,2i+d\,x\,1i}\left(\frac{3d\,(2b-d)}{b^2\,d\,128i-d^3\,32i} + \frac{e^{-a\,2i-b\,x\,2i}\,(24b^2-6d^2)}{b^2\,d\,128i-d^3\,32i} - \frac{3d\,e^{-a\,4i-b\,x\,4i}\,(2b+d)}{b^2\,d\,128i-d^3\,32i}\right) - e^{a\,2i-c\,3i+b\,x\,2i-d\,x\,3i}\left(-\frac{3d\,(2b+3d)}{b^2\,d\,384i-d^3\,864i} + \frac{e^{-a\,2i-b\,x\,2i}\,(8b^2-18d^2)}{b^2\,d\,384i-d^3\,864i} + \frac{3d\,e^{-a\,4i-b\,x\,4i}\,(2b-3d)}{b^2\,d\,384i-d^3\,864i}\right) + e^{a\,2i+c\,3i+b\,x\,2i+d\,x\,3i}\left(\frac{3d\,(2b-3d)}{b^2\,d\,384i-d^3\,864i} + \frac{e^{-a\,2i-b\,x\,2i}\,(8b^2-18d^2)}{b^2\,d\,384i-d^3\,864i} - \frac{3d\,e^{-a\,4i-b\,x\,4i}\,(2b+3d)}{b^2\,d\,384i-d^3\,864i}\right)$$

input
```
int(cos(a + b*x)^2*cos(c + d*x)^3,x)
```

output
```
exp(a*2i + c*1i + b*x*2i + d*x*1i)*((3*d*(2*b - d))/(b^2*d*128i - d^3*32i)
 + (exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d^3*32i) - (3*d*e
xp(- a*4i - b*x*4i)*(2*b + d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*1i +
 b*x*2i - d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d
^3*32i) - (3*d*(2*b + d))/(b^2*d*128i - d^3*32i) + (3*d*exp(- a*4i - b*x*4
i)*(2*b - d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*3i + b*x*2i - d*x*3i)
*((exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*(
2*b + 3*d))/(b^2*d*384i - d^3*864i) + (3*d*exp(- a*4i - b*x*4i)*(2*b - 3*d
))/(b^2*d*384i - d^3*864i)) + exp(a*2i + c*3i + b*x*2i + d*x*3i)*((3*d*(2*
b - 3*d))/(b^2*d*384i - d^3*864i) + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2)
)/(b^2*d*384i - d^3*864i) - (3*d*exp(- a*4i - b*x*4i)*(2*b + 3*d))/(b^2*d*
384i - d^3*864i))
```

3.244. $\int \cos^2(a+bx)\cos^3(c+dx)\,dx$

3.245 $\int \cos^2(a+bx)\cos^2(c+dx)\,dx$

3.245.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \frac{x}{4}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)} + \frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
1/4*x+1/8*sin(2*b*x+2*a)/b+1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sin(2*d*x
+2*c)/d+1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

3.245.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \frac{2d(b^2-d^2)\sin(2(a+bx))+bd(b+d)\sin(2(a-c+(b-d)x))+b(b-d)(2(b+d)\sin(2(c+dx))+d(4(b}{16b(b-d)d(b+d)}$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

output
```
(2*d*(b^2 - d^2)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*x)]
 + b*(b - d)*(2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a + c +
 (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

3.245.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{8}\cos(2(a-c)+2x(b-d))+\frac{1}{8}\cos(2(a+c)+2x(b+d))+\frac{1}{4}\cos(2a+2bx)+\frac{1}{4}\cos(2c+2dx)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

input
```
Int[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

output
```
x/4 + Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
 Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

3.245.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.245.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}+\frac{\sin(2xb+2a)}{8b}+\frac{\sin(2dx+2c)}{8d}+\frac{\sin((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sin((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sin((2b-2d)x+2a-2c)+4\left(\frac{bd\sin((2b+2d)x+2a+2c)}{4}+(b+d)\left(\frac{d\sin(2xb+2a)}{2}+b\left(dx+\frac{\sin(2dx+2c)}{2}\right)\right)\right)(b-d)}{16b^3d-16b\,d^3}$
risch	$\frac{x}{4}+\frac{\sin(2xb+2a)}{8b}+\frac{\sin(2dx+2c)b^2}{8(b-d)d(b+d)}-\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}+\frac{\sin(2xb-2dx+2a-2c)b}{16(b-d)(b+d)}+\frac{d\sin(2xb-2dx+2a-2c)}{16(b-d)(b+d)}+\frac{\sin(2xb-}{16(}$

input
```
int(cos(b*x+a)^2*cos(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d+1/8/(2*b-2*d)*sin((2*b-2*d
)*x+2*a-2*c)+1/8/(2*b+2*d)*sin((2*b+2*d)*x+2*a+2*c)
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = -\frac{\left(2\,bd^2\cos\left(bx+a\right)^2-b^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx+c\right)^2-c\right.}{4\left(b^3d-bd^3\right)}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*((2*b*d^2*cos(b*x + a)^2 - b^3)*cos(d*x + c)*sin(d*x + c) - (b^3*d -
b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a))*sin(b*
x + a))/(b^3*d - b*d^3)
```

3.245.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.

Time = 1.60 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*cos(d*x+c)**2,x)
```

output
```
Piecewise((x*cos(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a)**2, Eq
(b, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c
+ d*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*
sin(c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8
+ 3*sin(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) - sin(a - d*x)*cos(a -
 d*x)*cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8
*d), Eq(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**
2*cos(c + d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x
)/2 + x*sin(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*
x)**2/8 + 3*sin(a + d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) + sin(a + d*x)
*cos(a + d*x)*cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c +
 d*x)/(8*d), Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(
a + b*x)*cos(a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)*
*2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2
/(4*b**3*d - 4*b*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*
d - 4*b*d**3) + b**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d -
 4*b*d**3) + b**3*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*
b*d**3) + 2*b**2*d*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d ...
```

3.245.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 620, normalized size of antiderivative = 7.05

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx$$

$$= \frac{8\left(\left(b\cos(2c)^2 + b\sin(2c)^2\right)d^3 - \left(b^3\cos(2c)^2 + b^3\sin(2c)^2\right)d\right)x + \left(b^2 d\sin(2c) - bd^2\sin(2c)\right)\cos(2(b+}{}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c
)^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c)
 - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2
*c) + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*
d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*
cos(2*b*x + 2*a + 2*c) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a
 - 2*c) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) -
 b*d^2*sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(
2*(b + d)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)
*x + 2*a) + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c
) + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) - 2*(b^2*d*c
os(2*c) - d^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) - 2*(b^2*d*cos(2*c) - d^3*c
os(2*c))*sin(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*
d*x) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2
+ b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \frac{1}{4}x + \frac{\sin(2bx+2dx+2a+2c)}{16(b+d)} + \frac{\sin(2bx-2dx+2a-2c)}{16(b-d)} + \frac{\sin(2bx+2a)}{8b} + \frac{\sin(2dx+2c)}{8d}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d
*x + 2*a - 2*c)/(b - d) + 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

3.245.9 Mupad [B] (verification not implemented)

Time = 21.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.01

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx$$
$$= \frac{2\,b^3\,\sin\left(2\,c+2\,d\,x\right)-2\,d^3\,\sin\left(2\,a+2\,b\,x\right)+b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\,x\right.}{}$$

input
```
int(cos(a + b*x)^2*cos(c + d*x)^2,x)
```

output
```
(2*b^3*sin(2*c + 2*d*x) - 2*d^3*sin(2*a + 2*b*x) + b*d^2*sin(2*a - 2*c + 2
*b*x - 2*d*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c
 + 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) + 2*b^2*d*sin(2*a
 + 2*b*x) - 2*b*d^2*sin(2*c + 2*d*x) - 4*b*d^3*x + 4*b^3*d*x)/(16*b*d*(b^2
 - d^2))
```

3.246 $\int \cos^3(a+bx)\cos^3(c+dx)\,dx$

3.246.1 Optimal result

Integrand size = 17, antiderivative size = 195

$$\begin{aligned}\int \cos^3(a+bx)\cos^3(c+dx)\,dx &= \frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}\\ &+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\sin(3a-c+(3b-d)x)}{32(3b-d)}\\ &+\frac{9\sin(a+c+(b+d)x)}{32(b+d)}+\frac{\sin(3(a+c)+3(b+d)x)}{96(b+d)}\\ &+\frac{3\sin(3a+c+(3b+d)x)}{32(3b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}\end{aligned}$$

output
```
3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+1/96*sin(3*a
-3*c+3*(b-d)*x)/(b-d)+3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sin(a+c+(b+d)
*x)/(b+d)+1/96*sin(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b
+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.246.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \frac{1}{96}\left(\frac{9\sin(a-3c+bx-3dx)}{b-3d}+\frac{27\sin(a-c+bx-dx)}{b-d}+\frac{\sin(3(a-c+bx-dx))}{b-d}+\frac{9\sin(3a-c+3bx-dx)}{3b-d}+\frac{9\sin(3a+c+3bx+dx)}{3b+d}+\frac{9\sin(a+3c+bx+3dx)}{b+3d}+\frac{27\sin(a+c+(b+d)x)}{b+d}+\frac{\sin(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

output
```
((9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(b
 - d) + Sin[3*(a - c + b*x - d*x)]/(b - d) + (9*Sin[3*a - c + 3*b*x - d*x]
)/(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c +
b*x + 3*d*x])/(b + 3*d) + (27*Sin[a + c + (b + d)*x])/(b + d) + Sin[3*(a +
 c + (b + d)*x)]/(b + d))/96
```

3.246.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{3}{32}\cos(a+x(b-3d)-3c)+\frac{9}{32}\cos(a+x(b-d)-c)+\frac{1}{32}\cos(3(a-c)+3x(b-d))+\frac{3}{32}\cos(3a+x(3b-$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sin(a+x(b+d)+c)}{32(b+d)}+\frac{\sin(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\sin(3a+x(3b+d)+c)}{32(3b+d)}+\frac{3\sin(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

output
```
(3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])
/(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sin[3*a - c
 + (3*b - d)*x])/(32*(3*b - d)) + (9*Sin[a + c + (b + d)*x])/(32*(b + d))
+ Sin[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x
])/(32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

3.246.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.246.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{9\sin(a+c+(b+d)x)}{32(b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\sin((3b-3d)x+3a-3c)}{96b-96d}$
parallelrisch	$\frac{\frac{9(b-3d)(b+d)(b+3d)\left(b+\frac{d}{3}\right)(b-d)\sin(3a-c+(3b-d)x)}{32}+9\left(\frac{(b-3d)(b+d)(b+3d)\left(b+\frac{d}{3}\right)\sin((3b-3d)x+3a-3c)}{3}+\frac{(b-3d)(b+3d)\left(b+\frac{d}{3}\right)(b-d)\,\mathrm{s}}{3}\right.}{}$
risch	Expression too large to display

input
```
int(cos(b*x+a)^3*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+9/32*sin(a+c
+(b+d)*x)/(b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)+1/32/(3*b-3*d)*sin((3*b-
3*d)*x+3*a-3*c)+3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sin(3*a+c+(3*b+d)*x
)/(3*b+d)+1/32/(3*b+3*d)*sin((3*b+3*d)*x+3*a+3*c)
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.23

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx$$
$$= \frac{\left(\left(18\,b^5-2\,b^3d^2+\left(9\,b^5-82\,b^3d^2+9\,bd^4\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-6\left(20\,b^3d^2+\left(b^3d^2-9\,bd^4\right)\cos\left(bx\right.\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/3*(((18*b^5 - 2*b^3*d^2 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^2)
*cos(d*x + c)^3 - 6*(20*b^3*d^2 + (b^3*d^2 - 9*b*d^4)*cos(b*x + a)^2)*cos(
d*x + c))*sin(b*x + a) + (120*b^2*d^3*cos(b*x + a) + 2*(b^2*d^3 - 9*d^5)*c
os(b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos(b*x + a)^3 + 6*(9*b^4*
d - b^2*d^3)*cos(b*x + a))*cos(d*x + c)^2)*sin(d*x + c))/(9*b^6 - 91*b^4*d
^2 + 91*b^2*d^4 - 9*d^6)
```

3.246.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3584 vs. 2(172) = 344.

Time = 17.69 (sec) , antiderivative size = 3584, normalized size of antiderivative = 18.38

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**3*cos(d*x+c)**3,x)
```

output
```
Piecewise((x*cos(a)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x
)**3*sin(c + d*x)**3/32 - 9*x*sin(a - 3*d*x)**3*sin(c + d*x)*cos(c + d*x)*
*2/32 - 9*x*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/
32 + 3*x*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)**3/32 + 3*x*sin(a -
 3*d*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/32 - 9*x*sin(a - 3*d*x)*sin(c +
d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**2/32 - 9*x*sin(c + d*x)**2*cos(a - 3*
d*x)**3*cos(c + d*x)/32 + 3*x*cos(a - 3*d*x)**3*cos(c + d*x)**3/32 - 3*sin
(a - 3*d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(320*d) - sin(a - 3*d*x)**3*co
s(c + d*x)**3/(4*d) - 11*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*x)/
(320*d) - 3*sin(a - 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/
(20*d) - 117*sin(a - 3*d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**3/(320*d) - si
n(c + d*x)**3*cos(a - 3*d*x)**3/(30*d) - 61*sin(c + d*x)*cos(a - 3*d*x)**3
*cos(c + d*x)**2/(320*d), Eq(b, -3*d)), (-5*x*sin(a - d*x)**3*sin(c + d*x)
**3/16 - 3*x*sin(a - d*x)**3*sin(c + d*x)*cos(c + d*x)**2/16 + 9*x*sin(a -
 d*x)**2*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/16 + 3*x*sin(a - d*x)**
2*cos(a - d*x)*cos(c + d*x)**3/16 - 3*x*sin(a - d*x)*sin(c + d*x)**3*cos(a
 - d*x)**2/16 - 9*x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)
**2/16 + 3*x*sin(c + d*x)**2*cos(a - d*x)**3*cos(c + d*x)/16 + 5*x*cos(a -
 d*x)**3*cos(c + d*x)**3/16 + 3*sin(a - d*x)**3*sin(c + d*x)**2*cos(c + d*
x)/(16*d) + 5*sin(a - d*x)**3*cos(c + d*x)**3/(16*d) + sin(a - d*x)**2*...
```

3.246.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2614 vs. $2(179) = 358$.

Time = 0.42 (sec) , antiderivative size = 2614, normalized size of antiderivative = 13.41

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a +
 4*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a -
 2*c) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a
+ 4*c) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2
*d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a
 - 2*c) + 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30
*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a +
 6*c) - 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b
^2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) +
(9*b^5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(
3*c) + 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) - (
9*b^5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3
*c) + 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) + 27*(9*b^
5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c)
+ 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*
sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) +
9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5...
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(3\,bx+3\,dx+3\,a+3\,c)}{96\,(b+d)} + \frac{3\,\sin(3\,bx+dx+3\,a+c)}{32\,(3\,b+d)} + \frac{3\,\sin(3\,bx-dx+3\,a-c)}{32\,(3\,b-d)} + \frac{\sin(3\,bx-3\,dx+3\,a-3\,c)}{96\,(b-d)} + \frac{3\,\sin(bx+3\,dx+a+3\,c)}{32\,(b+3\,d)} + \frac{9\,\sin(bx+dx+a+c)}{32\,(b+d)} + \frac{9\,\sin(bx-dx+a-c)}{32\,(b-d)} + \frac{3\,\sin(bx-3\,dx+a-3\,c)}{32\,(b-3\,d)}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="giac")
```

output
```
1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a +
 c)/(3*b + d) + 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*x
 - 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d)
+ 9/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b - d
) + 3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

3.246. $\int \cos^3(a+bx)\cos^3(c+dx)\,dx$

3.246.9 Mupad [B] (verification not implemented)

Time = 25.16 (sec) , antiderivative size = 999, normalized size of antiderivative = 5.12

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = -\mathrm{e}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}} - \frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-81\,b^3-81\,b^2\,d+9\,b\,d^2+9\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-81\,b^3+81\,b^2\,d+9\,b\,d^2-9\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}\right)$$
$$- \mathrm{e}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}} - \frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-81\,b^3+81\,b^2\,d+9\,b\,d^2-9\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-81\,b^3-81\,b^2\,d+9\,b\,d^2+9\,d^3)}{b^4\,576\mathrm{i}-b^2\,d^2\,640\mathrm{i}+d^4\,64\mathrm{i}}\right)$$
$$- \mathrm{e}^{a\,3\mathrm{i}-c\,3\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{-b^3-b^2\,d+9\,b\,d^2+9\,d^3}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}} - \frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-b^3+b^2\,d+9\,b\,d^2-9\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-9\,b^3-27\,b^2\,d+9\,b\,d^2+27\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-9\,b^3+27\,b^2\,d+9\,b\,d^2-27\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}\right)$$
$$- \mathrm{e}^{a\,3\mathrm{i}+c\,3\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,3\mathrm{i}}\left(\frac{-b^3+b^2\,d+9\,b\,d^2-9\,d^3}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}} - \frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-b^3-b^2\,d+9\,b\,d^2+9\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-9\,b^3+27\,b^2\,d+9\,b\,d^2-27\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-9\,b^3-27\,b^2\,d+9\,b\,d^2+27\,d^3)}{b^4\,192\mathrm{i}-b^2\,d^2\,1920\mathrm{i}+d^4\,1728\mathrm{i}}\right)$$

input
```
int(cos(a + b*x)^3*cos(c + d*x)^3,x)
```

output
```
- exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/
(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b
^2*d - 9*b^3 - 3*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*2i -
 b*x*2i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*
d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(
b^4*576i + d^4*64i - b^2*d^2*640i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*(
(9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(b^4*576i + d^4*64i - b^2*d^2*640i) -
(exp(- a*6i - b*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(b^4*576i + d^4
*64i - b^2*d^2*640i) + (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3
- 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b
*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i)) -
exp(a*3i - c*3i + b*x*3i - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(b^4*1
92i + d^4*1728i - b^2*d^2*1920i) - (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d
- b^3 - 9*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) + (exp(- a*2i - b*x
*2i)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^
2*1920i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b
^4*192i + d^4*1728i - b^2*d^2*1920i)) - exp(a*3i + c*3i + b*x*3i + d*x*3i)
*((9*b*d^2 + b^2*d - b^3 - 9*d^3)/(b^4*192i + d^4*1728i - b^2*d^2*1920i) -
 (exp(- a*6i - b*x*6i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(b^4*192i + d^4*17
28i - b^2*d^2*1920i) + (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^2*d - 9*b^...
```

3.246. $\int \cos^3(a+bx)\cos^3(c+dx)\,dx$

3.247 $\int \cos(a+bx)\tan^3(c+bx)\,dx$

3.247.1 Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \frac{\cos(a+bx)}{b} + \frac{\cos(a-c)\sec(c+bx)}{b} + \frac{3\text{arctanh}(\sin(c+bx))\sin(a-c)}{2b} - \frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}$$

output
```
cos(b*x+a)/b+cos(a-c)*sec(b*x+c)/b+3/2*arctanh(sin(b*x+c))*sin(a-c)/b-1/2*
sec(b*x+c)*sin(a-c)*tan(b*x+c)/b
```

3.247.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \cos(a+bx)\tan^3(c+bx)\,dx$$
$$= \frac{(2\cos(a-2c-bx)+5\cos(a+bx)+\cos(a+2c+3bx))\sec^2(c+bx)+12\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right.\right.}{4b}$$

input
```
Integrate[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

output
```
((2*Cos[a - 2*c - b*x] + 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Sec[c + b*
x]^2 + 12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c])/(4*b)
```

3.247.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.867, Rules used = {5090, 3042, 3091, 3042, 4257, 5087, 3042, 3086, 24, 5090, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\tan^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 5090}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)^2dx$$

$$\Big\downarrow \text{ 3091}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{1}{2}\int \sec(c+bx)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{ 4257}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{ 5087}$$

$$-\int \cos(a+bx)\tan(c+bx)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \cos(a+bx)\tan(c+bx)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{ 3086}$$

$$-\int \cos(a+bx)\tan(c+bx)dx+\frac{\cos(a-c)\int 1d\sec(c+bx)}{b}-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \; 24$$

$$-\int \cos(a+bx)\tan(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \; 5090$$

$$\sin(a-c)\int \sec(c+bx)dx-\int \sin(a+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \; 3042$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\int \sin(a+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \; 3118$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \; 4257$$

$$\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b}-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

output
```
Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[
a - c])/b - Sin[a - c]*(-1/2*ArcTanh[Sin[c + b*x]]/b + (Sec[c + b*x]*Tan[c
 + b*x])/(2*b))
```

3.247. $\int \cos(a+bx)\tan^3(c+bx)\,dx$

3.247.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.247.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.51

method	result
risch	$\frac{e^{i(xb+a)}}{2b}+\frac{e^{-i(xb+a)}}{2b}+\frac{3\,e^{i(3xb+5a+2c)}+e^{i(3xb+3a+4c)}+e^{i(xb+5a)}+3\,e^{i(xb+3a+2c)}}{2b\left(e^{2i(xb+a+c)}+e^{2ia}\right)^{2}}-\frac{3\ln\left(e^{i(xb+a)}-ie^{i(a-c)}\right)\sin(a-c)}{2b}+\frac{3\ln}{}$

input
```
int(cos(b*x+a)*tan(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/2/b/(exp(2*I*(b*x+a+c))+exp(2
*I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a))
+3*exp(I*(b*x+3*a+2*c)))-3/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)+
3/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c)
```

3.247.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(68) = 136.

Time = 0.28 (sec) , antiderivative size = 366, normalized size of antiderivative = 5.08

$$\int \cos(a+bx)\tan^3(c+bx)\,dx$$

$$=\frac{16\,\cos\left(bx+a\right)^{3}\cos\left(-2\,a+2\,c\right)-4\left(4\,\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-4\left(\cos\left(-2\,a+2\,c\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x
+ a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) - 5)*cos(b*x + a) + 3*sqrt(2)*(2
*(cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*cos(b*x + a)^2*cos
(-2*a + 2*c) - cos(-2*a + 2*c) + 1)*sin(-2*a + 2*c))*log(-(2*cos(b*x + a)^
2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2
)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt
(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a +
2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))
/sqrt(cos(-2*a + 2*c) + 1))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(
b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

3.247.6 Sympy [F]

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \int \cos{(a+bx)}\tan^3{(bx+c)}\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)**3,x)
```

output
```
Integral(cos(a + b*x)*tan(b*x + c)**3, x)
```

3.247.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(68) = 136$.

Time $= 0.39$ (sec) , antiderivative size $= 1027$, normalized size of antiderivative $= 14.26$

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(
6*b*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) + 2*cos(4*b*x + 4*c) + 2*
cos(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) + 1)*cos(5*b*x + a + 4*c) + 10*(2*co
s(3*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*cos(3*b*x
 + a + 2*c) + cos(b*x + a))*cos(4*b*x + 4*c) + 4*(2*cos(2*b*x + 2*a) + 5*c
os(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x + a
) + 10*cos(2*b*x + 2*c)*cos(b*x + a) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a +
c) + 4*cos(3*b*x + a + 2*c)^2*sin(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x
 + a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*si
n(-a + c) + 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) + 4*sin(3*b*x + a + 2*c)*
sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) + 2*(2*cos(3*b*x + a
 + 2*c)*sin(-a + c) + cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) + 2*(
2*sin(3*b*x + a + 2*c)*sin(-a + c) + sin(b*x + a)*sin(-a + c))*sin(5*b*x +
 a + 4*c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + si
n(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + c
os(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*si
n(c) + sin(c)^2)) + 2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin
(b*x + a))*sin(6*b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*
b*x + 4*c) + 2*sin(2*b*x + 2*a) + 5*sin(2*b*x + 2*c))*sin(5*b*x + a + 4*c)
 + 10*(2*sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + ...
```

3.247.8 Giac [F]

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \int \cos(bx+a)\tan(bx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*tan(b*x + c)^3, x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)*tan(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.248 $\int \cos(a+bx)\tan^2(c+bx)\,dx$

3.248.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \cos(a+bx)\tan^2(c+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}-\frac{\sin(a+bx)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b-sin(b*x+a)/b
```

3.248.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = -\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} - \frac{\cos(bx)\sin(a)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}-\frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos
[b*x]*Sin[a])/b - (Sec[c + b*x]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

3.248.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5090, 3042, 3086, 24, 5087, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\tan^2(bx+c)\,dx$$

$$\Big\downarrow \text{5090}$$

$$\int \sin(a+bx)\tan(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\tan(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sin(a+bx)\tan(c+bx)dx-\frac{\sin(a-c)\int 1d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\int \sin(a+bx)\tan(c+bx)dx-\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{5087}$$

$$\cos(a-c)\int \sec(c+bx)dx-\int \cos(a+bx)dx-\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\int \sin\left(a+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

$$\downarrow \text{4257}$$

$$\frac{\cos(a-c)\text{arctanh}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
(ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b - Sin[a
 + b*x]/b
```

3.248.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.248.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.24

method	result
risch	$\frac{i e^{i(xb+a)}}{2b}-\frac{i e^{-i(xb+a)}}{2b}-\frac{i\left(-e^{i(xb+3a)}+e^{i(xb+a+2c)}\right)}{b\left(e^{2i(xb+a+c)}+e^{2ia}\right)}-\frac{\ln\left(e^{i(xb+a)}-i e^{i(a-c)}\right)\cos(a-c)}{b}+\frac{\ln\left(e^{i(xb+a)}+i e^{i(a-c)}\right)\cos(a-c)}{b}$

input
```
int(cos(b*x+a)*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))-I/b/(exp(2*I*(b*x+a+c))+exp
(2*I*a))*(-exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-ln(exp(I*(b*x+a))-I*exp(I*
(a-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

3.248.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.87

$$\int \cos(a+bx)\tan^2(c+bx)\,dx$$

$$= \frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\sin(bx+a)\sin(-2\,a+2\,c)-\left(\cos(-2\,a+2\,c)^2+2\,\cos(\right.\right.}{}}{4\left(b\sin\right.}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*
a + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2*c)^2 + 2*cos(-2
*a + 2*c) + 1)*cos(b*x + a))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos
(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*
sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - c
os(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin
(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1
) + 4*(cos(b*x + a)^2 - 2)*sin(-2*a + 2*c))/(b*sin(b*x + a)*sin(-2*a + 2*c
) - (b*cos(-2*a + 2*c) + b)*cos(b*x + a))
```

3.248.6 Sympy [F]

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = \int \cos(a+bx)\tan^2(bx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*tan(b*x + c)**2, x)
```

3.248.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(46) = 92$.

Time = 0.43 (sec) , antiderivative size = 526, normalized size of antiderivative = 11.43

$$\int \cos(a+bx)\tan^2(c+bx)\,dx$$
$$= \frac{(\sin(3\,bx+a+2\,c)+\sin(bx+a))\cos(4\,bx+2\,a+2\,c)-3\,(\sin(2\,bx+2\,a)-\sin(2\,bx+2\,c))\cos(3\,bx+}{}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 3*(sin
(2*b*x + 2*a) - sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a +
2*c)^2*cos(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos
(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 2*cos(-a +
c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log((co
s(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2
*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)
*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2))
- (cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + (3*cos(2*
b*x + 2*a) - 3*cos(2*b*x + 2*c) - 1)*sin(3*b*x + a + 2*c) - 3*cos(b*x + a)
*sin(2*b*x + 2*a) + 3*cos(b*x + a)*sin(2*b*x + 2*c) + 3*cos(2*b*x + 2*a)*s
in(b*x + a) - 3*cos(2*b*x + 2*c)*sin(b*x + a) - sin(b*x + a))/(b*cos(3*b*x
 + a + 2*c)^2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 +
 b*sin(3*b*x + a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(
b*x + a)^2)
```

3.248.8 Giac [F]

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = \int \cos(bx+a)\tan(bx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*tan(b*x + c)^2, x)
```

3.248.9 Mupad [B] (verification not implemented)

Time = 26.60 (sec) , antiderivative size = 285, normalized size of antiderivative = 6.20

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = -\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)}{b\,\left(e^{a\,2i-c\,2i}\,1i+e^{a\,2i+b\,x\,2i}\,1i\right)}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

input
```
int(cos(a + b*x)*tan(c + b*x)^2,x)
```

output
```
(exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1
i + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b*(exp(a*2i - c*2i)*1i + exp(a*2i + b
*x*2i)*1i)) + (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (e
xp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^
(1/2))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*
2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2)
 - exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1
))/(2*b*exp(a*2i - c*2i)^(1/2))
```

3.249 $\int \cos(a+bx)\tan(c+bx)\,dx$

3.249.1 Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \cos(a+bx)\tan(c+bx)\,dx = -\frac{\cos(a+bx)}{b} - \frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{b}$$

output
```
-cos(b*x+a)/b-arctanh(sin(b*x+c))*sin(a-c)/b
```

3.249.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.10

$$\int \cos(a+bx)\tan(c+bx)\,dx = -\frac{\cos(a)\cos(bx)}{b} + \frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b} + \frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Tan[c + b*x],x]
```

output
```
-((Cos[a]*Cos[b*x])/b) + ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*
Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c
])]*Sin[a - c])/b + (Sin[a]*Sin[b*x])/b
```

3.249.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5090, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \text{ 5090}$$

$$\int \sin(a+bx)dx-\sin(a-c)\int \sec(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)dx-\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3118}$$

$$-\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b}-\frac{\cos(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Tan[c + b*x],x]
```

output
```
-(Cos[a + b*x]/b) - (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

3.249.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.249.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.23

method	result	size
risch	$-\frac{e^{i(xb+a)}}{2b}-\frac{e^{-i(xb+a)}}{2b}+\frac{\ln\left(e^{i(xb+a)}-i e^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(xb+a)}+i e^{i(a-c)}\right)\sin(a-c)}{b}$	97

input
```
int(cos(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(I*(b*x+a))/b-1/2/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))-I*exp(I*(a-c
)))/b*sin(a-c)-ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c)
```

3.249.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 6.53

$$\int \cos(a+bx)\tan(c+bx)\,dx$$
$$= \frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)-3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)+1}\right)\mathrm{s}}{\sqrt{\cos(-2a+2c)+1}}}{4b}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*
x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) + c
os(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) -
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2
*a + 2*c) - cos(-2*a + 2*c) + 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1
) - 4*cos(b*x + a))/b
```

3.249.6 Sympy [F]

$$\int \cos(a+bx)\tan(c+bx)\,dx = \int \cos\left(a+bx\right)\tan\left(bx+c\right)dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c),x)
```

output
```
Integral(cos(a + b*x)*tan(b*x + c), x)
```

3.249.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(30) = 60$.

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.37

$$\int \cos(a+bx)\tan(c+bx)\,dx = -\frac{\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)\sin\left(-a+c\right)+2\cos\left(bx+a\right)}{2b}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

output
```
-1/2*(log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x
 + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)
^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c)
+ sin(c)^2))*sin(-a + c) + 2*cos(b*x + a))/b
```

3.249.8 Giac [F]

$$\int \cos(a+bx)\tan(c+bx)\,dx = \int \cos(bx+a)\tan(bx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*tan(b*x + c), x)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 25.71 (sec) , antiderivative size = 237, normalized size of antiderivative = 7.90

$$\begin{aligned}
&\int \cos(a+bx)\tan(c+bx)\,dx \\
&= -\frac{e^{-a\,1i-b\,x\,1i}}{2\,b} - \frac{e^{a\,1i+b\,x\,1i}}{2\,b} \\
&+ \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}\,1i-i) - \frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}-1)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,(e^{a\,2i-c\,2i}-1)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} \\
&- \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}\,1i-i) + \frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}-1)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,(e^{a\,2i-c\,2i}-1)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}
\end{aligned}$$

input
```
int(cos(a + b*x)*tan(c + b*x),x)
```

output
```
(log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) - (exp(a*2i)*e
xp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(e
xp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - exp(a*1i + b*x*1i)
/(2*b) - exp(- a*1i - b*x*1i)/(2*b) - (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i
)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i
)*(exp(a*2i)*exp(-c*2i)*1i - 1i))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i
- c*2i))^(1/2))
```

3.250 $\int \cos(a+bx)\cot(c+bx)\,dx$

3.250.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \cos(a+bx)\cot(c+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{b}+\frac{\cos(a+bx)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)/b+cos(b*x+a)/b
```

3.250.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.24

$$\int \cos(a+bx)\cot(c+bx)\,dx = -\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}+\frac{\cos(a)\cos(bx)}{b}-\frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Cot[c + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos
[a]*Cos[b*x])/b - (Sin[a]*Sin[b*x])/b
```

3.250.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5088, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \text{5088}$$

$$\cos(a-c)\int \csc(c+bx)dx - \int \sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)dx - \int \sin(a+bx)dx$$

$$\Big\downarrow \text{3118}$$

$$\cos(a-c)\int \csc(c+bx)dx + \frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(a+bx)}{b} - \frac{\cos(a-c)\text{arctanh}(\cos(bx+c))}{b}$$

input
```
Int[Cos[a + b*x]*Cot[c + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b
```

3.250.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

3.250.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

method	result	size
risch	$\frac{e^{i(xb+a)}}{2b}+\frac{e^{-i(xb+a)}}{2b}+\frac{\ln\left(e^{i(xb+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(e^{i(xb+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}$	93

input
```
int(cos(b*x+a)*cot(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))-exp(I*(a-c)))
/b*cos(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

3.250.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(29) = 58.

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 6.55

$$\int \cos(a+bx)\cot(c+bx)\,dx = \frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx+a}{\sqrt{\cos(-2a+}}}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\mathrm{co}}\right.}{4b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2
*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a +
 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*
c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b
*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)) + 4*cos(b*x +
 a))/b
```

3.250.6 Sympy [F]

$$\int \cos(a+bx)\cot(c+bx)\,dx = \int \cos{(a+bx)}\cot{(bx+c)}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+c),x)
```

output
```
Integral(cos(a + b*x)*cot(b*x + c), x)
```

3.250.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.62

$$\int \cos(a+bx)\cot(c+bx)\,dx = -\frac{\cos{(-a+c)}\log{\left(\cos{(bx)}^2+2\cos{(bx)}\cos{(c)}+\cos{(c)}^2+\sin{(bx)}^2-2\sin{(bx)}\sin{(c)}+\sin{(c)}^2\right)}-\mathrm{co}}{}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

output
```
-1/2*(cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)
^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - cos(-a + c)*log(cos(b*x)^2 - 2*cos(b*
x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*cos(
b*x + a))/b
```

3.250.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(29) = 58$.

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 8.07

$$\int \cos(a+bx)\cot(c+bx)\,dx =$$

$$-\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}c\right)}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}\right.\right.}{}}{b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(
1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1)
)/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + ta
n(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^
2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(2*tan(1/2*b*x)*tan(
1/2*a) + tan(1/2*a)^2 - 1)/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)))/b
```

3.250.9 Mupad [B] (verification not implemented)

Time = 26.57 (sec) , antiderivative size = 231, normalized size of antiderivative = 7.97

$$\begin{aligned}
&\int \cos(a+bx)\cot(c+bx)\,dx \\
&= \frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b} \\
&\quad -\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} \\
&\quad +\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}
\end{aligned}$$

input
```
int(cos(a + b*x)*cot(c + b*x),x)
```

output
```
exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) - (log(- exp(a*1i)*e
xp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2
i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1
))/(2*b*exp(a*2i - c*2i)^(1/2)) + (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*ex
p(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(ex
p(a*2i)*exp(-c*2i)*1i + 1i))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)
^(1/2))
```

3.250. $\int \cos(a+bx)\cot(c+bx)\,dx$

3.251 $\int \cos(a+bx)\cot^2(c+bx)\,dx$

3.251.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{\operatorname{arctanh}(\cos(c+bx))\sin(a-c)}{b} - \frac{\sin(a+bx)}{b}$$

output
```
-cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b-sin(b*x+a)/b
```

3.251.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = -\frac{\cos(a-c)\csc(c+bx)}{b} - \frac{\cos(bx)\sin(a)}{b} + \frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b} - \frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) - (Cos[b*x]*Sin[a])/b + ((2*I)*ArcTan[((Cos
[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos
[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

3.251.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5088, 3042, 25, 3086, 24, 5089, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^2(bx+c)\,dx$$

$$\Big\downarrow \text{5088}$$

$$\cos(a-c)\int \cot(c+bx)\csc(c+bx)dx - \int \cot(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx - \int \cot(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+bx)\sin(a+bx)dx - \cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int 1d\csc(c+bx)}{b} - \int \cot(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{24}$$

$$-\int \cot(c+bx)\sin(a+bx)dx - \frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{5089}$$

$$-\sin(a-c)\int \csc(c+bx)dx - \int \cos(a+bx)dx - \frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\sin(a-c)\int \csc(c+bx)dx - \int \sin\left(a+bx+\frac{\pi}{2}\right)dx - \frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\frac{\cos(a-c)\csc(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b - Si
n[a + b*x]/b
```

3.251.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.251.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.15

method	result	si
risch	$\frac{i e^{i(xb+a)}}{2b}-\frac{i e^{-i(xb+a)}}{2b}+\frac{i\left(e^{i(xb+3a)}+e^{i(xb+a+2c)}\right)}{b\left(-e^{2i(xb+a+c)}+e^{2ia}\right)}-\frac{\ln\left(e^{i(xb+a)}-e^{i(a-c)}\right)\sin(a-c)}{b}+\frac{\ln\left(e^{i(xb+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}$	14

input
```
int(cos(b*x+a)*cot(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+I/b/(-exp(2*I*(b*x+a+c))+ex
p(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-ln(exp(I*(b*x+a))-exp(I*(a
-c)))/b*sin(a-c)+ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)
```

3.251.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.87

$$\int \cos(a+bx)\cot^2(c+bx)\,dx$$

$$=\frac{4\left(\cos\left(-2a+2c\right)+1\right)\cos\left(bx+a\right)^2-4\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2a+2c\right)+\frac{\sqrt{2}\left(\left(\cos(-2a+2c)+1\right)\sin(b\right.}{}}{4\left(b\right.}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)*
sin(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2
*c) - (cos(-2*a + 2*c)^2 - 1)*cos(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*
a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-
2*a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a
 + 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*
cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos
(-2*a + 2*c) + 1) - 8*cos(-2*a + 2*c) - 8)/(b*cos(b*x + a)*sin(-2*a + 2*c)
 + (b*cos(-2*a + 2*c) + b)*sin(b*x + a))
```

3.251.6 Sympy [F]

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = \int \cos(a+bx)\cot^2(bx+c)\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*cot(b*x + c)**2, x)
```

3.251.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(46) = 92$.

Time = 0.25 (sec) , antiderivative size = 613, normalized size of antiderivative = 13.33

$$\int \cos(a+bx)\cot^2(c+bx)\,dx$$
$$= \frac{(\sin(3bx+a+2c) - \sin(bx+a))\cos(4bx+2a+2c) + 3(\sin(2bx+2a) + \sin(2bx+2c))\cos(3bx+$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((sin(3*b*x + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 3*(sin
(2*b*x + 2*a) + sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a +
2*c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos
(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(3*b*x
 + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log(cos
(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) +
sin(c)^2) + (cos(3*b*x + a + 2*c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*c
os(b*x + a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c
)^2*sin(-a + c) - 2*sin(3*b*x + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*
x + a)^2*sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(3*b*x + a + 2*c) - cos(b*x +
 a))*sin(4*b*x + 2*a + 2*c) - (3*cos(2*b*x + 2*a) + 3*cos(2*b*x + 2*c) - 1
)*sin(3*b*x + a + 2*c) - 3*cos(b*x + a)*sin(2*b*x + 2*a) - 3*cos(b*x + a)*
sin(2*b*x + 2*c) + 3*cos(2*b*x + 2*a)*sin(b*x + a) + 3*cos(2*b*x + 2*c)*si
n(b*x + a) - sin(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a +
 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin
(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

3.251.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 627, normalized size of antiderivative = 13.63

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(4*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*t
an(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a
)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) -
4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/
2*c))*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan
(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^4
- 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x)^3*tan(1/2*a)
*tan(1/2*c)^3 - 6*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)^
3*tan(1/2*c)^4 + tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*b*x)^3*t
an(1/2*a)^2 - 4*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*b*x)^2*ta
n(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*b*x)^3*tan(1/2*c)^2 + 2*tan(1/2*b*x)*tan
(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*b*x)^2*tan(1/2*c)^3 + 12*tan(1/2*b*x)*t
an(1/2*a)*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/
2*c)^4 - tan(1/2*b*x)^3 + tan(1/2*b*x)*tan(1/2*a)^2 - 6*tan(1/2*b*x)^2*tan
(1/2*c) - 12*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c) + 2*tan(1/2*a)^2*tan(1/2*c
) - 2*tan(1/2*b*x)*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*c
)^3 - tan(1/2*b*x) - 2*tan(1/2*c))/((tan(1/2*b*x)^4*tan(1/2*c) + tan(1/2*b
*x)^3*tan(1/2*c)^2 - tan(1/2*b*x)^3 + tan(1/2*b*x)*tan(1/2*c)^2 - tan(1/2*
b*x) - tan(1/2*c))*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c))))/b
```

3.251.9 Mupad [B] (verification not implemented)

Time = 27.02 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.28

$$\int \cos(a+bx)\cot^2(c+bx)\,dx$$
$$= -\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)}{b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)}$$
$$-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$
$$+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(cos(a + b*x)*cot(c + b*x)^2,x)
```

output
```
(exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1
i + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i)*1i - exp(a*2i + b
*x*2i)*1i)) - (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp
(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(
1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) + (log(exp(a
*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*2i)*(exp(a
*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i)
- 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

3.251. $\int \cos(a+bx)\cot^2(c+bx)\,dx$

3.252 $\int \cos(a+bx)\cot^3(c+bx)\,dx$

3.252.1 Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \frac{3\text{arctanh}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a+bx)}{b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} + \frac{\csc(c+bx)\sin(a-c)}{b}$$

output
```
3/2*arctanh(cos(b*x+c))*cos(a-c)/b-cos(b*x+a)/b-1/2*cos(a-c)*cot(b*x+c)*cs
c(b*x+c)/b+csc(b*x+c)*sin(a-c)/b
```

3.252.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \frac{12\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c) + (2\cos(a-2c-bx) - 5\cos(a+bx) + \cos(a+2c+3bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

output
```
(12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + (2*Cos[a - 2*c - b*
x] - 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Csc[c + b*x]^2)/(4*b)
```

3.252.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {5088, 3042, 3091, 3042, 4257, 5089, 3042, 25, 3086, 24, 5088, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^3(bx+c)\,dx$$

$$\Big\downarrow \text{5088}$$

$$\cos(a-c)\int \cot^2(c+bx)\csc(c+bx)dx-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)^2dx-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3091}$$

$$\cos(a-c)\left(-\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(-\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{5089}$$

$$-\int \cos(a+bx)\cot(c+bx)dx-\sin(a-c)\int \cot(c+bx)\csc(c+bx)dx+\cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \cos(a+bx)\cot(c+bx)dx-\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx+\cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(a+bx)\cot(c+bx)dx+\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx+\cos(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \; 3086$$

$$-\int \cos(a+bx)\cot(c+bx)dx+\frac{\sin(a-c)\int 1d\csc(c+bx)}{b}+\cos(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \; 24$$

$$-\int \cos(a+bx)\cot(c+bx)dx+\cos(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)+\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \; 5088$$

$$-\cos(a-c)\int \csc(c+bx)dx+\int \sin(a+bx)dx+\cos(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)+\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\cos(a-c)\int \csc(c+bx)dx+\int \sin(a+bx)dx+\cos(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)+\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \; 3118$$

$$-\cos(a-c)\int \csc(c+bx)dx+\cos(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)+\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \; 4257$$

$$\frac{\cos(a-c)\operatorname{arctanh}(\cos(bx+c))}{b}+\cos(a-c)\left(\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)+\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{\cos(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

3.252. $\int \cos(a+bx)\cot^3(c+bx)\,dx$

output
```
(ArcTanh[Cos[c + b*x]]*Cos[a - c])/b - Cos[a + b*x]/b + Cos[a - c]*(ArcTan
h[Cos[c + b*x]]/(2*b) - (Cot[c + b*x]*Csc[c + b*x])/(2*b)) + (Csc[c + b*x]
*Sin[a - c])/b
```

3.252.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

3.252.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.45

method	result
risch	$-\frac{e^{i(xb+a)}}{2b}-\frac{e^{-i(xb+a)}}{2b}-\frac{-3\,e^{i(3xb+5a+2c)}+e^{i(3xb+3a+4c)}+e^{i(xb+5a)}-3\,e^{i(xb+3a+2c)}}{2b\left(-e^{2i(xb+a+c)}+e^{2ia}\right)^{2}}-\frac{3\ln\left(e^{i(xb+a)}-e^{i(a-c)}\right)\cos(a-c)}{2b}+$

input
```
int(cos(b*x+a)*cot(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(I*(b*x+a))/b-1/2/b*exp(-I*(b*x+a))-1/2/b/(-exp(2*I*(b*x+a+c))+exp
(2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*
a))-3*exp(I*(b*x+3*a+2*c)))-3/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)
+3/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

3.252.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(69) = 138.

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 5.27

$$\int \cos(a+bx)\cot^3(c+bx)\,dx =$$

$$-\frac{16\,\cos\left(bx+a\right)^{3}\cos\left(-2\,a+2\,c\right)-4\left(4\,\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-4\left(\cos\left(-2\,a+\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x
 + a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) + 5)*cos(b*x + a) + 3*sqrt(2)*(
2*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*(cos
(-2*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2*a + 2*c)^2 + 2*c
os(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)
*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x +
 a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a +
 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)
*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1))/(2*b*c
os(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2
*c) - b*cos(-2*a + 2*c) - b)
```

3.252.6 Sympy [F]

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \int \cos(a+bx)\cot^3(bx+c)\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)**3,x)
```

output
```
Integral(cos(a + b*x)*cot(b*x + c)**3, x)
```

3.252.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(69) = 138$.

Time = 0.25 (sec) , antiderivative size = 1254, normalized size of antiderivative = 17.18

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*(cos(5*b*x + a + 4*c) - 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos
(6*b*x + 2*a + 4*c) - 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*x + 4*c) - 2
*cos(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) - 1)*cos(5*b*x + a + 4*c) + 10*(2*c
os(3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*cos(3*b*
x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 4*c) - 4*(2*cos(2*b*x + 2*a) - 5*
cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x +
a) - 10*cos(2*b*x + 2*c)*cos(b*x + a) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a +
 c) + 4*cos(3*b*x + a + 2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*
x + a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x +
a + 4*c)^2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*
x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x +
a + 2*c)*cos(-a + c) - cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*
(2*cos(-a + c)*sin(3*b*x + a + 2*c) - cos(-a + c)*sin(b*x + a))*sin(5*b*x
+ a + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2
*sin(b*x)*sin(c) + sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4*c
os(3*b*x + a + 2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*co
s(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^
2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*x + a + 2
*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x + a + 2*c)*
cos(-a + c) - cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*cos...
```

3.252.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(69) = 138$.

Time = 0.38 (sec) , antiderivative size = 963, normalized size of antiderivative = 13.19

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(12*(tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a
)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c
) - 1))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^
3 + tan(1/2*c)) - 12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2
*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(ta
n(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 16*(2*tan(1/2
*b*x)*tan(1/2*a) + tan(1/2*a)^2 - 1)/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 +
 1)) + (2*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/
2*a)^2*tan(1/2*c)^8 + 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^5 + 2*tan(1
/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^6 - 2*tan(1/2*b*x)^3*tan(1/2*c)^7 - 4*ta
n(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^7 - 2*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2
*c)^7 - tan(1/2*b*x)^2*tan(1/2*c)^8 - 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/
2*c)^3 + 16*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^4 - 22*tan(1/2*b*x)^2*tan
(1/2*a)^2*tan(1/2*c)^4 - 6*tan(1/2*b*x)^3*tan(1/2*c)^5 + 20*tan(1/2*b*x)^2
*tan(1/2*a)*tan(1/2*c)^5 - 14*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^5 - 2*t
an(1/2*b*x)^2*tan(1/2*c)^6 + 16*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^6 + 2*t
an(1/2*a)^2*tan(1/2*c)^6 + 2*tan(1/2*b*x)*tan(1/2*c)^7 - 2*tan(1/2*b*x)^3*
tan(1/2*a)^2*tan(1/2*c) + 2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 6*t
an(1/2*b*x)^3*tan(1/2*c)^3 - 20*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^3 + 1
4*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^3 + 22*tan(1/2*b*x)^2*tan(1/2*c)...
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)*cot(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.253 $\int \cos(a+bx)\tan(c+dx)\,dx$

3.253.1 Optimal result

Integrand size = 13, antiderivative size = 134

$$\int \cos(a+bx)\tan(c+dx)\,dx = \frac{e^{-i(a+bx)}}{2b} - \frac{e^{i(a+bx)}}{2b} - \frac{e^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b} + \frac{e^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}$$

output
```
1/2/b/exp(I*(b*x+a))-1/2*exp(I*(b*x+a))/b-hypergeom([1, -1/2*b/d],[1-1/2*b
/d],-exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))+exp(I*(b*x+a))*hypergeom([1, 1/2*b
/d],[1+1/2*b/d],-exp(2*I*(d*x+c)))/b
```

3.253.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int \cos(a+bx)\tan(c+dx)\,dx = \frac{e^{-i(a+bx)}\left(1-e^{2i(a+bx)}-2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)+2e^{2i(a+bx)}\,\text{Hypergeometric2F}\right.}{2b}$$

input
```
Integrate[Cos[a + b*x]*Tan[c + d*x],x]
```

output
```
(1 - E^((2*I)*(a + b*x)) - 2*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), -
E^((2*I)*(c + d*x))] + 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), -E^((2*I)*(c + d*x))])/(2*b*E^(I*(a + b*x)))
```

3.253.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5071, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\tan(c+dx)\,dx$$

$$\Big\downarrow \text{5071}$$

$$\int \left(\frac{ie^{-i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{ie^{i(a+bx)}}{1+e^{2i(c+dx)}}-\frac{1}{2}ie^{-i(a+bx)}-\frac{1}{2}ie^{i(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2i(c+dx)}\right)}{b}+\frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}$$

input
```
Int[Cos[a + b*x]*Tan[c + d*x],x]
```

output
```
1/(2*b*E^(I*(a + b*x))) - E^(I*(a + b*x))/(2*b) - Hypergeometric2F1[1, -1/
2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) + (E^(I*(a +
 b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))])/b
```

3.253.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5071
```
Int[Cos[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[(-I)*
(1/(E^(I*(a + b*x))*2)) - I*(E^(I*(a + b*x))/2) + I*(1/(E^(I*(a + b*x))*(1
+ E^(2*I*(c + d*x))))) + I*(E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x)))), x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.253.4 Maple [F]

$$\int \cos(xb+a)\tan(dx+c)\,dx$$

input
```
int(cos(b*x+a)*tan(d*x+c),x)
```

output
```
int(cos(b*x+a)*tan(d*x+c),x)
```

3.253.5 Fricas [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*tan(d*x + c), x)
```

3.253.6 Sympy [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(a+bx)\tan(c+dx)\,dx$$

input
```
integrate(cos(b*x+a)*tan(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*tan(c + d*x), x)
```

3.253.7 Maxima [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*tan(d*x + c), x)
```

3.253.8 Giac [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*tan(d*x + c), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(a+b\,x)\,\tan(c+d\,x)\,dx$$

input
```
int(cos(a + b*x)*tan(c + d*x),x)
```

output
```
int(cos(a + b*x)*tan(c + d*x), x)
```

3.254 $\int \cos(a+bx)\cot(c+dx)\,dx$

3.254.1 Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \cos(a+bx)\cot(c+dx)\,dx = -\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b} + \frac{e^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b} - \frac{e^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2i(c+dx)}\right)}{b}$$

output
```
-1/2/b/exp(I*(b*x+a))+1/2*exp(I*(b*x+a))/b+hypergeom([1, -1/2*b/d],[1-1/2*
b/d],exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))-exp(I*(b*x+a))*hypergeom([1, 1/2*b
/d],[1+1/2*b/d],exp(2*I*(d*x+c)))/b
```

3.254.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

$$\int \cos(a+bx)\cot(c+dx)\,dx = \frac{e^{-i(a+bx)}\left(-1+e^{2i(a+bx)}+2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)-2e^{2i(a+bx)}\,\text{Hypergeometric2F}\right.}{2b}$$

input
```
Integrate[Cos[a + b*x]*Cot[c + d*x],x]
```

output
```
(-1 + E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d),
E^((2*I)*(c + d*x))] - 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), E^((2*I)*(c + d*x))])/(2*b*E^(I*(a + b*x)))
```

3.254.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5069, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot(c+dx)\,dx$$

$$\Big\downarrow \text{5069}$$

$$\int \left(-\frac{ie^{-i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{ie^{i(a+bx)}}{1-e^{2i(c+dx)}}+\frac{1}{2}ie^{-i(a+bx)}+\frac{1}{2}ie^{i(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2i(c+dx)}\right)}{b}-\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}$$

input
```
Int[Cos[a + b*x]*Cot[c + d*x],x]
```

output
```
-1/2*1/(b*E^(I*(a + b*x))) + E^(I*(a + b*x))/(2*b) + Hypergeometric2F1[1,
-1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) - (E^(I*(a
 + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))])/
b
```

3.254.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5069
```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I*(1/
(E^(I*(a + b*x))*2)) + I*(E^(I*(a + b*x))/2) - I*(1/(E^(I*(a + b*x))*(1 - E
^(2*I*(c + d*x))))) - I*(E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x)))), x] /; Fr
eeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.254.4 Maple [F]

$$\int \cos(xb + a)\cot(dx + c)\,dx$$

input
```
int(cos(b*x+a)*cot(d*x+c),x)
```

output
```
int(cos(b*x+a)*cot(d*x+c),x)
```

3.254.5 Fricas [F]

$$\int \cos(a + bx)\cot(c + dx)\,dx = \int \cos(bx + a)\cot(dx + c)\,dx$$

input
```
integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(d*x + c), x)
```

3.254.6 Sympy [F]

$$\int \cos(a + bx)\cot(c + dx)\,dx = \int \cos(a + bx)\cot(c + dx)\,dx$$

input
```
integrate(cos(b*x+a)*cot(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*cot(c + d*x), x)
```

3.254.7 Maxima [F]

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos(bx+a)\cot(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*cot(d*x + c), x)
```

3.254.8 Giac [F]

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos(bx+a)\cot(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(d*x + c), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos(a+b\,x)\,\cot(c+d\,x)\,dx$$

input
```
int(cos(a + b*x)*cot(c + d*x),x)
```

output
```
int(cos(a + b*x)*cot(c + d*x), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```